Bulletin of Mathematical Analysis and Applications
ISSN: 1821-1291, URL: http://www.bmathaa.org
Volume 3 Issue 3(2011), Pages 273-281.

INTUITIONISTIC FUZZY BANACH ALGEBRA

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Abstract. In this paper, intuitionistic fuzzy Banach algebra is introduced and a few of its properties are studied. The properties of invertible elements and relation among invertible elements, open set, closed set are emphasized. Topological divisors of zero is defined and its relation with closed set are studied.

1. Introduction

Fuzzy set theory is a useful tool to describe the situation in which data are imprecise or vague or uncertain. Intuitionistic fuzzy set theory handle the situation by attributing a degree of membership and a degree of non-membership to which a certain object belongs to a set. The concept of intuitionistic fuzzy set, as a generalisation of fuzzy sets [10] was introduced by Atanassov in [1]. The concept of fuzzy norm was introduced by Katsaras [6] in 1984. In 1992, Felbin[5] introduced the idea of fuzzy norm on a linear space. Cheng-Moderson [3] introduced another idea of fuzzy norm on a linear space whose associated metric is same as the associated metric of Kramosil-Michalek [7]. Latter on Bag and Samanta [2] modified the definition of fuzzy norm on a linear space of Cheng-Moderson [3].
In this paper, we introduce intuitionistic fuzzy Banach algebra. In section 3 we define intuitionistic fuzzy normed algebra, intuitionistic fuzzy Banach algebra. In section 4 we study the properties of invertible elements and its relation with open sets and close sets. Lastly we define topological divisor of zero and study its relation with close sets.

2. Preliminaries

We quote some definitions and statements of a few theorems which will be needed in the sequel:

Definition 2.1([9]). A binary operation $* : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t - norm if $*$ satisfies the following conditions :

2000 *Mathematics Subject Classification.* 46G05, 03F55.
Key words and phrases. Intuitionistic fuzzy Banach algebra, invertible elements, open set, close set, topological divisors of zero.

Submitted December 22, 2010. Published August 8, 2011.

(i) $*$ is commutative and associative ,
(ii) $*$ is continuous ,
(iii) $a * 1 = a \quad \forall\, a \,\varepsilon\, [0, 1]$,
(iv) $a * b \le c * d$ whenever $a \le c$, $b \le d$ and $a, b, c, d \,\varepsilon\, [0, 1]$.

Example: $a \diamond b = a + b - ab$, $a \diamond b = \max\{a, b\}$, $a \diamond b = \min\{a + b, 1\}$.

Definition 2.2([9]). A binary operation $\diamond : [0, 1] \times [0, 1] \to [0, 1]$ is continuous t-conorm if $\diamond$ satisfies the following conditions :
(i) $\diamond$ is commutative and associative ,
(ii) $\diamond$ is continuous ,
(iii) $a \diamond 0 = a \quad \forall\, a \in [0, 1]$,
(iv) $a \diamond b \le c \diamond d$ whenever $a \le c$, $b \le d$ and $a, b, c, d \in [0, 1]$.

Example: $a * b = ab$, $a * b = \min\{a, b\}$.

Definition 2.3([8]). Let $*$ be a continuous t-norm , $\diamond$ be a continuous t-conorm and V be a linear space over the field $F\ (= \mathbb{R}\ or\ \mathbb{C}\ )$. An **intuitionistic fuzzy norm** on V is an object of the form
$A = \{ ((x, t), \mu(x, t), \nu(x, t)) : (x, t) \in V \times \mathbb{R}^+ \}$, where μ, ν *are fuzzy sets on* $V \times \mathbb{R}^+$, μ denotes the degree of membership and ν denotes the degree of non-membership. A is called an intuitionistic fuzzy norm if and only if $(x, t) \in V \times \mathbb{R}^+$ satisfying the following conditions :

(i) $\mu(x, t) + \nu(x, t) \le 1 \quad \forall\, (x, t) \in V \times \mathbb{R}^+$;
(ii) $\mu(x, t) > 0$;
(iii) $\mu(x, t) = 1$ if and only if $x = \theta$, θ is null vector ;
(iv) $\mu(cx, t) = \mu(x, \frac{t}{|c|}) \quad \forall\, c \in F$ and $c \neq 0$;
(v) $\mu(x, s) * \mu(y, t) \le \mu(x + y, s + t)$;
(vi) $\mu(x, \cdot)$ is non-decreasing function of $\mathbb{R}^+$ and $\lim_{t \to \infty} \mu(x, t) = 1$;
(vii) $\nu(x, t) < 1$;
$(viii)$ $\nu(x, t) = 0$ if and only if $x = \theta$;
(ix) $\nu(cx, t) = \nu(x, \frac{t}{|c|}) \quad \forall\, c \in F$ and $c \neq 0$;
(x) $\nu(x, s) \diamond \nu(y, t) \ge \nu(x + y, s + t)$;
(xi) $\nu(x, \cdot)$ is non-increasing function of $\mathbb{R}^+$ and $\lim_{t \to \infty} \nu(x, t) = 0$.

Definition 2.4([8]). If A is an intuitionistic fuzzy norm on a linear space V then (V, A) is called an intuitionistic fuzzy normed linear space.

Definition 2.5([8]). A sequence $\{x_n\}_n$ in an intuitionistic fuzzy normed linear space (V, A) is said to **converge** to $x \in V$ if for given $r > 0$, $t > 0$, $0 < r < 1$, there exists a positive integer n_0 such that
$\mu(x_n - x, t) > 1 - r$ and $\nu(x_n - x, t) < r$ for all $n \ge n_0$.

Definition 2.6([8]). Let (U, A) and (V, B) be two intuitionistic fuzzy normed linear space over the same field F. A mapping f from (U, A) to (V, B) is said to be **intuitionistic fuzzy continuous** at $x_0 \in U$, if for any given $\epsilon > 0$, $\alpha \in (0,1)$, $\exists\, \delta = \delta(\alpha, \epsilon) > 0$, $\beta = \beta(\alpha, \epsilon) \in (0,1)$ such that for all $x \in U$,

$$\mu_U(x - x_0 \, , \, \delta) \; > \; 1 - \beta \; \Rightarrow \; \mu_V(f(x) - f(x_0) \, , \, \epsilon) \; > \; 1 - \alpha$$

$$\nu_U(x - x_0 \, , \, \delta) \; < \; \beta \; \Rightarrow \; \nu_V(f(x) - f(x_0) \, , \, \epsilon) \; < \; \alpha \, .$$

Definition 2.7([4]). Let $0 < r < 1$, $t \in \mathbb{R}^+$ and $x \in V$. Then the set

$$B(x\,, r\,, t) \; = \; \{ \, y \, \in \, V \; : \; \mu(\, x \, - \, y \, , \, t \,) \; > \; 1 - r \; , \; \; \nu(\, x - y \, , \, t \,) \; < \; r \, \}$$

is called an **open ball** in $(V \, , \, A)$ with x as its center and r as its radious with respect to t.

3. Intuitionistic Fuzzy Banach Algebra.

Definition 3.1. Let $*$ be a continuous t-norm , $\diamond$ be a continuous t-conorm and $\mathcal{A}$ be an algebra over the field K $(= \; \mathbb{R} \; or \; \mathbb{C}\;)$. An **intuitionistic fuzzy normed algebra** is an object of the form $(\mathcal{A}, \mu, \nu, *, \diamond)$, where μ , ν are fuzzy sets on $V \times \mathbb{R}^+$, μ denotes the degree of membership and ν denotes the degree of non-membership satisfying the following conditions for every $x, y \in \mathcal{A}$ and $s, t \in \mathbb{R}^+$;

(*i*) $\mu\,(\,x\,,\,t\,) \, + \, \nu\,(\,x\,,\,t\,) \; \leq \; 1$
(*ii*) $\mu\,(\,x\,,\,t\,) \; > \; 0$;
(*iii*) $\mu\,(\,x\,,\,t\,) \; = \; 1$ if and only if $x \, = \, \theta$, θ is null vector;
(*iv*) $\mu\,(\,cx\,,\,t\,) \; = \; \mu\,(x, \frac{t}{|\,c\,|}) \quad \forall \, c \, \in \, F$ and $c \neq 0$;
(*v*) $\mu\,(\,x\,,\,s\,) * \mu\,(\,y\,,\,t\,) \; \leq \; \mu\,(\,x \, + \, y\,,\,s \, + \, t\,)$;
(*vi*) $\max\{\mu(x,s), \mu(y,t)\} \leq \mu(xy, s+t)$;
(*vii*) $\lim\limits_{t \, \to \, \infty} \; \mu\,(\,x\,,\,t\,) = 1$ and $\lim\limits_{t \, \to \, 0} \; \mu\,(\,x\,,\,t\,) = 0$
(*viii*) $\nu\,(\,x\,,\,t\,) \; < \; 1$;
(*ix*) $\nu\,(\,x\,,\,t\,) \; = \; 0$ if and only if $x \, = \, \theta$, θ is null vector;
(*x*) $\nu\,(\,cx\,,\,t\,) \; = \; \nu\,(\,x\,, \frac{t}{|\,c\,|}) \quad \forall \, c \, \in \, F$ and $c \neq 0$;
(*xi*) $\nu\,(\,x\,,\,s\,) \diamond \nu\,(\,y\,,\,t\,) \; \geq \; \nu\,(\,x \, + \, y\,,\,s \, + \, t\,)$;
(*xii*) $\min\{\nu(x,s), \nu(y,t)\} \geq \nu(xy, s+t)$;
(*xiii*) $\lim\limits_{t \, \to \, \infty} \; \nu\,(\,x\,,\,t\,) = 0$ and $\lim\limits_{t \, \to \, 0} \; \nu\,(\,x\,,\,t\,) = 1$.

For an intuitionistic fuzzy normed algebra $(\mathcal{A}, \mu, \nu, *, \diamond)$ we further assume that: (*xiv*) $\begin{smallmatrix} a \, \diamond \, a \, = \, a \\ a \, * \, a \, = \, a \end{smallmatrix}$ }, for all $a \in [0\,,1]$.

Definition 3.2. A sequence $\{x_n\}_n$ in an intuitionistic fuzzy normed algebra $(\mathcal{A}, \mu, \nu, *, \diamond)$ is said to **converge** to $x \in \mathcal{A}$ if for given $r > 0$, $t > 0$, $0 < r < 1$, there exists a positive integer n_0 such that
$\mu\,(\,x_n \, - \, x\,,\,t\,) \; > \; 1 \, - \, r$ and $\nu\,(\,x_n \, - \, x\,,\,t\,) \; < \; r$ for all $n \geq n_0$.

Theorem 3.3. In an intuitionistic fuzzy normed algebra $(\mathcal{A}, \mu, \nu, *, \diamond)$, a sequence $\{\,x_n\,\}_n$ converges to $x \in \mathcal{A}$ if and only if $\lim\limits_{n \, \to \, \infty} \mu\,(\,x_n \, - \, x\,,\,t\,) \; = \; 1$ and $\lim\limits_{n \, \to \, \infty} \nu\,(\,x_n \, - \, x\,,\,t\,) \; = \; 0$.

Definition 3.4. A sequence $\{x_n\}_n$ in an intuitionistic fuzzy normed algebra $(\mathcal{A}, \mu, \nu, *, \diamond)$ is said to be **cauchy sequence** if $\lim\limits_{n \, \to \, \infty} \mu(x_{n+p} - x_n, t) \; = \; 1$

and $\lim_{n \to \infty} \nu(x_{n+p} - x_n, t) = 0$, $t \in \mathbb{R}^+$, $p = 1, 2, 3, ...$

Definition 3.5. An intuitionistic fuzzy normed algebra $(\mathcal{A}, \mu, \nu, *, \diamond)$ is said to be **complete** if every cauchy sequence in $\mathcal{A}$ converges to an element of $\mathcal{A}$.

Definition 3.6. A complete intuitionistic fuzzy normed algebra is called intuitionistic fuzzy Banach algebra.

Theorem 3.7. In an intuitionostic fuzzy Banach algebra $(\mathcal{A}, \mu, \nu, *, \diamond)$ two sequences $\{x_n\}_n$ and $\{y_n\}_n$ be such that $x_n \to x$ and $y_n \to y$ then $x_n y_n \to xy$.

Proof. For any $t > 0$, $x_n \to x$ implies

$$\lim_{n \to \infty} \mu(x_n - x, t) = 1, \quad \lim_{n \to \infty} \nu(x_n - x, t) = 0$$

and $y_n \to y$ implies

$$\lim_{n \to \infty} \mu(y_n - y, t) = 1, \quad \lim_{n \to \infty} \nu(y_n - y, t) = 0$$

Now

$$\lim_{n \to \infty} \mu(x_n y_n - xy, t) \geq \lim_{n \to \infty} \mu\left((x_n - x)y_n, \frac{t}{2}\right) * \lim_{n \to \infty} \mu\left((y_n - y)x, \frac{t}{2}\right)$$

$$\geq \max\left\{\lim_{n \to \infty} \mu\left(x_n - x, \frac{t}{4}\right), \lim_{n \to \infty} \mu\left(y_n, \frac{t}{4}\right)\right\}$$

$$* \max\left\{\lim_{n \to \infty} \mu\left(y_n - y, \frac{t}{4}\right), \lim_{n \to \infty} \mu\left(x, \frac{t}{4}\right)\right\}$$

$$= \max\left\{1, \lim_{n \to \infty} \mu\left(y_n, \frac{t}{4}\right)\right\} * \max\left\{1, \lim_{n \to \infty} \mu\left(x, \frac{t}{4}\right)\right\}$$

$= 1 * 1 = 1$.

and

$$\lim_{n \to \infty} \nu(x_n y_n - xy, t) \leq \lim_{n \to \infty} \nu\left((x_n - x)y_n, \frac{t}{2}\right) \diamond \lim_{n \to \infty} \nu\left((y_n - y)x, \frac{t}{2}\right)$$

$$\leq \min\left\{\lim_{n \to \infty} \nu\left(x_n - x, \frac{t}{4}\right), \lim_{n \to \infty} \nu\left(y_n, \frac{t}{4}\right)\right\}$$

$$\diamond \min\left\{\lim_{n \to \infty} \nu\left(y_n - y, \frac{t}{4}\right), \lim_{n \to \infty} \nu\left(x, \frac{t}{4}\right)\right\}$$

$$= \min\left\{0, \lim_{n \to \infty} \nu\left(y_n, \frac{t}{4}\right)\right\} \diamond \min\left\{0, \lim_{n \to \infty} \nu\left(x, \frac{t}{4}\right)\right\}$$

$= 0 \diamond 0 = 0$.

Hence the proof.

4. Invertible elements and Topological divisors of Zero.

Theorem 4.1. Let $(\mathcal{A}, \mu, \nu, *, \diamond)$ be an intuitionistic fuzzy Banach algebra. If $x \in \mathcal{A}$ be such that $x \in B(\theta, r, t)$, $0 < r < 1$ then $(e - x)$ is invertible and $(e-x)^{-1} = e + \sum_{(n=1)}^{\infty} x^n$.

Proof. Since $x \in B(\theta, r, t)$ we have for any $t \in \mathbb{R}^+$,
$\mu(x,t) > 1-r,\ \nu(x,t) < r$
To prove the theorem we first show that the series $\sum_{(n=1)}^{\infty} x^n$ is convergent to some element of $\mathcal{A}$. Let $s_n = \sum_{(k=1)}^{n} x^k$, then it is sufficient to prove that

$$\mu\left(s_{n+p} - s_n, t\right) > 1 - r \quad and \quad \nu\left(s_{n+p} - s_n, t\right) < r$$

Now

$$\begin{aligned}
\mu\left(s_{n+p} - s_n, t\right) &= \mu\left(x^{n+1} + x^{n+2} + \cdots + x^{n+p}, t\right) \\
&\geq \mu(x^{n+1}, t_1) * \mu(x^{n+2}, t_2) * \cdots \mu(x^{n+p}, t_p), \quad where \;\; t_1 + t_2 + \cdots + t_p = t \\
&\geq \max\{\mu(x,t_{11}), \mu(x,t_{12}), \cdots \mu(x,t_{1n+1})\} * \max\{\mu(x,t_{21}), \mu(x,t_{22}), \cdots \mu(x,t_{2n+2})\} \\
&\quad * \cdots\cdots * \max\{\mu(x,t_{p1}), \mu(x,t_{p2}), \cdots \mu(x,t_{pn+p})\}, \\
&\qquad where \sum_{(j=1)}^{n+i} t_{ij} = t_i,\ i = 1, 2, \cdots, p \\
&> (1-r) * (1-r) * \cdots * (1-r) = (1-r)
\end{aligned}$$

and

$$\begin{aligned}
\nu\left(s_{n+p} - s_n, t\right) &= \nu\left(x^{n+1} + x^{n+2} + \cdots + x^{n+p}, t\right) \\
&\leq \nu(x^{n+1}, t_1) \diamond \nu(x^{n+2}, t_2) \diamond \cdots \nu(x^{n+p}, t_p), \quad where \;\; t_1 + t_2 + \cdots + t_p = t \\
&\leq \min\{\nu(x,t_{11}), \nu(x,t_{12}), \cdots \nu(x,t_{1n+1})\} \diamond \min\{\nu(x,t_{21}), \nu(x,t_{22}), \cdots \nu(x,t_{2n+2})\} \\
&\quad \diamond \cdots\cdots \diamond \min\{\nu(x,t_{p1}), \nu(x,t_{p2}), \cdots \nu(x,t_{pn+p})\}, \\
&\qquad where \sum_{(j=1)}^{n+i} t_{ij} = t_i,\ i = 1, 2, \cdots, p \\
&< r \diamond r \diamond \cdots \diamond r = r
\end{aligned}$$

Thus the series $\sum_{(n=1)}^{\infty} x^n$ is convergent. Since $\mathcal{A}$ is complete, $\sum_{(n=1)}^{\infty} x^n$ converges to some element of $\mathcal{A}$.
Let $s = e + \sum_{(n=1)}^{\infty} x^n$.
Now $(e-x)(e + x + \cdots + x^n) = (e + x + \cdots + x^n)(e - x) = e - x^{n+1}$.
Also, $\mu\left(x^{n+1}, t\right) \geq \max\{\mu(x,t_1), \mu(x,t_2), \cdots, \mu(x,t_{n+1})\} > 1-r$, and $\nu\left(x^{n+1}, t\right) \leq \min\{\nu(x,t_1), \nu(x,t_2), \cdots, \nu(x,t_{n+1})\} < r$, $where\ t_1 + t_2 + \cdots + t_{n+1} = t$.
So, $x^{n+1} \to \theta \ \ as\ n \to \infty$. Therefore letting $n \to \infty$ and remembering that multiplication on $\mathcal{A}$ is continuous, we obtain $(e-x)s = s(e-x) = e$.
Hence $s = (e-x)^{-1}$. Hence the proof.

Corollary 4.2. Let $x \in \mathcal{A}$ be such that $e - x \in B(\theta, r, t)$, $0 < r < 1$ then x^{-1} exist and $x^{-1} = e + \sum_{(n=1)}^{\infty} (e-x)^n$.

Corollary 4.3. Let $x \in \mathcal{A}$ and $\lambda(\neq 0)$ be a scaler such that $\frac{x}{\lambda} \in B(\theta, r, t)$, $0 < r < 1$. Then $(\lambda\, e - x)^{-1}$ exists and $(\lambda\, e - x)^{-1} = \sum_{(n=1)}^{\infty} \lambda^{-n} x^{n-1}$.

Proof. Since $\frac{x}{\lambda} \in B(\theta, r, t)$, for any $t \in \mathbb{R}^{+}$ we have $\mu(\frac{x}{\lambda}, t) > 1 - r$ *and* $\nu(\frac{x}{\lambda}, t) < r$.
If $y \in \mathcal{A}$ be such that $y^{-1} \in \mathcal{A}$ and $c(\neq 0)$ be a scaler then it is clear that $(cy)^{-1}$ exist and $(cy)^{-1} = c^{-1} y^{-1}$.
Now $(\lambda\, e - x) = \lambda(e - \frac{x}{\lambda})$ and we show that $(e - \frac{x}{\lambda})^{-1}$ exists. $\mu\left(e - (e - \frac{x}{\lambda}), t\right) = \mu(\frac{x}{\lambda}, t) > 1 - r$ *and* $\nu\left(e - (e - \frac{x}{\lambda}), t\right) = \nu(\frac{x}{\lambda}, t) < r$, by hypothesis.
So, by corollary **??** $(e - \frac{x}{\lambda})^{-1}$ exists and so $(\lambda\, e - x)^{-1}$ exists.

$$(\lambda\, e - x)^{-1} = \left\{ \lambda(e - \frac{x}{\lambda}) \right\} = \lambda^{-1} \left(e - \lambda^{-1} x\right)^{-1}$$

$$= \lambda^{-1} \left(e + \sum_{(n=1)}^{\infty} [e - (e - \lambda^{-1} x)]^{n} \right)$$

$$= \lambda^{-1} \left(e + \sum_{(n=1)}^{\infty} \lambda^{-n} x^{n} \right) = \sum_{(n=1)}^{\infty} \lambda^{-n} x^{n-1}$$

Hence the proof.

Theorem 4.4. The set of all invertible elements of an intuitionistic fuzzy Banach algebra $(\mathcal{A}, \mu, \nu, *, \diamond)$ is an open subset of $\mathcal{A}$.

Proof. Let G be the set of all invertible elements of $\mathcal{A}$. Let $x_0 \in G$, we have to show that the open ball with centre at x_0 and radious $r\,(0 < r < 1)$ contain in G i.e., every point x of the open ball $B(x_0, r, t)$ satisfies the inequality

$$\mu(x - x_0, t) > 1 - r \ \ , \ \ \nu(x - x_0, t) < r. \tag{4.1}$$

Let us choose r such that

$$r < \min\{\mu(x_0^{-1}, t),\, 1 - \nu(x_0^{-1}, t)\}, \quad 0 < \mu(x_0^{-1}, t),\, \nu(x_0^{-1}, t) < 1$$

Then $\mu(x - x_0, t) > 1 - \left(1 - \mu(x_0^{-1}, t)\right) = \mu(x_0^{-1}, t)$, $\nu(x - x_0, t) < \nu(x_0^{-1}, t)$.
Let $y = x_0^{-1} x$ *and* $z = e - y$. Then

$$\mu(z, t) = \mu(e - y, t) = \mu(y - e, t) = \mu(x_0^{-1} x - x_0^{-1} x_0, t) = \mu\left(x_0^{-1}(x - x_0), t\right)$$

$$\geq \max\{\mu(x_0^{-1}, \frac{t}{2}),\, \mu(x - x_0, \frac{t}{2})\} = \mu(x - x_0, \frac{t}{2}) > 1 - r.$$

$$\nu(z, t) = \nu(e - y, t) = \nu(y - e, t) = \nu(x_0^{-1} x - x_0^{-1} x_0, t) = \nu\left(x_0^{-1}(x - x_0), t\right)$$

$$\leq \min\{\nu(x_0^{-1}, \frac{t}{2}),\, \nu(x - x_0, \frac{t}{2})\} = \nu(x - x_0, \frac{t}{2}) < r.$$

Thus $z \in B(\theta, r, t)$ and hence $e - z$ is invertible. That is, y is invertible. So, $y \in G$.
Now $x - \theta \in G$ and $y \in G$ and by our earlier verification G is a group. So, $x_0 y = x_0 x_0^{-1} x = x$. So, any point x satisfies (4.1) belongs to G; that is, $B(x_0, r, t) \subseteq G$. Hence, G is an open subset of $\mathcal{A}$.

Corollary 4.5. The set of all non-invertible elements of an intuitionistic fuzzy Banach algebra $(\mathcal{A}, \mu, \nu, *, \diamond)$ is a closed subset of $\mathcal{A}$.

Theorem 4.6. Let G be the set of all invertible elements of $(\mathcal{A}, \mu, \nu, *, \diamond)$. The mapping $x \to x^{-1}$ of G into G is strongly intuitionistic fuzzy continuous.

Proof. Let x_0 be an element of G. Let $x \in G$ be such that $x \in B(x_0, r, t)$, where $r < \min\{1 - \mu(x_0^{-1}, t),\ \nu(x_0^{-1}, t)\}$. Therefore,
$\mu(x - x_0, t) > 1 - r > \mu(x_0^{-1}, t)$ *and* $\nu(x - x_0, t) < r < \nu(x_0^{-1}, t)$
Since $x_0 \in G$ is arbitrary $\forall x \in G$, we have

$$\mu(x - x_0, t) > \mu(x^{-1}, t) \ and \ \nu(x - x_0, t) > \nu(x^{-1}, t)$$

Let $x \in G$ then $T(x) = x^{-1}$. Now

$$\begin{aligned}
\mu(T(x) - T(x_0), \epsilon) &= \mu(x^{-1} - {x_0}^{-1}, \epsilon) = \mu\left((x^{-1}x_0 - e){x_0}^{-1}, \epsilon\right) \\
&\geq \max\{\mu(x^{-1}x_0 - e, \frac{\epsilon}{2}), \mu({x_0}^{-1}, \frac{\epsilon}{2})\} \\
&\geq \max\{\mu\left(x^{-1}(x_0 - x), \frac{\epsilon}{2}\right), \mu({x_0}^{-1}, \frac{\epsilon}{2})\} \\
&\geq \max\left\{\max\{\mu(x^{-1}, \frac{\epsilon}{4}),\ \mu(x_0 - x, \frac{\epsilon}{4})\}, \mu({x_0}^{-1}, \frac{\epsilon}{2})\right\} \\
&= \max\{\mu(x - x_0, \frac{\epsilon}{4}), \mu({x_0}^{-1}, \frac{\epsilon}{2})\} \\
&= \mu(x - x_0, \frac{\epsilon}{4})
\end{aligned}$$

$$\begin{aligned}
\nu(T(x) - T(x_0), \epsilon) &= \nu(x^{-1} - {x_0}^{-1}, \epsilon) = \nu\left((x^{-1}x_0 - e){x_0}^{-1}, \epsilon\right) \\
&\leq \min\{\nu(x^{-1}x_0 - e, \frac{\epsilon}{2}), \nu({x_0}^{-1}, \frac{\epsilon}{2})\} \\
&\leq \min\{\nu\left(x^{-1}(x_0 - x), \frac{\epsilon}{2}\right), \nu({x_0}^{-1}, \frac{\epsilon}{2})\} \\
&\leq \min\left\{\min\{\nu(x^{-1}, \frac{\epsilon}{4}),\ \nu(x_0 - x, \frac{\epsilon}{4})\}, \nu({x_0}^{-1}, \frac{\epsilon}{2})\right\} \\
&= \min\{\nu(x - x_0, \frac{\epsilon}{4}), \nu({x_0}^{-1}, \frac{\epsilon}{2})\} \\
&= \nu(x - x_0, \frac{\epsilon}{4})
\end{aligned}$$

Hence T is strongly intuitionistic fuzzy continuous at $x_0 \in G$. Since x_0 is arbitrary, T is strongly intuitionistic fuzzy continuous on G.

Definition 4.7. Let $(\mathcal{A}, \mu, \nu, *, \diamond)$ be an intuitionistic fuzzy Banach algebra. An element $z \in \mathcal{A}$ is called a **topological divisor of zero** if there exist a sequence $\{z_n\}_n$, $z_n \in \mathcal{A}$ satisfies $z_n \notin B(\theta, r, t)$, $0 < r < 1$ (*i.e.*, $\mu(z_n, t) < 1 - r$ *or* $\nu(z_n, t) > r$ *or both*) be such that, **either**

$$\lim_{n \to \infty} \mu(z_n z, t) = 1 \ and \ \lim_{n \to \infty} \nu(z_n z, t) = 0$$

or

$$\lim_{n \to \infty} \mu(z z_n, t) = 1 \ and \ \lim_{n \to \infty} \nu(z z_n, t) = 0.$$

Theorem 4.8. Let Z be the set of all topological divisors of zero in $\mathcal{A}$. Then $Z \subseteq S$, where S the set of non-invertible elements of $\mathcal{A}$.

Proof. Let $z \in Z$ then there exist a sequence $\{z_n\}_n$, $z_n \in \mathcal{A}$ satisfies $z_n \notin B(\theta, r, t)$, $0 < r < 1$ (*i.e.*, $\mu(z_n, t) < 1 - r$ *or* $\nu(z_n, t) > r$ *or both*) be such that, **either**

$$\lim_{n \to \infty} \mu\,(\,z_n\, z\,,\,t\,) \,=\, 1 \;\; and \;\; \lim_{n \to \infty} \nu\,(\,z_n\, z\,,\,t\,) \,=\, 0$$

or

$$\lim_{n \to \infty} \mu\,(\,z\, z_n\,,\,t\,) \,=\, 1 \;\; and \;\; \lim_{n \to \infty} \nu\,(\,z\, z_n\,,\,t\,) \,=\, 0\,.$$

Suppose $\lim_{n \to \infty} \mu\,(\,z\, z_n\,,\,t\,) \,=\, 1$ *and* $\lim_{n \to \infty} \nu\,(\,z\, z_n\,,\,t\,) \,=\, 0$.
Let G be the set of non-invertible elements of $\mathcal{A}$. If possible let $z \in G$. Then z^{-1} exists. Now since multiplicatin is a continuous operation, we should have

$$z_n \,=\, (z^{-1} z) z_n \,=\, z^{-1}(z z_n) \to z^{-1}.\theta \,=\, 0 \quad as \;\; n \to \infty\,.$$

which contradicts the fact that $z_n \notin B(\theta, r, t)$, $0 < r < 1$ (*i.e.*, $\mu(z_n, t) < 1 - r$ *or* $\nu(z_n, t) > r$ *or both*).
So, $z \in S$ and hence the proof.

5. Conclusions

In this work, we dealt with invertible elements and and topological divisors of zero in an intuitionistic fuzzy Banach algebra. It will be natural to continue the study of the spectral theory.

Acknowledgements: The authors are grateful to the referees and the Editors for their fruitful comments, valuable suggestions.

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