Divulgaciones Matemáticas Vol. 8 No. 1 (2000), pp. 31–42

A Note on Prime Modules

Una Nota sobre Módulos Primos

Christian Lomp (clomp@fc.up.pt)
Centro de Matemática da Universidade do Porto
Praça Gomes Teixeira
4099-002 Porto, Portugal.

Alirio J. Peña P. (apena@luz.ve)
Laboratorio de Álgebra Teórica y Computacional (LATyC)
Departamento de Matemática y Computación
Facultad Experimental de Ciencias.
La Universidad del Zulia.
Apartado Postal 526
Maracaibo 4001. Venezuela.

Humberto Fernández-Morán Villalobos in memoriam

Abstract

In this note we compare some notions of primeness for modules existing in the literature. We characterize the prime left R-modules such that the left annihilator of every element is a (two-sided) ideal of R, where R is an associative ring with unity, and we prove that if M is such a left R-module then M is strongly prime. These two notions are studied by Beachy [B75]. Furthermore, if M is projective as left $R/(0 : M)$-module then M is B-prime in the sense of Bican et al. [BJKN]. On the other hand, if M is faithful then M is (strongly) prime if and only if M is strongly prime (or an SP-module) in the sense of Handelman-Lawrence [HL] if and only if M is torsionfree and R is a domain. In particular this happens if R is commutative.
Key words: bounded kernel functor, annihilating set for a module, prime module, strongly prime module, SP-module, torsionfree module.

Resumen

En esta nota se comparan algunas nociones de primitud para módulos existentes en la literatura. Se caracterizan los R-módulos a izquierda

Received: 1999/06/01. Revised: 2000/02/23. Accepted: 2000/03/02.
MSC (2000): 16D80, 16D20.

primos para los cuales el anulador a izquierda de cada elemento es un ideal (bilátero) de R, donde R es un anillo asociativo con unidad, y se prueba que si M es un tal R-módulo izquierda, entonces M es fuertemente primo. Estas dos nociones son estudiadas por Beachy [B75]. Además, si M es proyectivo como $R/(0:M)$-módulo a izquierda, entonces M es B-primo en el sentido de Bican et al. [BJKN]. Por otro lado, si M es fiel, entonces M es (fuertemente) primo si y sólo si M es fuertememte primo (o un SP-módulo) en el sentido de Handelman-Lawrence [HL] si y sólo si M es libre de torsión y R es un dominio. En particular, esto ocurre si R es conmutativo.
Palabras y frases clave: funtor núcleo acotado, conjunto anulador para un módulo, módulo primo, módulo fuertemente primo, SP-módulo, módulo libre de torsión.

Introduction

In the category of left R-modules there exist several notions of prime objects which generalize the well known notion of prime (two-sided) ideal of an associative ring R with unity. In each case, an ideal P of a ring R is prime if and only if R/P is prime as left R-module. In this paper four of these notions are considered: prime and strongly prime due to Beachy [B75], B-prime due to Bican et al. [BJKN] and strongly prime (or SP-module) due to Handelman-Lawrence [HL].

In section 1 we give some terminology, notation and basic results. In section 2 we consider some primeness conditions for left R-modules existing in the literature and some of its relations are pointed out. Each of these conditions generalizes the well known notion of prime ideal in a commutative ring. In section 3 we characterize the prime left R-modules such that the left annihilator of every element is a two-sided ideal of R (Proposition 3.1) and we show some properties of this special type of left R-modules (Proposition 3.2). In particular, if M is such a prime left R-module which is projective as left $R/(0:M)$-module then M is B-prime (Corollary 3.1). We consider also the commutative case (Corollary 3.2) and the faithful case (Corollary 3.3). Furthermore, we prove that if M is a nonzero left R-module such that the left annihilator of every element is a two-sided ideal of R then M is prime if and only if M is strongly prime (Theorem 3.1).

1 Terminology, notation and basic results

By a *ring* we shall understand an associative ring with unit $1 \neq 0$. R will always denote a ring. By an *ideal* of R we shall understand a two-sided ideal of R. By a *module* we mean a unitary left R-module. We shall denote by $R-\mathrm{Mod}$ the category of modules with zero object (0) whose morphisms act on the right side. If M is a module and X is a set then we shall denote by M^X (respectively, $M^{(X)}$) the direct product (respectively, direct sum) of $|X|$ (cardinal of X) copies of M. We shall use the notations ${}_RM$ and $N \leq M$ to indicate that M is a module and N a submodule of M, respectively. Let ${}_RM$, let $X \subseteq M$ and let $N \leq M$. Then the set $(N : X) := \{a \in R : aX \subseteq N\}$ is a left ideal of R. In particular, if $m \in M$ then $(0 : m) := ((0) : \{m\})$ is called the *left annihilator of* m and $(0 : M) := ((0) : M)$ is an ideal of R called the *annihilating ideal of* M. Furthermore M is said to be *faithful* if and only if $(0 : M) = (0)$; M is said to be *cofaithful* if and only if there exist elements $m_1, ..., m_n \in M$ such that $\bigcap_{i=1}^{n}(0 : m_i) = (0)$, or equivalently, if R can be embedded in a finite direct sum M^n of n copies of M. It is clear that every cofaithful module is also faithful. The question about when every faithful left ideal of R is cofaithful was considered by Beachy-Blair in [BB]. On the other hand, every faithful module is cofaithful if and only if R contains an essential artinian left ideal if and only if R has an essential and finitely generated socle (see [B71] and [V]).

Following Peña [P94] (see also [Ri]), we shall define the *annihilating set for a module* M, which will be denoted by $\mathrm{A}(M)$, as

$$\mathrm{A}(M) := \{a \in R : aRm = (0) \text{ for some } 0 \neq m \in M\}.$$

Note that $\mathrm{A}((0)) = \varnothing$ and if M is a nonzero module then $(0 : M) \subseteq \mathrm{A}(M)$ and $\mathrm{A}(M) = \bigcup\{(0 : N) : (0) \neq N \leq M\}$ is a union of ideals of R. Thus, $\mathrm{A}(M)$ is closed under left and right multiplication by elements of R, but $\mathrm{A}(M)$ is not necessarily an ideal of R. Recently Dauns [D] considered for which modules M the set $\mathrm{A}(M)$ is an ideal of R (called *primal modules*), and proved that M is a primal module if and only if the complete lattice formed by all the (left, right or two-sided) ideals of R which are contained in $\mathrm{A}(M)$ has a unique coatom (Proposition B in [D]). Note that if N is a proper submodule of a module M then the annihilating set $\mathrm{A}(M/N)$ is called the *adjoint of* N in [D], and it is denoted by $adj(N)$. Independently of the work of Dauns, Peña in [P99a] asks about the modules having zero annihilating sets, obtaining the following result in the language of the kernel functors of Goldman [G]:

Theorem 1.1 (Teorema (3.1) in [P99a])
Let M be a nonzero left R-module, let τ_M be the kernel functor associated to

M, and let $\mathbb{K}(R)$ be the complete lattice formed by all the kernel functors over the category $R-\mathrm{Mod}$. Then the following conditions are equivalent:

1. $\mathrm{A}(M) = (0)$*;*
2. *M is a faithful prime module;*
3. *Every nonzero submodule of M is faithful;*
4. *M is σ-torsion-free for every proper bounded kernel functor σ;*
5. *τ_M is an upper bound in $\mathbb{K}(R)$ for the proper bounded kernel functors.*

Let M and N be modules. We say that M *is cogenerated by* N (or M *is N-cogenerated*) if and only if there is an exact sequence of modules of type $0 \longrightarrow M \longrightarrow N^X$ for some index set X. Dually, we say that M *is generated by* N (or M *is N-generated*) if and only if there is an exact sequence of modules of type $N^{(X)} \longrightarrow M \longrightarrow 0$ for some index set X. We shall denote by $Cog(N)$ the class of modules formed by all the N-cogenerated modules. It is easy to see that $Cog(N)$ is a class of modules closed under taking submodules, isomorphic images and direct products. A module M is faithful if and only if every projective module is M-cogenerated (see [B75]). Now, following Wisbauer [W91], we say that M *is subgenerated by* N (or M *is N-subgenerated*) if and only if M is isomorphic to a submodule of an N-generated module. The category of N-subgenerated modules is denoted by $\sigma[N]$.

In this work we shall suppose a certain familiarity with the notions and basic constructions about kernel functors and its corresponding filters (see Goldman [G]). Let M be a module and let σ be a kernel functor. We say that M is *σ-torsion* (respectively, *σ-torsion-free*) if and only if $\sigma(M) = M$ (respectively, $\sigma(M) = (0)$). Furthermore, we say that M is *σ-decisive* if and only if M is either σ-torsion or σ-torsion-free. This notion of relatively decisive modules has been considered recently in Peña [P99c]. For more details on ring and module theory, refer to Anderson-Fuller [AF].

2 Some primeness conditions for modules

Let M be a nonzero module. Following Beachy [B75], we say that M is *prime* if and only if $(0 : M) = (0 : N)$ for every nonzero submodule N of M. In Dauns [D] it is pointed out that M is prime if and only if $(0 : M) = \mathrm{A}(M)$. It is easy to see also that M is prime if and only if the quotient module $R/(0 : M)$ is cogenerated by every nonzero submodule of M. This condition was studied by Johnson in [J]. See also [W83]. Recently, Peña in [P99c] showed that M is prime if and only if M is σ-decisive for every bounded kernel functor σ. Recall that a kernel functor is said to be *bounded* if and only if its corresponding filter is *bounded* (i.e., contains a cofinal set of ideals of R). This characterization of prime modules leads us to define *prime object* in categories of Grothendieck (or more generally, in any category where there exist bounded localizant subcategories) and the notion of a Φ-*prime* module in Ore extensions relative to a subset Φ of the set of additive endomorphisms of R (see Lam-Leroy-Matczuk [LLM]). Now, following Bican et al. [BJKN], we say that M is B-*prime* if and only if M is cogenerated by each of its nonzero submodules. It is easy to see that B-prime implies prime. In [W83] it is pointed out that M is B-prime if and only if $L \cdot \mathrm{Hom}_R(M, N) \neq (0)$ for every pair L, N of nonzero submodules of M. Finally, following Beachy [B75], we say that M is *strongly prime* if and only if for every nonzero submodule N of M and $m \in M$ there exist elements $n_1, \ldots, n_k \in N$ such that $\bigcap_{i=1}^{k}(0 : n_i) \leq (0 : m)$, or equivalently, if M is subgenerated by every of its nonzero submodules (see [W83]). It is easy to see that every strongly prime module is prime and that every simple module is strongly prime. Thus an strongly prime module doesn't need to be faithful. The rings for which there exists a faithful strongly prime module were considered by Desale-Nicholson in [DN]. It is well known that M is strongly prime if and only if M is σ-decisive for every kernel funtor σ (Proposition 1.2 in [B75]).

In the literature on module theory there is another notion of strongly primeness due to Handelman-Lawrence [HL]: a nonzero module M is an *SP-module* ('SP' by strongly prime) if and only if for every $0 \neq m \in M$ there exist elements $r_1, ..., r_n \in R$ such that $\bigcap_{i=1}^{n}(0 : r_i m) = (0)$. It is easy to see that every SP-module is a faithful strongly prime module. Furthermore, it is well known that R is an SP-module (and in such a case, R is said to be a *left strongly prime* ring in [HL]) if and only if there exists an SP-module if and only if R is an strongly prime module in the sense of Beachy (Proposition 1.3 in [B75]) if and only if there exists a cofaitful strongly prime module (see [W83]). On the other hand, in [W83] it is pointed out that R is a prime ring if and only if there exists a faithfull (cofaithful) prime module.

The left strongly prime rings of Handelman-Lawrence coincide with the *left absolutely torsion free* rings (i.e., rings which are torsion-free relative to every proper kernel functor). These rings were studied by Rubin in [Ru] and by Viola-Prioli in [VP75] and [VP73]. Using these ideas and concepts, it is not difficult to prove that a nonzero module M is an SP-module if and only if M is σ-torsion-free for every proper (idempotent) kernel functor σ (see [P99a]) if and only if the quotient module $R/(0 : M)$ is finitely cogenerated by every nonzero submodule of M. The last condition was pointed out by Wisbauer in [W83].

Let M be a nonzero module and let us consider the following conditions:

(I) M is cogenerated by each of its nonzero submodules (B-prime);

(II) $L \cdot \mathrm{Hom}_R(M,N) \neq (0)$ for every pair L, N of nonzero submodules of M;

(III) $(0:M) = (0:N)$ for every nonzero submodule N of M (prime);

(IV) $R/(0:M)$ is cogenerated for every nonzero submodule of M;

(V) $(0:M) = \mathrm{A}(M)$;

(VI) M is σ-decisive for every bounded kernel functor;

(VII) M is σ-decisive for every kernel functor σ (strongly prime);

$(VIII)$ M is σ-torsion-free for every proper kernel functor σ (SP-prime).

Then $(I) \Leftrightarrow (II) \Rightarrow (III) \Leftrightarrow (IV) \Leftrightarrow (V) \Leftrightarrow (VI) \Leftarrow (VII) \Leftarrow (VIII)$. In this work it is shown that if M is a nonzero module such that the left annihilator of every element of M is an ideal of R then the conditions $(III)-(VII)$ above are equivalent. If M is projective as left $R/(0:M)$-module then we can add the conditions (I) and (II). Furthermore, if M is faithful then we can add the condition $(VIII)$. In particular, this last holds if R is commutative.

3 The main results

Let M be a nonzero module. Recall that M is said to be *torsionfree* if for every non zero-divisor $r \in R$ and $0 \neq m \in M$ we have $rm \neq 0$. If R is a *domain*, i.e., every $0 \neq r \in R$ is a non zero-divisor, and M is torsionfree then M is an SP-module and for every $0 \neq m \in M$ we have $(0:m) = (0)$. Note also that if $m \in M$ such that $(0:m)$ is an ideal of R then $(0:m) = (0:Rm)$. Now, M is called *compressible* if and only if M can be embedded in any nonzero submodule of M, and M is said to be *semi-compressible* if and only if M is finitely cogenerated by each of its nonzero submodules (see [BB]). We have the obvious chain of conditions:

$$\text{compressible} \Rightarrow \text{semi-compressible} \Rightarrow \text{B-prime}.$$

Proposition 3.1 *Let M be a nonzero module, and let $S = R/(0:M)$. Then the following conditions are equivalent:*

(a) M is prime and the left annihilator of every element of M is an ideal;

(b) $(0:m) = (0:M)$ for every $0 \neq m \in M$;

(c) *for every* $0 \neq m \in M$ *there exists an isomorphism of left* R*-modules* $f\colon Rm \to S$ *such that* $(m)f = \bar{1}$*;*

(d) *for every* $0 \neq m \in M$ *there exists a homomorphism of left* R*-modules* $f\colon Rm \to S$ *such that* $(m)f = \bar{1}$*;*

(e) *every non-zero cyclic* R*-submodule of* M *is isomorphic to* ${}_RS$*;*

(f) M *is a torsionfree left* S*-module and* S *is a domain.*

Proof. It is easy to see that $(a) \Leftrightarrow (b) \Leftrightarrow (e)$ and $(c) \Rightarrow (d)$.
$(b) \Leftrightarrow (f)$ If M satisfies (b) then S has no zero-divisors, because $\bar{r}\bar{s} = \bar{0}$ (where $\bar{r} = r + (0 : M)$) implies $\bar{r}\bar{s}m = rsm = 0$ for any $0 \neq m \in M$. Thus, $r \in (0 : sm) = (0 : M)$ or $s \in (0 : m) = (0 : M)$ implies $\bar{r} = \bar{0}$ or $\bar{s} = \bar{0}$. Also ${}_SM$ is torsionfree, because if $0 = \bar{s}m = sm$ then $s \in (0 : m) = (0 : M)$ and so, $\bar{s} = \bar{0}$. If M satisfies (f) then certainly for all $r \in R$ and $0 \neq m \in M$ we have $rm = 0 \Rightarrow \bar{r}m = 0 \Rightarrow \bar{r} = \bar{0} \Rightarrow r \in (0 : M)$. Hence $(0 : m) = (0 : M)$ for every $0 \neq m \in M$.
$(a) \Rightarrow (c)$: Suppose that the condition (a) holds and define $f\colon Rm \to S$ by $(rm)f := \bar{r}$ for each $r \in R$. Then it is easy to see that f is well defined and f is an isomorphism of left R-modules.
$(d) \Rightarrow (b)$: Suppose the condition (d) holds and let $0 \neq m \in M$. Then there exists a homomorphism of modules $f\colon Rm \to S$ such that $(m)f = \bar{1}$. Now, let $r \in R$ such that $rm = 0$. Then $\bar{0} = (rm)f = r(m)f = \bar{r}$ in S. Hence, $(0 : m) = (0 : M)$ and the condition (b) holds. □

Note that if M is a module satisfying some of the conditions of Proposition 3.1 then all non-zero cyclic submodules of M are isomorphic to each other and the natural projection $S \to Rm$ with $\bar{s} \mapsto sm$ for every $0 \neq m \in M$ is an isomorphism of left R-modules. Hence the nonzero cyclic submodules of ${}_RM$ and ${}_RS$ share the same properties. Recall that a domain D is called a *left Ore domain* if for every pair of elements $x, y \in D$ there exist $s, r \in D$ such that $xs = yr \neq 0$. Recall also that, following Fleury [F], a nonzero module M is said to be *hollow* if and only if every proper submodule of M is small in M.

Proposition 3.2 *Let* M *be a prime left* R*-module such that the left annihilators of elements of* M *are ideals of* R *and let* $S = R/(0 : M)$*. Then the following hold:*

1. M *contains a simple submodule* $\Leftrightarrow S$ *is a divison ring.*

2. M *contains a uniform submodule* $\Leftrightarrow S$ *is a left Ore domain.*

3. M *contains a hollow submodule* $\Leftrightarrow S$ *is a local domain.*

4. *M contains a nonzero artinian (noetherian) submodule $\Leftrightarrow$ S is a left artinian (noetherian) ring.*

Proof. All the observations follow from the fact that, by hypothesis, every cyclic R-submodule of M is isomorphic to ${}_RS$. So, if M contains a simple submodule then ${}_RS$ is simple and hence also S is simple as a left S-module. Thus it is a division ring. If M contains a uniform (cyclic) submodule then ${}_SS$ is uniform and by [GW, Lemma 5.15] the result follows. The rest is similar. □

Theorem 3.1 *Let M be a nonzero left R-module such that the left annihilators of elements of M are ideals of R, and let $S = R/(0 : M)$. Then the following hold:*

1. *${}_RM$ is prime if and only if ${}_RM$ is strongly prime.*
2. *If ${}_RM$ is prime then ${}_SM$ is an SP-module.*
3. *${}_RM$ is B-prime if and only if ${}_RM$ is prime and ${}_SM$ is cogenerated by S as S-module.*
4. *${}_RM$ is semi-compressible if and only if ${}_RM$ is prime and ${}_SM$ is finitely cogenerated by S as S-module.*
5. *${}_RM$ is compressible if and only if ${}_RM$ is prime and ${}_SM$ is isomorphic to a left ideal of S as left S-module.*

Proof. (1) It is well known that strongly prime implies prime. Suppose that M is prime and let σ be a proper kernel functor such that $\sigma(M) \neq (0)$. If $0 \neq m \in \sigma(M)$ then, by hypothesis, the ideal $(0 : M) = (0 : Rm) = (0 : m)$ is in the corresponding filter of σ. Thus, $\sigma(M) = M$ and by Proposition 1.2 in [B75], M is strongly prime.
(2) Clear by definition and Proposition 3.1.
(3) Suppose that M is prime as left R-module and cogenerated by S as left S-module. Then M is also cogenerated by S as left R-module. Now, by hypothesis, every nonzero cyclic submodule of M is isomorphic to S and so, every nonzero (cyclic) submodule of M cogenerates M. Therefore, M is B-prime. Conversely, if M is B-prime and every nonzero cyclic submodule is isomorphic to S then M is cogenerated by S as left R-module and also as left S-module.

Finally, the proof of (4) and (5) are analogous to the proof of (3). □

Corollary 3.1 *Let M be a prime left R-module such that the left annihilators of elements of M are ideals of R, and let $S = R/(0 : M)$. Then the following hold:*

1. *If ${}_SM$ is projective then ${}_RM$ is B-prime.*
2. *If ${}_SM$ is finitely generated and self-projective then ${}_RM$ is semi-compressible.*
3. *If ${}_SM$ is cyclic and self-projective then ${}_RM$ is compressible.*

Proof. All the assertions follow from the Theorem above since in case ${}_SM$ is projective, M is cogenerated by S. Note also that for ${}_SM$ finitely generated and self-projective, we have that ${}_SM$ is projective in the category $\sigma[{}_SM]$ of M-subgenerated S-modules (see [W91, 18.3]). As S embeds into ${}_SM$, we have $\sigma[{}_SM] = S$-Mod and hence ${}_SM$ is projective. □

To underline the result look at the following example: the rational numbers $\mathbb{Q}$ form a flat prime $\mathbb{Z}$-module that is not B-prime, because $\mathrm{Hom}_{\mathbb{Z}}(\mathbb{Q}, \mathbb{Z}) = (0)$, and hence $\mathbb{Q}$ is not cogenerated by $\mathbb{Z}$.

Corollary 3.2 *Let R be a commutative ring, let M be a nonzero R-module, and let $S = R/(0 : M)$. Then the following conditions are equivalent:*

(a) *M is prime;*

(b) *M is strongly prime;*

(c) *S is an integral domain.*

Corollary 3.3 *Let M be a faithful left R-module such that the left annihilator of every element of M is an ideal of R. Then the following conditions are equivalent:*

(a) *M is prime;*

(b) *M is strongly prime;*

(c) *M is an SP-module;*

(d) *M is torsionfree and R is a domain.*

Moreover, if M is prime then

1. *M is B-prime if and only if M is cogenerated by R.*

2. M is semi-compressible if and only if M is finitely cogenerated by R.

3. M is compressible if and only if M is isomorphic to a left ideal of R.

Some final remarks: (1) If R is a commutative ring then the notions of prime, strongly prime and SP-modules coincide for faithful modules. The existence of a faithful prime module over a commutative ring R is therefore also equivalent to the R being an integral domain.
(2) Prime modules were studied for many years, in particular over left noetherian rings. The annihilator $P = (0 : M)$ of a prime module M is a prime ideal of R called the *affiliated prime* of M (see [GW], [McR]). Over left noetherian rings, prime modules occur plentiful: from [GW, 2.13], we know that every finitely generated nonzero left R-module M over a left noetherian ring R has a finite ascending series of submodules $(0) = M_0 \subset M_1 \subset \cdots \subset M_n = M$ such that M_{i+1}/M_i are prime modules. Also, from [McR, 4.3.5], we know the following: suppose ${}_SM_R$ is a bimodule, where S is a left noetherian ring, ${}_SM$ is finitely generated and M_R is prime with affiliated prime $P = (0 : M_R)$. If R/P is a right Goldie domain then M is torsionfree over R/P. From our Propositon 3.1 we know that $(0 : m)$ is an ideal of R for every $m \in M$ and by Theorem 3.1, M_R is strongly prime.

References

[AF] Anderson, F. and Fuller, K., *Rings and categories of modules*, Graduate Texts in Mathematics, Springer-Verlag, Berlin-New York, 1974.

[B75] Beachy, J., *Some aspects of noncommutative localization*, in Noncommutative Ring Theory, Kent State University, Lecture Notes in Mathematics, Vol. 545, Springer-Verlag, Berlin-New York, 1975.

[B71] Beachy, J., *On quasi-artinian rings*, J. London Math. Soc. **3**(2) (1971), 449–452.

[BB] Beachy, J. and Blair, W. D., *Rings whose faithful left ideals are cofaithful*, Pacific J. Math. **58**(1) (1975), 1–13.

[BJKN] Bican, L., Jampor, P., Kepka, T. and Nemec, P., *Prime and coprime modules*, Fund. Math. **57** (1980), 33–45.

[D] Dauns, J., *Primal modules*, Comm. Algebra **25**(8) (1997), 2409–2435.

[DN] Desale, G. and Nicholson, W.K., *Endoprimitive rings*, J. Algebra **70** (1981), 548–560.

[F] Fleury, P., *A note on dualizing Goldie dimension*, Canad. Math. Bull. **17** (1974), 511–517.

[G] Goldman, O., *Rings and modules of quotients*, J. Algebra **13** (1969), 10–40.

[GW] Goodearl, K. R. and Warfield, R. B., *An introduction to noncommutative Noetherian rings.*, London Math. Soc. Student Texts, 16., 1989.

[HL] Handelman, D. and Lawrence, J., *Strongly prime rings*, Trans. Amer. Math. Soc. **211** (1975), 209–223.

[J] Johnson, R. E., *Representations of prime modules*, Trans. Amer. Math. Soc. **74** (1953), 351–357.

[LLM] Lam., T. Y., Leroy, A. and Matczuk, J., *Primeness, semiprimeness and prime radical of Ore extensions*, Comm. Algebra **25**(8) (1997), 2459–2506.

[McR] McConnel, J. C. and Robson, J. C., *Noncommutative Noetherian Rings*, New York 1987, Wiley-Interscience.

[P99a] Peña, A. J., *Anillos y módulos con conjunto anulador nulo*, submitted, 1999.

[P99b] Peña, A. J., *Prime modules, bounded kernel functors and Gabriel's support over left locally prime rings*, in preparation, 1999.

[P99c] Peña, A. J., *Relative decisiveness and cocriticality in modules*, in preparation, 1999.

[P94] Peña, A. J., *Filtros idempotentes y conjuntos anuladores*, Divulg. Mat. **2**(1) (1994), 11–35.

[Ri] Riley, J., *Axiomatic primary and tertiary decomposition theory*, Trans. Amer. Math. Soc. **105**(1962), 177–201.

[Ru] Rubin, R. A., *Absolutely torsion-free rings*, Pacific J. Math. **46** (2) (1973), 503–514.

[V] Vámos, P., *On the dual of the notion of "finitely generated"*, J. London Math. Soc. **43**(1968), 643–646.

[VP75] Viola-Prioli, J., *On absolutely torsion-free rings*, Pacific J. Math. **56**(1) (1975), 275–283.

[VP73] Viola-Prioli, J., *On absolutely torsion-free rings and kernel functors*, Ph.D. Thesis, Rutgers University, New Brunswick, N.J., 1973.

[W91] Wisbauer, R., *Foundations of Module and Ring Theory*, Gordon and Breach, Reading, 1991.

[W83] Wisbauer, R., *On prime modules and rings*, Comm. Algebra **11**(20) (1983), 2249–2265.

[W77] Wisbauer, R., *Co-semisimple modules and nonassociative V-rings*, Comm. Algebra **5**(11) (1977), 1193–1209.