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SOME COINCIDENCE AND COMMON FIXED POINT THEOREMS IN CONE METRIC SPACES

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Abstract. The purpose of this paper is to prove some coincidence and common fixed point theorems for three mappings with a new type of contractive conditions in cone metric spaces. Our results generalize and extend some recent results in cone metric spaces.

1. Introduction and Preliminaries

K-metric and K-normed spaces were introduced in the mid-20th century (see [4, 10, 16, 18], see also [11], [12]) by using an ordered Banach space instead of the set of real numbers, as the codomain for a metric. Huang and Zhang [6] re-introduced such spaces under the name of cone metric spaces, defining convergent and Cauchy sequence in terms of interior points of underlying cone. They proved the basic version of the fixed point theorem in cone metric spaces. Subsequently, Abbas and Jungck [2], Abbas and Rhoades [1] and Pasquale Verto [17] have studied common fixed point theorems in cone metric spaces . Stojan Radenović [13] has obtained coincidence point result for two mappings in cone metric spaces which satisfy new contractive conditions. Recently, M. Rangamma and K. Prudhvi [14] extended coincidence point results of Stojan Radenović [13] for three maps which satisfy generalized contractive condition without exploiting the notion of continuity.

Rangamma and Prudhvi proved the following theorems.

Theorem 1.1. *Let (X, d) be a cone metric space, and P a normal cone with normal constant L. Suppose the self maps f, g, h on X satisfy the condition*

$$\|d(fx, gy)\| \leq \lambda \|d(hx, hy)\|, \text{ for all } x, y \in X,$$

where $\lambda \in (0, 1)$ is a constant. If $f(X) \cup g(X) \subset h(X)$ and $h(X)$ is a complete subspace of X.Then the maps f, g and h have a coincidence point p in X. Moreover

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if (f,h) and (g,h) are (IT)-Commuting at p, then f,g and h have a unique common fixed point.

Theorem 1.2. *Let (X,d) be a cone metric space, and P a normal cone with normal constant L. Suppose the self maps f,g,h on X satisfy the condition*

$$\|d(fx,gy)\| \leq \lambda(\|d(fx,hx)\| + \|d(gy,hy)\|), \text{ for all } x,y \in X,$$

where $\lambda \in [0,\frac{1}{2})$ is a constant. If $f(X) \cup g(X) \subset h(X)$ and $h(X)$ is a complete subspace of X.Then the maps f,g and h have a coincidence point p in X. Moreover if (f,h) and (g,h) are (IT)-Commuting at p, then f,g and h have a unique common fixed point.

Note that in above theorems, to generalize the contractive condition, the norm function is used which is defined from the real Banach space in to the set of real numbers i.e. $\mathbb{R}$, and $\mathbb{R}$ is also a Banach space. In this paper we extend and generalize the result of Rangamma and Prudhvi [14] as well as Huang and Zhang [6] and Vetro [17] with a new type of contractive condition, by introducing a function which is defined from a Banach space in to another Banach space.

We need the following definitions and results, consistent with [5] and [6].

Definition 1. [6] *Let E be a real Banach space and P be a subset of E. The set P is called a cone if:*
(i) P is closed, nonempty and $P \neq \{0_E\}$, here 0_E is the zero vector of E;
(ii) $a,b \in \mathbb{R},\ a,b \geq 0,\ x,y \in P$ implies $ax+by \in P$;
(iii) $x \in P$ and $-x \in P$ implies $x = 0_E$.
Given a cone $P \subset E$, we define a partial ordering "$\preceq$" with respect to P by $x \preceq y$ if and only if $y - x \in P$. We write $x \prec y$ to indicate that $x \preceq y$ but $x \neq y$. While $x \ll y$ if and only if $y - x \in P^0$, where P^0 denotes the interior of P.
Let P be a cone in a real Banach space E, then P is called normal, if there exist a constant $K > 0$ such that for all $x,y \in E$,

$$0_E \preceq x \preceq y \text{ implies } \|x\| \leq K\|y\|.$$

The least positive number K satisfying the above inequality is called the normal constant of P. P is called solid if $P^0 \neq \emptyset$ i.e. interior of P is nonempty.

Definition 2. [6] *Let X be a nonempty set, E be a real Banach space. Suppose that the mapping $d : X \times X \to E$ satisfies:*

(1) *$0_E \preceq d(x,y)$, for all $x,y \in X$ and $d(x,y) = 0_E$ if and only if $x = y$;*
(2) *$d(x,y) = d(y,x)$ for all $x,y \in X$;*
(3) *$d(x,y) \preceq d(x,z) + d(y,z)$, for all $x,y,z \in X$.*

Then d is called a cone metric on X, and (X,d) is called a cone metric space. If the underlying cone is normal then (X,d) is called a normal cone metric space.

The concept of cone metric space is more general then that of a metric space, because each metric space is a cone metric space with $E = \mathbb{R}$ and $P = [0,+\infty)$.

For examples and basic properties of normal and non-normal cones and cone metric spaces we refer [3, 6, 8] and [15].
The following remark will be useful in sequel.

Remark 1. [9] *Let P be a cone in a real Banach space E with zero vector 0_E and $a,b,c \in P$, then:*

(a) *If $a \preceq b$ and $b \ll c$ then $a \ll c$.*

(b) *If* $a \ll b$ *and* $b \ll c$ *then* $a \ll c$.
(c) *If* $0_E \preceq u \ll c$ *for each* $c \in P^0$ *then* $u = 0_E$.
(d) *If* $c \in P^0$ *and* $a_n \to 0_E$ *then there exist* $n_0 \in \mathbb{N}$ *such that, for all* $n > n_0$ *we have* $a_n \ll c$.
(e) *If* $0_E \preceq a_n \preceq b_n$ *for each* n *and* $a_n \to a$, $b_n \to b$ *then* $a \preceq b$.
(f) *If* $a \preceq \lambda a$ *where* $0 \leq \lambda < 1$ *then* $a = 0_E$.

Definition 3. [6] *Let* (X, d) *be a cone metric space. Let* $\{x_n\}$ *be a sequence in* X *and* $x \in X$.

(1) *If for every* $c \in E$ *with* $0 \ll c$ *(or equivalently* $c \in P^0$ *) there is a positive integer* n_0 *such that,* $d(x_n, x) \ll c$ *for all* $n > n_0$, *then the sequence* $\{x_n\}$ *is said to be convergent and converges to* x. *We denote this by* $\lim_{n\to\infty} x_n = x$ *or* $x_n \to x$ *as* $n \to \infty$.
(2) *If for every* $c \in E$ *with* $0 \ll c$ *there is a positive integer* n_0 *such that,* $d(x_n, x_m) \ll c$ *for all* $n, m > n_0$, *then the sequence* $\{x_n\}$ *is called a Cauchy sequence in* X.

(X, d) *is called a complete cone metric space, if every Cauchy sequence in* X *is convergent in* X.

Lemma 1.3. [6] *Let* (X, d) *be a cone metric space,* P *be a normal cone with normal constant* K. *Let* $\{x_n\}$ *and* $\{y_n\}$ *be two sequences in* X.
(1) $\{x_n\}$ *is Cauchy sequence if and only if* $d(x_n, x_m) \to 0_E$ *as* $n \to \infty$.
(2) If $x_n \to x, y_n \to y$ *as* $n \to \infty$, *then* $d(x_n, y_n) \to d(x, y)$ *as* $n \to \infty$.

Let E, B be two real Banach spaces, P and C normal cones in E and B respectively. Let "$\preceq$" and "$\leq$" be the partial orderings induced by P and C in E and B respectively. Let $\phi : P \to C$ be a function satisfying:

(i) if $a, b \in P$ with $a \preceq b$ then $\phi[a] \leq k\phi[b]$, for some positive real k;
(ii) $\phi[a + b] \leq \phi[a] + \phi[b]$ for all $a, b \in P$;
(iii) ϕ is sequentially continuous i.e. if $a_n, a \in P$ and $\lim_{n\to\infty} a_n = a$, then $\lim_{n\to\infty} \phi[a_n] = \phi[a]$;
(iv) if $\phi[a_n] \to 0_B$ then $a_n \to 0_E$, where 0_E and 0_B are the zero vectors of E and B respectively.

We denote the set of all such functions by $\Phi(P, C)$ i.e. $\phi \in \Phi(P, C)$ if ϕ satisfies all above properties. It is clear that $\phi[a] = 0_B$ if and only if $a = 0_E$.

Let (X, d) be a cone metric space with normal cone P and $\phi \in \Phi(P, C)$. Since $d(x, y) \preceq d(x, z) + d(z, y)$ for all $x, y, z \in X$, therefore

$$\phi[d(x, y)] \leq k\phi[d(x, z)] + k\phi[d(z, y)]. \tag{1.1}$$

Following are some examples of function ϕ.

Example 1. *Let* E *be any real Banach space with normal cone* P *and normal constant* K. *Define* $\phi : P \to P$ *by* $\phi[a] = a$, *for all* $a \in P$. *Then* $\phi \in \Phi(P, C)$ *with* $E = B, P = C$ *and* $k = 1$.

Example 2. *Let* $E = \mathbb{R}^2, P = \{(a, b) : a, b \in \mathbb{R}$ *and* $a, b \geq 0\}$ *and* $B = \mathbb{R}^3, C = \{(a, b, c) : a, b, c \in \mathbb{R}$ *and* $a, b, c \geq 0\}$, *then* P *and* C *are normal cones and both have normal constant* $K = 1$. *Define* $\phi : P \to C$ *by* $\phi[(a, b)] = (a, b, a + 2b)$, *then* $\phi \in \Phi(P, C)$ *with* $k = 1$.

Example 3. *Let E be any real Banach space with normal cone P and normal constant K. Define $\phi : P \to [0, \infty)$ by $\phi[a] = \|a\|$, for all $a \in P$. Then $\phi \in \Phi(P, C)$ with $B = \mathbb{R}, C = [0, \infty)$ and $k = K$.*

In the following we always suppose that E, B are two real Banach spaces, P and C are normal cones in E and B respectively and " $\preceq$ " and " $\leq$ " are partial orderings in E and B with respect to P and C respectively.

Definition 4. *Let X be a nonempty set and f, g be self maps on X and $x, z \in X$. Then x is called coincidence point of pair (f, g) if $fx = gx$, and z is called point of coincidence of pair (f, g) if $fx = gx = z$.*

Definition 5. *Let X be a nonempty set and f, g be self maps on X. Pair (f, g) is called weakly compatible if f and g commutes at their coincidence point, i.e. $fgx = gfx$, whenever $fx = gx$ for some $x \in X$.*

For results on weakly compatible mappings in cone metric spaces see [7, 9].

Now we can state our main results.

2. Main Results

Theorem 2.1. *Let (X, d) be a cone metric space and P a normal cone with normal constant K. Suppose f, g, h be self maps of X satisfy the condition*

$$\phi[d(fx, gy)] \leq a\phi[d(hx, hy)] + b\phi[d(hx, fx)] + c\phi[d(hy, gy)] \tag{2.1}$$

where $\phi \in \Phi(P, C)$ and a, b, c are nonnegative reals with $a + b + c < 1$. If $f(X) \cup g(X) \subset h(X)$ and $h(X)$ is a complete subspace of X, then the maps f, g and h have a unique point of coincidence in X. Moreover if (f, h) and (g, h) are weakly compatible pairs then f, g and h have a unique common fixed point.

Proof. Suppose x_0 be any arbitrary point of X. Since $f(X) \cup g(X) \subset h(X)$, starting with x_0 we define a sequence $\{y_n\}$ such that

$$y_{2n} = fx_{2n} = hx_{2n+1} \text{ and } y_{2n+1} = gx_{2n+1} = hx_{2n+2},$$

for all $n \geq 0$. We shall prove that $\{y_n\}$ is a Cauchy sequence in X.
If $y_n = y_{n+1}$ for some n, e.g. if $y_{2n} = y_{2n+1}$, then from (2.1) we obtain

$$\begin{aligned}
\phi[d(y_{2n+2}, y_{2n+1})] &= \phi[d(fx_{2n+2}, gx_{2n+1})] \\
&\leq a\phi[d(hx_{2n+2}, hx_{2n+1})] + b\phi[d(hx_{2n+2}, fx_{2n+2})] \\
&\quad + c\phi[d(hx_{2n+1}, gx_{2n+1})] \\
&= a\phi[d(y_{2n+1}, y_{2n})] + b\phi[d(y_{2n+1}, y_{2n+2})] + c\phi[d(y_{2n}, y_{2n+1})]
\end{aligned}$$

since $y_{2n} = y_{2n+1}$ it follows from above inequality that

$$\phi[d(y_{2n+2}, y_{2n+1})] \leq b\phi[d(y_{2n+1}, y_{2n+2})].$$

As $b < 1$ from (f) of remark 1 we obtain $\phi[d(y_{2n+2}, y_{2n+1})] = 0_B$ also $\phi \in \Phi(P, C)$ therefore we have $d(y_{2n+2}, y_{2n+1}) = 0_E$ i.e. $y_{2n+2} = y_{2n+1}$. Similarly we obtain that

$$y_{2n} = y_{2n+1} = y_{2n+2} = \ldots = v \text{ (say)} .$$

Therefore $\{y_n\}$ is a Cauchy sequence.
Suppose $y_n \neq y_{n+1}$ for all n. Then from (2.1) it follows that

$$\begin{aligned}
\phi[d(y_{2n}, y_{2n+1})] &= \phi[d(fx_{2n}, gx_{2n+1})] \\
&\leq a\phi[d(hx_{2n}, hx_{2n+1})] + b\phi[d(hx_{2n}, fx_{2n})] + c\phi[d(hx_{2n+1}, gx_{2n+1})] \\
&= a\phi[d(y_{2n-1}, y_{2n})] + b\phi[d(y_{2n-1}, y_{2n})] + c\phi[d(y_{2n}, y_{2n+1})] \\
&= (a+b)\phi[d(y_{2n-1}, y_{2n})] + c\phi[d(y_{2n}, y_{2n+1})]
\end{aligned}$$

i.e.

$$\phi[d(y_{2n}, y_{2n+1})] \leq \frac{a+b}{1-c}\phi[d(y_{2n-1}, y_{2n})] = \lambda\phi[d(y_{2n-1}, y_{2n})],$$

where $\lambda = \frac{a+b}{1-c} < 1$ (since $a + b + c < 1$).
Writing $d_n = \phi[d(y_n, y_{n+1})]$, we obtain

$$d_{2n} \leq \lambda d_{2n-1}. \tag{2.2}$$

Again

$$\begin{aligned}
\phi[d(y_{2n+2}, y_{2n+1})] &= \phi[d(fx_{2n+2}, gx_{2n+1})] \\
&\leq a\phi[d(hx_{2n+2}, hx_{2n+1})] + b\phi[d(hx_{2n+2}, fx_{2n+2})] \\
&\quad + c\phi[d(hx_{2n+1}, gx_{2n+1})] \\
&= a\phi[d(y_{2n+1}, y_{2n})] + b\phi[d(y_{2n+1}, y_{2n+2})] + c\phi[d(y_{2n}, y_{2n+1})] \\
&= (a+c)\phi[d(y_{2n+1}, y_{2n})] + b\phi[d(y_{2n+1}, y_{2n+2})]
\end{aligned}$$

i.e.

$$\phi[d(y_{2n+2}, y_{2n+1})] \leq \frac{a+c}{1-b}\phi[d(y_{2n+1}, y_{2n})] = \mu\phi[d(y_{2n+1}, y_{2n})],$$

where $\mu = \frac{a+c}{1-b} < 1$ (since $a + b + c < 1$).
Therefore

$$d_{2n+1} \leq \mu d_{2n}. \tag{2.3}$$

From (2.2) and (2.3) we get

$$d_{2n} \leq \lambda d_{2n-1} \leq \lambda\mu d_{2n-2} \leq \cdots \leq \lambda^n \mu^n d_0,$$

and

$$d_{2n+1} \leq \mu d_{2n} \leq \lambda\mu d_{2n-1} \leq \cdots \leq \lambda^n \mu^{n+1} d_0.$$

Thus

$$\begin{aligned}
d_{2n} + d_{2n+1} &\leq \lambda^n \mu^n (1+\mu) d_0, && (2.4) \\
d_{2n+1} + d_{2n+2} &\leq \lambda^n \mu^{n+1} (1+\lambda) d_0. && (2.5)
\end{aligned}$$

Let $n, m \in \mathbb{N}$, then for the sequence $\{y_n\}$ we consider $\phi[d(y_n, y_m)]$ in two cases.
If n is even and $m > n$, then using (1.1) and (2.4) we obtain

$$\begin{aligned}
\phi[d(y_n, y_m)] &\leq k\phi[d(y_n, y_{n+1})] + k\phi[d(y_{n+1}, y_{n+2})] + \cdots + k\phi[d(y_{m-1}, y_m)] \\
&\leq k[d_n + d_{n+1} + d_{n+2} + d_{n+3} + \cdots] \\
&\leq k[\lambda^{\frac{n}{2}}\mu^{\frac{n}{2}}(1+\mu)d_0 + \lambda^{\frac{n+2}{2}}\mu^{\frac{n+2}{2}}(1+\mu)d_0 + \cdots] \\
\phi[d(y_n, y_m)] &\leq \frac{k(\lambda\mu)^{\frac{n}{2}}(1+\mu)}{1-\lambda\mu}d_0.
\end{aligned}$$

If n is odd and $m > n$, then again using (1.1) and (2.5) we obtain

$$\begin{aligned}
\phi[d(y_n, y_m)] &\leq k\phi[d(y_n, y_{n+1})] + k\phi[d(y_{n+1}, y_{n+2})] + \cdots + k\phi[d(y_{m-1}, y_m)] \\
&\leq k[d_n + d_{n+1} + d_{n+2} + d_{n+3} + \cdots] \\
&\leq k[\lambda^{\frac{n-1}{2}} \mu^{\frac{n-1}{2}+1}(1+\lambda)d_0 + \lambda^{\frac{n+1}{2}} \mu^{\frac{n+1}{2}+1}(1+\lambda)d_0 + \cdots] \\
\phi[d(y_n, y_m)] &\leq \frac{k(\lambda\mu)^{\frac{n-1}{2}}(1+\lambda)}{1-\lambda\mu} d_0.
\end{aligned}$$

Since $\lambda < 1, \mu < 1$ therefore $\lambda\mu < 1$, so in both the cases $\phi[d(y_n, y_m)] \to 0_B$ as $n \to \infty$, and since $\phi \in \Phi(P, C)$, we have $d(y_n, y_m) \to 0_E$ as $n \to \infty$. So by lemma 1.3 $\{y_n\} = \{hx_{n-1}\}$ is a Cauchy sequence.
Since $h(X)$ is complete, there exist $v \in h(X)$ and $u \in X$ such that $\lim\limits_{n\to\infty} y_n = v$ and $v = hu$.
We shall show that u is a coincidence point of pairs (f, h) and (g, h), i.e. $fu = gu = hu$.
If $fu \neq hu$ then $0_E \prec d(fu, hu)$. Using (2.1) we obtain

$$\begin{aligned}
\phi[d(fu, y_{2n+1})] &= \phi[d(fu, gx_{2n+1})] \\
&\leq a\phi[d(hu, hx_{2n+1})] + b\phi[d(hu, fu)] + c\phi[d(hx_{2n+1}, gx_{2n+1})] \\
&= a\phi[d(hu, y_{2n})] + b\phi[d(hu, fu)] + c\phi[d(y_{2n}, y_{2n+1})] \\
&= a\phi[d(hu, y_{2n})] + b\phi[d(hu, fu)] + cd_{2n}
\end{aligned}$$

Since $y_{2n} \to hu, d_{2n} \to 0_B$, $d(fu, y_{2n+1}) \to d(fu, hu)$ as $n \to \infty$ and $\phi \in \Phi(P, C)$, therefore letting $n \to \infty$ in above inequality and using remark 1 we get

$$\begin{aligned}
\phi[d(fu, hu)] &\leq b\phi[d(hu, fu)] \\
&< \phi[d(hu, fu)] \text{ (since } b < 1),
\end{aligned}$$

a contradiction. Therefore $fu = hu$. Similarly it can be shown that $gu = hu$. Therefore

$$fu = gu = hu = v. \tag{2.6}$$

Thus v is point of coincidence of pairs (f, h) and (g, h). We shall show that it is unique.
Suppose w is another point of coincidence of these pairs i.e. $fz = gz = hz = w$ for some $z \in X$. Then from (2.1) it follows that

$$\begin{aligned}
\phi[d(w, v)] &= \phi[d(fz, gu)] \\
&\leq a\phi[d(hz, hu)] + b\phi[d(hz, fz)] + c\phi[d(hu, gu)] \\
&= a\phi[d(w, v)] + b\phi[d(w, w)] + c\phi[d(v, v)] \\
&= a\phi[d(w, v)].
\end{aligned}$$

Since $a < 1$, by remark 1 we obtain $\phi[d(w, v)] = 0_B$ i.e. $w = v$. Thus point of coincidence is unique.
If pairs (f, h) and (g, h) are weakly compatible, from (2.6) we have $fv = fhu = hfu = hv$, and $gv = ghu = hgu = hv$, therefore $fv = gv = hv = p$ (say). This shows that p is another point of coincidence, therefore by uniqueness we must have $p = v$ i.e.

$$fv = gv = hv = v.$$

Thus v is unique common fixed point of self maps f, g and h. □

Taking $B = \mathbb{R}, C = [0, \infty)$ and $\phi[a] = \|a\|$ for all $a \in P$. Then C is normal cone with normal constant 1, $\phi \in \Phi(P, C)$ with $k = K$ =normal constant of P. Therefore we have the following generalization of Theorems **2.1** and **2.3** of Rangamma and Prudhvi [14].

Corollary 2.2. *Let (X, d) be a cone metric space and P a normal cone with normal constant K. Suppose f, g, h be self maps of X satisfy the condition*

$$\|d(fx, gy)\| \leq a\|d(hx, hy)\| + b\|d(hx, fx)\| + c\|d(hy, gy)\|$$

where a, b, c are nonnegative reals with $a + b + c < 1$. If $f(X) \cup g(X) \subset h(X)$ and $h(X)$ is a complete subspace of X, then the maps f, g and h have a unique point of coincidence in X. Moreover if (f, h) and (g, h) are weakly compatible pairs then f, g and h have a unique common fixed point.

Note that Theorems **2.1** and **2.3** of Rangamma and Prudhvi [14] can be obtained by taking $b = c = 0$ and $a = 0, b = c$ respectively in above corollary.

Taking $E = B, P = C$ and defining $\phi : P \to P$ by $\phi[a] = a$, for all $a \in P$, in theorem 2.1, we get following generalization of main results of Vetro [17] and Huang and Zhang [6].

Corollary 2.3. *Let (X, d) be a cone metric space and P a normal cone with normal constant K. Suppose f, g, h be self maps of X satisfy the condition*

$$d(fx, gy) \preceq ad(hx, hy) + bd(hx, fx) + cd(hy, gy)$$

where a, b, c are nonnegative reals with $a + b + c < 1$. If $f(X) \cup g(X) \subset h(X)$ and $h(X)$ is a complete subspace of X, then the maps f, g and h have a unique point of coincidence in X. Moreover if (f, h) and (g, h) are weakly compatible pairs then f, g and h have a unique common fixed point.

Above corollary gives the Result of Vetro [17] with $h = I_X$ and gives a generalization of Theorems **1** and **3** of Huang and Zhang [6] with $f = g, h = I_X$.

Remark 2. *In normal cones the condition $a \preceq b$, does not implies $\|a\| \leq \|b\|$ (see Example* **2.2** *of* [3]*), hence the results of Huang and Zhang* [6] *can not be obtained by the results of Rangamma and Prudhvi* [14]*. In this paper a more competent function ϕ instead of $\|\cdot\|$ is used, so that the results of Rangamma and Prudhvi* [14] *as well as Huang and Zhang* [6] *are the consequences of our results.*

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