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SOME PROPERTIES OF LP-SASAKIAN MANIFOLDS EQUIPPED WITH $m-$PROJECTIVE CURVATURE TENSOR

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Abstract. In the present paper we studied the properties of the $m-$projective curvature tensor in LP-Sasakian, Einstein LP-Sasakian and $\eta-$Einstein LP-Sasakian manifolds.

1. Introduction

The notion of Lorentzian para contact manifold was introduced by K. Matsumoto [3]. The properties of Lorentzian para contact manifolds and their different classes, viz LP-Sasakian and LSP-Sasakian manifolds, have been studied by several authors since then. In [13], M. Tarafdar and A. Bhattacharya proved that a LP-Sasakian manifold with conformally flat and quasi-conformally flat curvature tensor is locally isometric with a unit sphere $S^n(1)$. Further, they obtained that a LP-Sasakian manifold with $R(X,Y).C = 0$ is locally isometric with a unit sphere $S^n(1)$, where C is the conformal curvature tensor of type $(1,3)$ and $R(X,Y)$ denotes the derivation of the tensor algebra at each point of the tangent space. J. P. Singh [10] proved that an $m-$projectively flat para-Sasakian manifold is an Einstein manifold. He has also shown that, if in an Einstein P-Sasakian manifold $R(\xi,X).W^* = 0$ holds, then it is locally isometric with a unit sphere $H^n(1)$. Also, an n-dimensional $\eta-$Einstein P-Sasakian manifold satisfies $W^*(\xi,X).R = 0$ if and only if either the manifold is locally isometric to the hyperbolic space $H^n(-1)$ or the scalar curvature tensor r of the manifold is $-n(n-1)$. LP-Sasakian manifolds have also studied by Matsumoto and Mihai [4], Takahashi [11], De, Matsumoto and Shaikh [2], Prasad and Ojha [8], Shaikh and De [9], Venkatesha and Bagewadi [14].

In this paper, we studied the properties of LP-Sasakian manifolds equipped with $m-$projective curvature tensor. Section 2 deals with brief account of Lorentzian para-contact manifolds, LP-Sasakian manifolds and $m-$projective curvature tensor. It has also shown that $m-$projective curvature tensor and concircular curvature tensor coincide in an Einstein LP-Sasakian manifold. In section 3, we proved that an $m-$projectively flat LP-Sasakian manifold is locally isometric to a unit sphere $S^n(1)$. Also, a LP-Sasakian manifold M_n is $m-$projectively flat if and only if it has

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constant curvature 1. In section 4, we prove that an Einstein LP-Sasakian manifold satisfies $R(X,Y).W^{*}=0$ is $m-$projectively flat if and only if it is locally isometric with a unit sphere $S^{n}(1)$. In section 5, we have shown that an $n-$dimensional $\eta-$Einstein LP-Sasakian manifold satisfies $W^{*}(\xi,X).R=0$ if and only if either the manifold is locally isometric to a unit sphere $S^{n}(1)$ or it has constant scalar curvature $n(n-1)$. In the last, we proved that an $n-$dimensional LP-Sasakian manifold is $m-$projectively semi-symmetric if and only if it is concircularly semi-symmetric.

2. Preliminaries

If on an $n-$dimensional differentiable manifold M_n of differentiability class C^{r+1}, there exist a vector valued linear function ϕ, a $1-$form η, the associated vector field ξ and the Lorentzian metric g satisfying

$$\phi^{2}X=X+\eta(X)\xi, \tag{2.1}$$

$$\eta(\phi X)=0, \tag{2.2}$$

$$g(\phi X,\phi Y)=g(X,Y)+\eta(X)\eta(Y) \tag{2.3}$$

for arbitrary vector fields X and Y, then (M_n,g) is said to be Lorentzian almost para contact manifold and the structure $\{\phi,\eta,\xi,g\}$ is called Lorentzian almost para contact structure on M_n [3].

In view of (2.1), (2.2) and (2.3), we find

$$\eta(\xi)=-1,\quad g(X,\xi)=\eta(X),\quad \phi(\xi)=0. \tag{2.4}$$

If moreover,

$$(D_X\phi)(Y)=\left[g(X,Y)+\eta(X)\eta(Y)\right]\xi+\left[X+\eta(X)\xi\right]\eta(Y), \tag{2.5}$$

$$D_X\xi=\phi X, \tag{2.6}$$

where D denotes the operator of covariant differentiation with respect to the Lorentzian metric g, then (M_n,ϕ,ξ,η,g) is called Lorentzian para Sasakian manifold [3], [4]. Also, the following relations hold in an LP-Sasakian manifold [2], [8], [9]

$$R(X,Y)\xi=\eta(Y)X-\eta(X)Y, \tag{2.7}$$

$$R(\xi,X)Y=g(X,Y)\xi-\eta(Y)X, \tag{2.8}$$

$$S(X,\xi)=(n-1)\eta(X), \tag{2.9}$$

$$\eta(R(X,Y)Z)=\eta(X)g(Y,Z)-\eta(Y)g(X,Z), \tag{2.10}$$

for arbitrary vector fields X, Y, Z.

A LP-Sasakian manifold M_n is said to be $\eta-$Einstein if its Ricci tensor S is of the form

$$S(X,Y)=ag(X,Y)+b\eta(X)\eta(Y), \tag{2.11}$$

for arbitrary vector fields X and Y, where a and b are smooth functions on (M_n,g) [1], [15]. If $b=0$, then $\eta-$Einstein manifold becomes Einstein manifold.

In view of (2.4) and (2.11), we have

$$QX=aX+b\eta(X)\xi, \tag{2.12}$$

where Q is the Ricci operator defined by

$$S(X,Y)\overset{\text{def}}{=}g(QX,Y).$$

Again, contracting (2.12) with respect to X and using (2.4), we have

$$r = na - b. \tag{2.13}$$

Now, substituting $X = \xi$ and $Y = \xi$ in (2.11) and then using (2.4) and (2.9), we obtain

$$a - b = (n-1). \tag{2.14}$$

Equations (2.13) and (2.14) give

$$a = \left(\frac{r}{n-1} - 1\right) \quad \textit{and} \quad b = \left(\frac{r}{n-1} - n\right). \tag{2.15}$$

In 1971, G. P. Pokhariyal and R. S. Mishra [7] defined a tensor field W^* on a Riemannian manifold as

$$\begin{aligned} W^*(X,Y)Z &= R(X,Y)Z - \frac{1}{2(n-1)}[S(Y,Z)X \\ &- S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] \end{aligned} \tag{2.16}$$

so that

$$'W^*(X,Y,Z,U) \overset{\text{def}}{=} g(W^*(X,Y)Z,U) =' W^*(Z,U,X,Y)$$

and

$$'W^*_{ijkl}w^{ij}w^{kl} =' W_{ijkl}w^{ij}w^{kl},$$

where $'W^*_{ijkl}$ and $'W_{ijkl}$ are components of $'W^*$ and $'W$ and w^{kl} is a skew-symmetric tensor [5], [12]. Such a tensor field W^* is known as $m-$projective curvature tensor.

On an $n-$dimensional LP-Sasakian manifold, the concircular curvature tensor $\tilde{C}$ is defined as

$$\tilde{C}(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}\left\{g(Y,Z)X - g(X,Z)Y\right\}, \tag{2.17}$$

where

$$'\tilde{C}(X,Y,Z,U) \overset{\text{def}}{=} g(\tilde{C}(X,Y)Z,U). \tag{2.18}$$

Now, in view of $S(X,Y) = \frac{r}{n}g(X,Y)$, (2.16) becomes

$$W^*(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}\left\{g(Y,Z)X - g(X,Z)Y\right\}$$

$$\iff \quad W^*(X,Y)Z = \tilde{C}(X,Y)Z.$$

Thus, in an Einstein LP-Sasakian manifold, the $m-$projective curvature tensor W^* and concircular curvature tensor $\tilde{C}$ coincide.

It is well known that

Proposition 2.1. [16] *Let M_n be an $n-$dimensional LP-Sasakian manifold. Then M_n is Ricci-symmetric if and only if it is an Einstein manifold.*

Proposition 2.2. [16] *Let M_n be an $n-$dimensional LP-Sasakian manifold. Then M_n satisfies the condition $\tilde{C}(\xi,X).S = 0$, if and only if either M_n is Einstein manifold or M_n has scalar curvature $r = n(n-1)$.*

Proposition 2.3. [17] *In an $n-$dimensional Riemannian manifold M_n, the following are equivalent*

(i) M_n is an Einstein manifold,

(ii) $m-$projective and Weyl projective curvature tensors are linearly dependent.

(iii) $m-$projective and concircular curvature tensors are linearly dependent.

(iv) $m-$projective and conformal curvature tensors are linearly dependent.

In consequence of Prepositions (2.1), (2.2) and (2.3), we state

Theorem 2.4. *On an $n-$dimensional LP-Sasakian manifold, the following are equivalent*
(i) M_n is Ricci-semi symmetric, i. e., $R(X,Y).S = 0$,
(ii) M_n satisfies $\tilde{C}(\xi, X).S = 0$,
(ii) $m-$projective and Weyl projective curvature tensors are linearly dependent.
(iii) $m-$projective and concircular curvature tensors are linearly dependent.
(iv) $m-$projective and conformal curvature tensors are linearly dependent.

3. LP-Sasakian manifolds satisfying $W^* = 0$

In view of $W^* = 0$, (2.16) becomes

$$\begin{aligned} R(X,Y)Z &= \frac{1}{2(n-1)}[S(Y,Z)X - S(X,Z)Y \\ &+ g(Y,Z)QX - g(X,Z)QY]. \end{aligned} \tag{3.1}$$

Replacing Z by ξ in (3.1) and then using (2.4), (2.7) and (2.9), we obtain

$$(n-1)\,(\eta(Y)X - \eta(X)Y) = \eta(Y)QX - \eta(X)QY.$$

Again putting $Y = \xi$ in the above relation and using (2.4) and (2.9), we have

$$QX = (n-1)X \quad \Longleftrightarrow \quad S(X,Y) = (n-1)g(X,Y) \tag{3.2}$$

and

$$r = n(n-1).$$

In consequence of (3.2), (3.1) becomes

$$R(X,Y)Z = g(Y,Z)X - g(X,Z)Y, \tag{3.3}$$

which shows that an $m-$projectively flat LP-Sasakian manifold is of constant curvature. The value of this constant is $+1$ [13]. Hence we can state

Theorem 3.1. *A LP-Sasakian manifold M_n is $m-$projectively flat if and only if it has constant curvature $+1$.*

Theorem 3.2. *An $n-$dimensional LP-Sasakian manifold M_n is $m-$projectively flat if and only if it is locally isometric to a unit sphere $S^n(1)$.*

A. Taleshian and N. Asghari [16] proved

Proposition 3.3. *An $n-$dimensional LP-Sasakian manifold M_n satisfies $R(\xi, X).\tilde{C} = 0$ if and only if M_n is locally isometric to the unit sphere $S^n(1)$.*

In view of Theorem (3.2) and Proposition (3.3), we have

Theorem 3.4. *An $n-$dimensional LP-Sasakian manifold M_n satisfies the condition $R(\xi, X).\tilde{C} = 0$ if and only if M_n is $m-$projectively flat.*

4. An Einstein LP-Sasakian manifold satisfying $R(X,Y).W^{*}=0$

In consequence of $S(X,Y)=kg(X,Y)$, (2.16) becomes

$$W^{*}(X,Y)Z=R(X,Y)Z-\frac{k}{n-1}\left\{g(Y,Z)X-g(X,Z)Y\right\}. \tag{4.1}$$

In view of (2.4), (2.10) and (4.1), we find

$$\eta(W^{*}(X,Y)Z)=\left(1-\frac{k}{n-1}\right)\left\{\eta(X)g(Y,Z)-\eta(Y)g(X,Z)\right\}. \tag{4.2}$$

Replacing Z by ξ in (4.2) and using (2.4), we have

$$\eta(W^{*}(X,Y)\xi)=0. \tag{4.3}$$

Now,

$$\begin{aligned}\left(R(X,Y).W^{*}\right)(Z,U)V &= R(X,Y)W^{*}(Z,U)V-W^{*}\left(R(X,Y)Z,U\right)V\\ &- W^{*}\left(Z,R(X,Y)U\right)V-W^{*}\left(Z,U\right)R(X,Y)V\end{aligned} \tag{4.4}$$

Using $R(X,Y).W^{*}=0$ in the above equation, we obtain

$$\begin{aligned}R(X,Y)W^{*}(Z,U)V &- W^{*}\left(R(X,Y)Z,U\right)V\\ &- W^{*}\left(Z,R(X,Y)U\right)V-W^{*}\left(Z,U\right)R(X,Y)V=0.\end{aligned}$$

With the help of (2.4), above equation becomes

$$\begin{aligned}g\left(R(X,Y)W^{*}(Z,U)V,\xi\right) &- g\left(W^{*}\left(R(X,Y)Z,U\right)V,\xi\right)\\ &- g\left(W^{*}\left(Z,R(X,Y)U\right)V,\xi\right)-g\left(W^{*}\left(Z,U\right)R(X,Y)V,\xi\right)=0.\end{aligned}$$

Putting $X=\xi$ in the above equation and then using (2.4) and (2.8), we obtain

$$\begin{aligned}-\eta(Y)\eta(W^{*}(Z,U)V) &- {}'W^{*}\left(Z,U,V,Y\right)+\eta(Z)\eta(W^{*}(Y,U)V)\\ &- g(Y,Z)\eta(W^{*}(\xi,U)V)+\eta(U)\eta(W^{*}(Z,Y)V)\\ &- g(Y,U)\eta(W^{*}(Z,\xi)V)+\eta(V)\eta(W^{*}(Z,U)Y)\\ &- g(Y,V)\eta(W^{*}(Z,U)\xi)=0.\end{aligned}$$

In consequence of (2.4) and (4.2), above equation becomes

$$\begin{aligned}-{}'W^{*}\left(Z,U,V,Y\right) &- \eta(Y)\left[\left(1-\frac{k}{n-1}\right)\left\{\eta(Z)g(U,V)-\eta(U)g(V,Z)\right\}\right]\\ &+ \eta(U)\left[\left(1-\frac{k}{n-1}\right)\left\{\eta(Z)g(Y,V)-\eta(Y)g(V,Z)\right\}\right]\\ &+ \eta(Z)\left[\left(1-\frac{k}{n-1}\right)\left\{\eta(Y)g(U,V)-\eta(U)g(Y,V)\right\}\right]\\ &+ \eta(V)\left[\left(1-\frac{k}{n-1}\right)\left\{\eta(Z)g(U,Y)-\eta(U)g(Y,Z)\right\}\right]\\ &- g(Y,Z)\left[\left(1-\frac{k}{n-1}\right)\left\{\eta(\xi)g(U,V)-\eta(U)g(\xi,V)\right\}\right]\\ &- g(Y,U)\left[\left(1-\frac{k}{n-1}\right)\left\{\eta(Z)g(\xi,V)-\eta(\xi)g(Z,V)\right\}\right]\\ &- g(Y,V)\left[\left(1-\frac{k}{n-1}\right)\left\{\eta(Z)g(U,\xi)-\eta(U)g(Z,\xi)\right\}\right]=0.\end{aligned}$$

or,

$$'W^*(Z,U,V,Y) = \left(1 - \frac{k}{n-1}\right)[g(Y,Z)g(U,V) - g(Y,U)g(Z,V)], \tag{4.5}$$

which gives

$$W^*(Z,U)V = \left(1 - \frac{k}{n-1}\right)[g(U,V)Z - g(Z,V)U]. \tag{4.6}$$

In view of (4.1) and (4.6), we obtain

$$R(Z,U)V = \{g(U,V)Z - g(Z,V)U\}. \tag{4.7}$$

Thus, we state the following

Theorem 4.1. *An Einstein LP-Sasakian manifold M_n satisfies $R(X,Y).W^* = 0$ if and only if it is locally isometric to a unit sphere $S^n(1)$.*

Contracting (4.7) with respect to Z, we get

$$S(U,V) = (n-1)g(U,V) \tag{4.8}$$

and

$$QU = (n-1)U, \tag{4.9}$$

which gives

$$r = n(n-1). \tag{4.10}$$

In consequence of (2.16), (4.7), (4.8) and (4.9), we obtain

$$W^*(X,Y)Z = 0. \tag{4.11}$$

Again, equations (4.4) and (4.11) give

$$R(X,Y).W^* = 0. \tag{4.12}$$

Hence, we say

Theorem 4.2. *An Einstein LP-Sasakian manifold M_n satisfies $R(X,Y).W^* = 0$ if and only if it is $m-$projectively flat.*

In view of the Theorems (4.1) and (4.2), we state

Corollary 4.3. *An Einstein LP-Sasakian manifold M_n satisfies $R(X,Y).W^* = 0$ if and only if either M_n is $m-$projectively flat or it is locally isometric to a unit sphere $S^n(1)$.*

5. $\eta-$Einstein LP-Sasakian manifold satisfying $W^*(\xi,X).R = 0$

Replacing X by ξ in (2.16) and then using (2.4), (2.8), (2.11), (2.12) and (2.15), we obtain

$$W^*(\xi,Y)Z = \frac{1}{2}\left[1 - \frac{1}{(n-1)}\left\{\frac{r}{n-1} - 1\right\}\right]\{g(Y,Z)\xi - \eta(Z)Y\}. \tag{5.1}$$

Also, we have

$$\begin{aligned}(W^*(\xi,X).R)(Y,Z)U &= W^*(\xi,X)R(Y,Z)U - R(W^*(\xi,X)Y,Z)U \\ &- R(Y,W^*(\xi,X)Z)U - R(Y,Z)W^*(\xi,X)U.\end{aligned}$$

Using $W^*(\xi,X).R = 0$ in the above relation, we get

$$\begin{aligned}W^*(\xi,X)R(Y,Z)U &- R(W^*(\xi,X)Y,Z)U \\ &- R(Y,W^*(\xi,X)Z)U - R(Y,Z)W^*(\xi,X)U = 0.\end{aligned}$$

In view of (2.4), (2.7), (2.8), (2.10) and (5.1), last result becomes

$$\begin{aligned}&\frac{1}{2}\left[1-\frac{1}{(n-1)}\left\{\frac{r}{n-1}-1\right\}\right]({}'R(Y,Z,U,X)\xi+\eta(Z)g(Y,U)X\\&-\eta(Y)g(Z,U)X+\eta(Y)R(X,Z)U+g(X,Y)\eta(U)Z\\&-g(X,Y)g(Z,U)\xi+\eta(Z)R(Y,X)U-g(X,Z)R(Y,\xi)U\\&+\eta(U)R(Y,Z)X+g(X,U)\eta(Y)Z-g(X,U)\eta(Z)Y)=0,\end{aligned} \tag{5.2}$$

where

$${}'R(X,Y,Z,U)\overset{\text{def}}{=}g(R(X,Y)Z,U). \tag{5.3}$$

With the help of (2.4), (2.10) and (5.2), we find

$$\left[1-\frac{1}{(n-1)}\left\{\frac{r}{n-1}-1\right\}\right]\{-{}'R(Y,Z,U,X)+g(X,Y)g(Z,U)-g(Y,U)g(X,Z)\}=0,$$

which gives

$${}'R(Y,Z,U,X)=g(X,Y)g(Z,U)-g(Y,U)g(X,Z).$$

In consequence of (2.4) and (5.3), above equation becomes

$$R(Y,Z)U=g(Z,U)Y-g(Y,U)Z. \tag{5.4}$$

Contracting equation (5.4) with respect to Y, we have

$$S(Z,U)=(n-1)g(Z,U),$$

which gives

$$QZ=(n-1)Z$$

and

$$r=n(n-1).$$

Thus, we can state

Theorem 5.1. *An $n-$dimensional $\eta-$Einstein LP-Sasakian manifold M_n satisfies $W^*(\xi,X).R=0$ if and only if either M_n is locally isometric to a unit sphere $S^n(1)$ or M_n has constant scalar curvature $n(n-1)$.*

Theorem 5.2. *An $n-$dimensional $\eta-$Einstein LP-Sasakian manifold M_n satisfies $W^*(\xi,X).R=0$ if and only if it is $m-$projectively flat.*

6. Some more results-

Definition 6.1. *If an $n-$dimensional LP-Sasakian manifold M_n satisfies the relation*

$$R(X,Y).W^*=0, \tag{6.1}$$

then M_n is said to be $m-$projective semi-symmetric, where $R(X,Y)$ denotes the derivation of the tensor algebra at each point of the manifold for the tangent vectors X and Y.

Theorem 6.1. *An $n-$dimensional LP-Sasakian manifold M_n satisfies*

$$R.W^*=R.R. \tag{6.2}$$

Proof. We have,

$$\begin{aligned}(R(X,Y).W^*)(Z,U)V &= R(X,Y)W^*(Z,U)V - W^*(R(X,Y)Z,U)V \\ &- W^*(Z,R(X,Y)U)V - W^*(Z,U)R(X,Y)V.\end{aligned}$$

In consequence of (2.16), above equation becomes

$$\begin{aligned}(R(X,Y).W^*)(Z,U)V &= R(X,Y)R(Z,U)V - R(R(X,Y)Z,U)V \\ &- R(Z,R(X,Y)U)V - R(Z,U)R(X,Y)V. \quad (6.3)\end{aligned}$$

Also,

$$\begin{aligned}(R(X,Y).R)(Z,U)V &= R(X,Y)R(Z,U)V - R(R(X,Y)Z,U)V \\ &- R(Z,R(X,Y)U)V - R(Z,U)R(X,Y)V. \quad (6.4)\end{aligned}$$

Equations (6.3) and (6.4) give the statement of the theorem. □

It is well known that if an $n-$dimensional LP-Sasakian manifold M_n satisfies the relation $R(X,Y).R = 0$, then M_n is said to be semi-symmetric. Thus, in consequence of (6.1), (6.2) and the above result, we state

Corollary 6.2. *Let M_n be an $n-$dimensional LP-Sasakian manifold, then the necessary and sufficient condition for M_n to be semi-symmetric is that it is $m-$projectively semi-symmetric.*

Now, in consequence of Theorem 3.3 of [16], Theorem (6.1) and Corollary (6.2), we say

Theorem 6.3. *An $n-$dimensional LP-Sasakian manifold M_n is $m-$projectively semi-symmetric if and only if it is concircularly semi-symmetric.*

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