# Appendix to

# "Linear programming and the worst- case analysis of greedy algorithms on cubic graphs"

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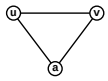
## MMM

In all operations considered, the edge $uv$ is the one that could possibly be selected by "some" algorithm for inclusion in the matching. The priorities are

- edges with an end-point that has a neighbour of degree 1 over
- edges with an end-point that has a neighbour of degree 2

In each case, choose the edge that, if added to the set, would give the smallest ratio of edges added to the set (which includes the edge $uv$ itself and all isolated edges created) to edges removed from the graph. Where there are two edges with the same ratio, choose the edge with the fewest vertices neighbouring its end-points, otherwise, ties are broken arbitrarily. Operations written in red (with "***" in the last column) are excluded from $OPS_2$.

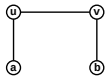
E01:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E01-00: | $V_2$ | | | | : | 0 | 3 | 0 | 0 | 0 | 0 | 1 |
| E01-01: | $V_3$ | | | | : | 1 | 2 | 0 | 0 | 0 | 1 | 1 |

It is assumed here that the minimum degree is 2.

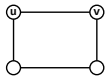
E02:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E02-00: | $V_2$ | $V_2$ | | | : | 0 | 4 | 0 | 0 | 0 | 2 | 1 |
| E02-01: | $V_2$ | $V_3$ | | | : | 1 | 3 | 0 | 0 | 1 | 1 | 1 |
| E02-02: | $V_3$ | $V_3$ | | | : | 2 | 2 | 0 | 0 | 2 | 0 | 1*** |

It is assumed here that the minimum degree is 2. It is also assumed that no isolated edges are generated. If the degree of both $a$ and $b$ is 3, there is always an edge that gives a ratio of less than 1/3 since it is assumed here that the minimum degree is 2.

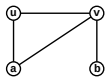
E03:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E03-00: | | | | | : | 0 | 4 | 0 | 0 | 0 | 0 | 2 |

It is assumed here that the minimum degree is 2.

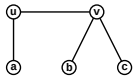
E04:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E04-00: | $V_2$ | $V_2$ | | | : | 1 | 3 | 0 | 0 | 0 | 1 | 1 |
| E04-01: | $V_2$ | $V_3$ | | | : | 2 | 2 | 0 | 0 | 1 | 0 | 1 |
| E04-02: | $V_3$ | $V_2$ | | | : | 2 | 2 | 0 | 0 | 0 | 2 | 1 |
| E04-03: | $V_3$ | $V_3$ | | | : | 3 | 1 | 0 | 0 | 1 | 1 | 1 |

It is assumed here that the minimum degree is 2. If any isolated edges were created, there would be another edge to select with smaller ratio.

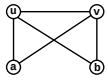
E05:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E05-00: | $V_2$ | $V_2$ | $V_2$ | | : | 1 | 4 | 0 | 0 | 0 | 3 | 1 |
| E05-01: | $V_2$ | $V_2$ | $V_3$ | | : | 2 | 3 | 0 | 0 | 1 | 2 | 1 |
| E05-02: | $V_2$ | $V_3$ | $V_3$ | | : | 3 | 2 | 0 | 0 | 2 | 1 | 1 |
| E05-03: | $V_3$ | $V_2$ | $V_2$ | | : | 2 | 3 | 0 | 0 | 1 | 2 | 1 |
| E05-04: | $V_3$ | $V_2$ | $V_3$ | | : | 3 | 2 | 0 | 0 | 2 | 1 | 1 |
| E05-05: | $V_3$ | $V_3$ | $V_3$ | | : | 4 | 1 | 0 | 0 | 3 | 0 | 1*** |

It is assumed here that the minimum degree is 2. If $a$, $b$ and $c$ have degree 3, there exists another edge with ratio at least 1/4, that has fewer vertices neighbouring its end-points. If any isolated edges were created, there would be another edge to select with smaller ratio.

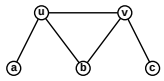
E06:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E06-00: | $V_2$ | $V_2$ | | | : | 2 | 2 | 0 | 0 | 0 | 0 | 1 |
| E06-01: | $V_2$ | $V_3$ | | | : | 3 | 1 | 0 | 0 | 0 | 1 | 1 |
| E06-02: | $V_3$ | $V_3$ | | | : | 4 | 0 | 0 | 0 | 0 | 2 | 1*** |

It is assumed here that the minimum degree is 2. The selected edge has to have an end-point that is incident with a vertex of current minimum degree as its end-points are both of degree 3. Cannot isolate an edge as $a$ or $b$ has degree 2.

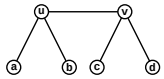
E07:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E07-00: | $V_2$ | $V_2$ | $V_2$ | | : | 2 | 3 | 0 | 0 | 0 | 2 | 1 |
| E07-01: | $V_2$ | $V_2$ | $V_3$ | | : | 3 | 2 | 0 | 0 | 1 | 1 | 1 |
| E07-02: | $V_2$ | $V_3$ | $V_2$ | | : | 3 | 2 | 0 | 0 | 1 | 1 | 1 |
| E07-03: | $V_2$ | $V_3$ | $V_3$ | | : | 4 | 1 | 0 | 0 | 2 | 0 | 1 |
| E07-04: | $V_3$ | $V_2$ | $V_3$ | | : | 4 | 1 | 0 | 0 | 1 | 2 | 1 |
| E07-05: | $V_3$ | $V_3$ | $V_3$ | | : | 5 | 0 | 0 | 0 | 2 | 1 | 1*** |

It is assumed here that the minimum degree is 2. The selected edge has to have an end-point that is incident with a vertex of current minimum degree as its end-points are both of degree 3. If any isolated edges were created, there would be another edge to select with smaller ratio.

E08:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E08-00: | $V_2$ | $V_2$ | $V_2$ | $V_2$ | : | 2 | 4 | 0 | 0 | 0 | 4 | 1 |
| E08-01: | $V_2$ | $V_2$ | $V_2$ | $V_3$ | : | 3 | 3 | 0 | 0 | 1 | 3 | 1 |
| E08-02: | $V_2$ | $V_2$ | $V_3$ | $V_3$ | : | 4 | 2 | 0 | 0 | 2 | 2 | 1 |
| E08-03: | $V_2$ | $V_3$ | $V_2$ | $V_3$ | : | 4 | 2 | 0 | 0 | 2 | 2 | 1 |
| E08-04: | $V_2$ | $V_3$ | $V_3$ | $V_3$ | : | 5 | 1 | 0 | 0 | 3 | 1 | 1 |
| E08-05: | $V_3$ | $V_3$ | $V_3$ | $V_3$ | : | 6 | 0 | 0 | 0 | 4 | 0 | 1*** |

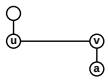
It is assumed here that the minimum degree is 2. The selected edge has to have an end-point that is incident with a vertex of current minimum degree as its end-points are both of degree 3. If any isolated edges were created, there would be another edge to select with smaller ratio.

H01:


| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H01-00: | | | | | : | 0 | 1 | 2 | 0 | 0 | 0 | 1 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

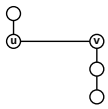
H02:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H02-00: | $V_1$ | | | | : | 0 | 2 | 2 | 0 | 0 | 0 | 1 |
| H02-01: | $V_2$ | | | | : | 0 | 3 | 1 | 0 | 0 | 1 | 1 |
| H02-02: | $V_3$ | | | | : | 1 | 2 | 1 | 0 | 1 | 0 | 1 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. It is also assumed that no isolated edges are generated.

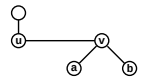
H03:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H03-00: | | | | | : | 0 | 3 | 2 | 0 | 0 | 0 | 2 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

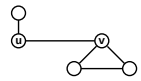
H04:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H04-00: | $V_1$ | $V_1$ | | | : | 1 | 1 | 3 | 0 | 0 | 0 | 1 |
| H04-01: | $V_1$ | $V_2$ | | | : | 1 | 2 | 2 | 0 | 0 | 1 | 1 |
| H04-02: | $V_1$ | $V_3$ | | | : | 2 | 1 | 2 | 0 | 1 | 0 | 1 |
| H04-03: | $V_2$ | $V_2$ | | | : | 1 | 3 | 1 | 0 | 0 | 2 | 1 |
| H04-04: | $V_2$ | $V_3$ | | | : | 2 | 2 | 1 | 0 | 1 | 1 | 1 |
| H04-05: | $V_3$ | $V_3$ | | | : | 3 | 1 | 1 | 0 | 2 | 0 | 1 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. It is also assumed that no isolated edges are generated.

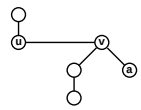
H05:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H05-00: | | | | | : | 1 | 3 | 1 | 0 | 0 | 0 | 2 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

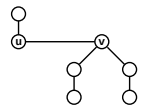
H06:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H06-00: | $V_1$ | | | | : | 1 | 2 | 3 | 0 | 0 | 0 | 2 |
| H06-01: | $V_2$ | | | | : | 1 | 3 | 2 | 0 | 0 | 1 | 2 |
| H06-02: | $V_3$ | | | | : | 2 | 2 | 2 | 0 | 1 | 0 | 2 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

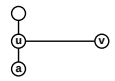
H07:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H07-00: | | | | | : | 1 | 3 | 3 | 0 | 0 | 0 | 3 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

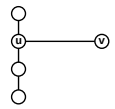
H08:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H08-00: | $V_1$ | | | | : | 1 | 0 | 3 | 0 | 0 | 0 | 1 |
| H08-01: | $V_2$ | | | | : | 1 | 1 | 2 | 0 | 0 | 1 | 1 |
| H08-02: | $V_3$ | | | | : | 2 | 0 | 2 | 0 | 1 | 0 | 1 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. It is also assumed that no isolated edges are generated.

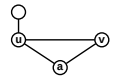
H09:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H09-00: | | | | | : | 1 | 1 | 3 | 0 | 0 | 0 | 2 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

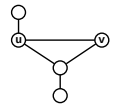
H10:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H10-00: | $V_2$ | | | | : | 1 | 2 | 1 | 0 | 0 | 0 | 1 |
| H10-01: | $V_3$ | | | | : | 2 | 1 | 1 | 0 | 0 | 1 | 1 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. It is also assumed that no isolated edges are generated.

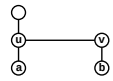
H11:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H11-00: | | | | | : | 2 | 1 | 2 | 0 | 0 | 0 | 2 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

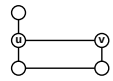
H12:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H12-00: | $V_1$ | $V_1$ | | | : | 1 | 2 | 3 | 0 | 0 | 0 | 1 |
| H12-01: | $V_1$ | $V_2$ | | | : | 1 | 3 | 2 | 0 | 0 | 1 | 1 |
| H12-02: | $V_1$ | $V_3$ | | | : | 2 | 2 | 2 | 0 | 1 | 0 | 1 |
| H12-03: | $V_2$ | $V_2$ | | | : | 1 | 4 | 1 | 0 | 0 | 2 | 1 |
| H12-04: | $V_2$ | $V_3$ | | | : | 2 | 3 | 1 | 0 | 1 | 1 | 1 |
| H12-05: | $V_3$ | $V_3$ | | | : | 3 | 2 | 1 | 0 | 2 | 0 | 1 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. It is also assumed that no isolated edges are generated.

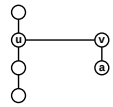
H13:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H13-00: | | | | | : | 1 | 3 | 1 | 0 | 0 | 0 | 2 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

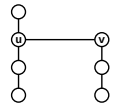
H14:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H14-00: | $V_1$ | | | | : | 1 | 2 | 3 | 0 | 0 | 0 | 2 |
| H14-01: | $V_2$ | | | | : | 1 | 3 | 2 | 0 | 0 | 1 | 2 |
| H14-02: | $V_3$ | | | | : | 2 | 2 | 2 | 0 | 1 | 0 | 2 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

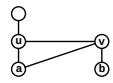
H15:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H15-00: | | | | | : | 1 | 3 | 3 | 0 | 0 | 0 | 3 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

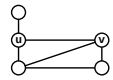
H16:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H16-00: | $V_2$ | $V_1$ | | | : | 2 | 1 | 2 | 0 | 0 | 0 | 1 |
| H16-01: | $V_2$ | $V_2$ | | | : | 2 | 2 | 1 | 0 | 0 | 1 | 1 |
| H16-02: | $V_2$ | $V_3$ | | | : | 3 | 1 | 1 | 0 | 1 | 0 | 1 |
| H16-03: | $V_3$ | $V_1$ | | | : | 3 | 0 | 2 | 0 | 0 | 1 | 1 |
| H16-04: | $V_3$ | $V_2$ | | | : | 3 | 1 | 1 | 0 | 0 | 2 | 1 |
| H16-05: | $V_3$ | $V_3$ | | | : | 4 | 0 | 1 | 0 | 1 | 1 | 1 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. It is also assumed that no isolated edges are generated.

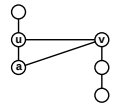
H17:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H17-00: | | | | | : | 3 | 1 | 1 | 0 | 0 | 0 | 2 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

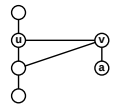
H18:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H18-00: | $V_2$ | | | | : | 2 | 2 | 2 | 0 | 0 | 0 | 2 |
| H18-01: | $V_3$ | | | | : | 3 | 1 | 2 | 0 | 0 | 1 | 2 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

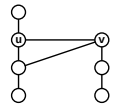
H19:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H19-00: | $V_1$ | | | | : | 3 | 0 | 4 | 0 | 0 | 0 | 2 |
| H19-01: | $V_2$ | | | | : | 3 | 1 | 3 | 0 | 0 | 1 | 2 |
| H19-02: | $V_3$ | | | | : | 4 | 0 | 3 | 0 | 1 | 0 | 2 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

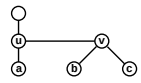
H20:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H20-00: | | | | | : | 3 | 1 | 3 | 0 | 0 | 0 | 3 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

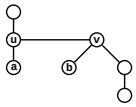
H21:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H21-00: | $V_1$ | $V_1$ | $V_1$ | | : | 2 | 0 | 4 | 0 | 0 | 0 | 1 |
| H21-01: | $V_1$ | $V_1$ | $V_2$ | | : | 2 | 1 | 3 | 0 | 0 | 1 | 1 |
| H21-02: | $V_1$ | $V_1$ | $V_3$ | | : | 3 | 0 | 3 | 0 | 1 | 0 | 1 |
| H21-03: | $V_1$ | $V_2$ | $V_2$ | | : | 2 | 2 | 2 | 0 | 0 | 2 | 1 |
| H21-04: | $V_1$ | $V_2$ | $V_3$ | | : | 3 | 1 | 2 | 0 | 1 | 1 | 1 |
| H21-05: | $V_1$ | $V_3$ | $V_3$ | | : | 4 | 0 | 2 | 0 | 2 | 0 | 1 |
| H21-06: | $V_2$ | $V_1$ | $V_1$ | | : | 2 | 1 | 3 | 0 | 0 | 1 | 1 |
| H21-07: | $V_2$ | $V_1$ | $V_2$ | | : | 2 | 2 | 2 | 0 | 0 | 2 | 1 |
| H21-08: | $V_2$ | $V_1$ | $V_3$ | | : | 3 | 1 | 2 | 0 | 1 | 1 | 1 |
| H21-09: | $V_2$ | $V_2$ | $V_2$ | | : | 2 | 3 | 1 | 0 | 0 | 3 | 1 |
| H21-10: | $V_2$ | $V_2$ | $V_3$ | | : | 3 | 2 | 1 | 0 | 1 | 2 | 1 |
| H21-11: | $V_2$ | $V_3$ | $V_3$ | | : | 4 | 1 | 1 | 0 | 2 | 1 | 1 |
| H21-12: | $V_3$ | $V_1$ | $V_1$ | | : | 3 | 0 | 3 | 0 | 1 | 0 | 1 |
| H21-13: | $V_3$ | $V_1$ | $V_2$ | | : | 3 | 1 | 2 | 0 | 1 | 1 | 1 |
| H21-14: | $V_3$ | $V_1$ | $V_3$ | | : | 4 | 0 | 2 | 0 | 2 | 0 | 1 |
| H21-15: | $V_3$ | $V_2$ | $V_2$ | | : | 3 | 2 | 1 | 0 | 1 | 2 | 1 |
| H21-16: | $V_3$ | $V_2$ | $V_3$ | | : | 4 | 1 | 1 | 0 | 2 | 1 | 1 |
| H21-17: | $V_3$ | $V_3$ | $V_3$ | | : | 5 | 0 | 1 | 0 | 3 | 0 | 1 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. It is also assumed that no isolated edges are generated.

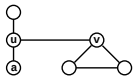
H22:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H22-00: | $V_1$ | $V_1$ | | | : | 2 | 1 | 4 | 0 | 0 | 0 | 2 |
| H22-01: | $V_1$ | $V_2$ | | | : | 2 | 2 | 3 | 0 | 0 | 1 | 2 |
| H22-02: | $V_1$ | $V_3$ | | | : | 3 | 1 | 3 | 0 | 1 | 0 | 2 |
| H22-03: | $V_2$ | $V_2$ | | | : | 2 | 3 | 2 | 0 | 0 | 2 | 2 |
| H22-04: | $V_2$ | $V_3$ | | | : | 3 | 2 | 2 | 0 | 1 | 1 | 2 |
| H22-05: | $V_3$ | $V_3$ | | | : | 4 | 1 | 2 | 0 | 2 | 0 | 2 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

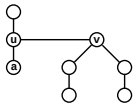
H23:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H23-00: | $V_1$ | | | | : | 2 | 2 | 2 | 0 | 0 | 0 | 2 |
| H23-01: | $V_2$ | | | | : | 2 | 3 | 1 | 0 | 0 | 1 | 2 |
| H23-02: | $V_3$ | | | | : | 3 | 2 | 1 | 0 | 1 | 0 | 2 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

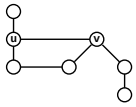
H24:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H24-00: | $V_1$ | | | | : | 2 | 2 | 4 | 0 | 0 | 0 | 3 |
| H24-01: | $V_2$ | | | | : | 2 | 3 | 3 | 0 | 0 | 1 | 3 |
| H24-02: | $V_3$ | | | | : | 3 | 2 | 3 | 0 | 1 | 0 | 3 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

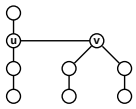
H25:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H25-00: | | | | | : | 2 | 3 | 2 | 0 | 0 | 0 | 3 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

H26:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H26-00: | | | | | : | 2 | 3 | 4 | 0 | 0 | 0 | 4 |

```
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 \  MAPLE  /  reserved. Maple and Maple V are registered trademarks of
 <____ ____>  Waterloo Maple Inc.
      |       Type ? for help.
> with(simplex):
Warning, new definition for maximize
Warning, new definition for minimize
>
> obj:= 1*E01_00 +1*E01_01 +1*E02_00 +1*E02_01 +2*E03_00 +1*E04_00
+1*E04_01 +1*E04_02 +1*E04_03 +1*E05_00 +1*E05_01 +1*E05_02 +1*E05_03
+1*E05_04 +1*E06_00 +1*E06_01 +1*E07_00 +1*E07_01 +1*E07_02 +1*E07_03
+1*E07_04 +1*E08_00 +1*E08_01 +1*E08_02 +1*E08_03 +1*E08_04 +1*H01_00
+1*H02_00 +1*H02_01 +1*H02_02 +2*H03_00 +1*H04_00 +1*H04_01 +1*H04_02
+1*H04_03 +1*H04_04 +1*H04_05 +2*H05_00 +2*H06_00 +2*H06_01 +2*H06_02
+3*H07_00 +1*H08_00 +1*H08_01 +1*H08_02 +2*H09_00 +1*H10_00 +1*H10_01
+2*H11_00 +1*H12_00 +1*H12_01 +1*H12_02 +1*H12_03 +1*H12_04 +1*H12_05
+2*H13_00 +2*H14_00 +2*H14_01 +2*H14_02 +3*H15_00 +1*H16_00 +1*H16_01
+1*H16_02 +1*H16_03 +1*H16_04 +1*H16_05 +2*H17_00 +2*H18_00 +2*H18_01
+2*H19_00 +2*H19_01 +2*H19_02 +3*H20_00 +1*H21_00 +1*H21_01 +1*H21_02
+1*H21_03 +1*H21_04 +1*H21_05 +1*H21_06 +1*H21_07 +1*H21_08 +1*H21_09
+1*H21_10 +1*H21_11 +1*H21_12 +1*H21_13 +1*H21_14 +1*H21_15 +1*H21_16
+1*H21_17 +2*H22_00 +2*H22_01 +2*H22_02 +2*H22_03 +2*H22_04 +2*H22_05
+2*H23_00 +2*H23_01 +2*H23_02 +3*H24_00 +3*H24_01 +3*H24_02 +3*H25_00
+4*H26_00;
>
obj := E01_01 + E01_00 + E02_01 + E02_00 + 2 E03_00 + E04_00 + E04_01 + E04_02
     + E04_03 + E05_00 + E05_01 + E05_02 + E05_03 + E05_04 + E06_00 + E06_01
     + E07_00 + E07_01 + E07_02 + E07_03 + E07_04 + E08_00 + E08_01 + E08_02
     + E08_03 + E08_04 + H01_00 + H02_00 + H02_01 + H02_02 + 2 H03_00 + H04_00
     + H04_01 + H04_02 + H04_03 + H04_04 + H04_05 + 2 H05_00 + 2 H06_00
     + 2 H06_01 + 2 H06_02 + 3 H07_00 + H08_00 + H08_01 + H08_02 + 2 H09_00
     + H10_00 + H10_01 + 2 H11_00 + H12_00 + H12_01 + H12_02 + H12_03 + H12_04
     + H12_05 + 2 H13_00 + 2 H14_00 + 2 H14_01 + 2 H14_02 + 3 H15_00 + H16_00
     + H16_01 + H16_02 + H16_03 + H16_04 + H16_05 + 2 H17_00 + 2 H18_00
     + 2 H18_01 + 2 H19_00 + 2 H19_01 + 2 H19_02 + 3 H20_00 + H21_00 + H21_01
     + H21_02 + H21_03 + H21_04 + H21_05 + H21_06 + H21_07 + H21_08 + H21_09
     + H21_10 + H21_11 + H21_12 + H21_13 + H21_14 + H21_15 + H21_16 + H21_17
     + 2 H22_00 + 2 H22_01 + 2 H22_02 + 2 H22_03 + 2 H22_04 + 2 H22_05
     + 2 H23_00 + 2 H23_01 + 2 H23_02 + 3 H24_00 + 3 H24_01 + 3 H24_02
     + 3 H25_00 + 4 H26_00
>
> cnsts:={
>  +1*E02_00 +1*E04_02 +2*E05_00 +1*E05_01 +1*E05_03 +1*E07_00
+1*E07_04 +3*E08_00 +2*E08_01 +1*E08_02 +1*E08_03 -1*H01_00 -1*H02_00
-1*H03_00 -2*H04_00 -1*H04_01 -1*H04_02 +1*H04_03 -2*H06_00 -1*H06_01
-1*H06_02 -2*H07_00 -2*H08_00 -1*H08_01 -1*H08_02 -2*H09_00 -1*H11_00
-2*H12_00 -1*H12_01 -1*H12_02 +1*H12_03 -2*H14_00 -1*H14_01 -1*H14_02
-2*H15_00 -1*H16_00 -1*H16_03 +1*H16_04 -1*H18_00 -1*H18_01 -3*H19_00
-2*H19_01 -2*H19_02 -2*H20_00 -3*H21_00 -2*H21_01 -2*H21_02 -1*H21_04
-1*H21_05 -2*H21_06 -1*H21_08 +2*H21_09 +1*H21_10 -2*H21_12 -1*H21_13
-1*H21_14 +1*H21_15 -3*H22_00 -2*H22_01 -2*H22_02 -1*H22_04 -1*H22_05
-1*H23_00 -3*H24_00 -2*H24_01 -2*H24_02 -1*H25_00 -3*H26_00 >= 0,
>
>  +1*E05_00 +2*E08_00 +1*E08_01 -1*H04_00 +1*H04_03 -1*H06_00
-1*H07_00 -1*H08_00 -1*H09_00 -1*H12_00 +1*H12_03 -1*H14_00 -1*H15_00
+1*H16_04 -2*H19_00 -1*H19_01 -1*H19_02 -1*H20_00 -2*H21_00 -1*H21_01
-1*H21_02 -1*H21_06 +2*H21_09 +1*H21_10 -1*H21_12 +1*H21_15 -2*H22_00
-1*H22_01 -1*H22_02 -2*H24_00 -1*H24_01 -1*H24_02 -2*H26_00 >= 0,
```

```
>
>  +1*E08_00 +1*H04_03 +1*H12_03 +1*H16_04 -1*H19_00 -1*H21_00
+2*H21_09 +1*H21_10 +1*H21_15 -1*H22_00 -1*H24_00 -1*H26_00 >= 0,
>
>  +1*E01_01 +2*E02_00 +1*E02_01 +1*E04_00 +2*E04_02 +1*E04_03
+3*E05_00 +2*E05_01 +1*E05_02 +2*E05_03 +1*E05_04 +1*E06_01 +2*E07_00
+1*E07_01 +1*E07_02 +2*E07_04 +4*E08_00 +3*E08_01 +2*E08_02 +2*E08_03
+1*E08_04 -2*H01_00 -2*H02_00 -1*H02_02 -2*H03_00 -3*H04_00 -1*H04_01
-2*H04_02 +1*H04_03 -1*H04_05 -1*H05_00 -3*H06_00 -1*H06_01 -2*H06_02
-3*H07_00 -3*H08_00 -1*H08_01 -2*H08_02 -3*H09_00 -1*H10_00 -2*H11_00
-3*H12_00 -1*H12_01 -2*H12_02 +1*H12_03 -1*H12_05 -1*H13_00 -3*H14_00
-1*H14_01 -2*H14_02 -3*H15_00 -2*H16_00 -1*H16_02 -1*H16_03 +1*H16_04
-1*H17_00 -2*H18_00 -1*H18_01 -4*H19_00 -2*H19_01 -3*H19_02 -3*H20_00
-4*H21_00 -2*H21_01 -3*H21_02 -1*H21_04 -2*H21_05 -2*H21_06 -1*H21_08
+2*H21_09 +1*H21_10 -3*H21_12 -1*H21_13 -2*H21_14 +1*H21_15 -1*H21_17
-4*H22_00 -2*H22_01 -3*H22_02 -1*H22_04 -2*H22_05 -2*H23_00 -1*H23_02
-4*H24_00 -2*H24_01 -3*H24_02 -2*H25_00 -4*H26_00 >= 0,
>
> -1*E02_00 -1*E04_02 -1*E05_00 -1*E05_01 -1*E05_03 -1*E07_00
-1*E07_04 -2*E08_00 -1*E08_01 -1*E08_02 -1*E08_03 +1*H01_00 +1*H02_00
+1*H03_00 +1*H04_00 +1*H04_02 -1*H04_03 +1*H06_00 +1*H06_02 +1*H07_00
+1*H08_00 +1*H08_02 +1*H09_00 +1*H11_00 +1*H12_00 +1*H12_02 -1*H12_03
+1*H14_00 +1*H14_02 +1*H15_00 +1*H16_00 -1*H16_04 +1*H18_00 +2*H19_00
+1*H19_01 +1*H19_02 +1*H20_00 +2*H21_00 +1*H21_01 +1*H21_02 +1*H21_05
+1*H21_06 -1*H21_09 -1*H21_10 +1*H21_12 +1*H21_14 -1*H21_15 +2*H22_00
+1*H22_01 +1*H22_02 +1*H22_05 +1*H23_00 +2*H24_00 +1*H24_01 +1*H24_02
+1*H25_00 +2*H26_00  <= 0,
>
> -1*E05_00 -1*E08_00 -1*E08_01 +1*H04_00 -1*H04_03 +1*H06_00
+1*H07_00 +1*H08_00 +1*H09_00 +1*H12_00 -1*H12_03 +1*H14_00 +1*H15_00
-1*H16_04 +1*H19_00 +1*H19_02 +1*H20_00 +1*H21_00 +1*H21_02 -1*H21_09
-1*H21_10 +1*H21_12 -1*H21_15 +1*H22_00 +1*H22_02 +1*H24_00 +1*H24_02
+1*H26_00  <= 0,
>
> -1*E08_00 -1*H04_03 -1*H12_03 -1*H16_04 +1*H19_00 +1*H21_00
-1*H21_09 -1*H21_10 -1*H21_15 +1*H22_00 +1*H24_00 +1*H26_00  <= 0,
>
> -1*E01_01 -1*E02_01 -1*E04_00 -2*E04_01 -2*E04_02 -3*E04_03
-1*E05_00 -2*E05_01 -3*E05_02 -2*E05_03 -3*E05_04 -2*E06_00 -3*E06_01
-2*E07_00 -3*E07_01 -3*E07_02 -4*E07_03 -4*E07_04 -2*E08_00 -3*E08_01
-4*E08_02 -4*E08_03 -5*E08_04 -1*H02_02 -1*H04_00 -1*H04_01 -2*H04_02
-1*H04_03 -2*H04_04 -3*H04_05 -1*H05_00 -1*H06_00 -1*H06_01 -2*H06_02
-1*H07_00 -1*H08_00 -1*H08_01 -2*H08_02 -1*H09_00 -1*H10_00 -2*H10_01
-2*H11_00 -1*H12_00 -1*H12_01 -2*H12_02 -1*H12_03 -2*H12_04 -3*H12_05
-1*H13_00 -1*H14_00 -1*H14_01 -2*H14_02 -1*H15_00 -2*H16_00 -2*H16_01
-3*H16_02 -3*H16_03 -3*H16_04 -4*H16_05 -3*H17_00 -2*H18_00 -3*H18_01
-3*H19_00 -3*H19_01 -4*H19_02 -3*H20_00 -2*H21_00 -2*H21_01 -3*H21_02
-2*H21_03 -3*H21_04 -4*H21_05 -2*H21_06 -2*H21_07 -3*H21_08 -2*H21_09
-3*H21_10 -4*H21_11 -3*H21_12 -3*H21_13 -4*H21_14 -3*H21_15 -4*H21_16
-5*H21_17 -2*H22_00 -2*H22_01 -3*H22_02 -2*H22_03 -3*H22_04 -4*H22_05
-2*H23_00 -2*H23_01 -3*H23_02 -2*H24_00 -2*H24_01 -3*H24_02 -2*H25_00
-2*H26_00  >= -1,
>
> -3*E01_00 -2*E01_01 -4*E02_00 -2*E02_01 -4*E03_00 -3*E04_00
-1*E04_01 -2*E04_02 -4*E05_00 -2*E05_01 -2*E05_03 -2*E06_00 -1*E06_01
-3*E07_00 -1*E07_01 -1*E07_02 +1*E07_03 -4*E08_00 -2*E08_01 +2*E08_04
-1*H01_00 -2*H02_00 -3*H02_01 -1*H02_02 -3*H03_00 -1*H04_00 -2*H04_01
-3*H04_03 -1*H04_04 +1*H04_05 -3*H05_00 -2*H06_00 -3*H06_01 -1*H06_02
-3*H07_00 -1*H08_01 +1*H08_02 -1*H09_00 -2*H10_00 -1*H10_01 -1*H11_00
```

```
-2*H12_00 -3*H12_01 -1*H12_02 -4*H12_03 -2*H12_04 -3*H13_00 -2*H14_00
-3*H14_01 -1*H14_02 -3*H15_00 -1*H16_00 -2*H16_01 -1*H16_04 +1*H16_05
-1*H17_00 -2*H18_00 -1*H18_01 -1*H19_01 +1*H19_02 -1*H20_00 -1*H21_01
+1*H21_02 -2*H21_03 +2*H21_05 -1*H21_06 -2*H21_07 -3*H21_09 -1*H21_10
+1*H21_11 +1*H21_12 +2*H21_14 -1*H21_15 +1*H21_16 +3*H21_17 -1*H22_00
-2*H22_01 -3*H22_03 -1*H22_04 +1*H22_05 -2*H23_00 -3*H23_01 -1*H23_02
-2*H24_00 -3*H24_01 -1*H24_02 -3*H25_00 -3*H26_00  >= 0,
>
> +1*E01_01 +2*E02_00 +1*E02_01 +1*E04_00 +2*E04_02 +1*E04_03
+3*E05_00 +2*E05_01 +1*E05_02 +2*E05_03 +1*E05_04 +1*E06_01 +2*E07_00
+1*E07_01 +1*E07_02 +2*E07_04 +4*E08_00 +3*E08_01 +2*E08_02 +2*E08_03
+1*E08_04 -2*H01_00 -2*H02_00 -1*H02_02 -2*H03_00 -3*H04_00 -1*H04_01
-2*H04_02 +1*H04_03 -1*H04_05 -1*H05_00 -3*H06_00 -1*H06_01 -2*H06_02
-3*H07_00 -3*H08_00 -1*H08_01 -2*H08_02 -3*H09_00 -1*H10_00 -2*H11_00
-3*H12_00 -1*H12_01 -2*H12_02 +1*H12_03 -1*H12_05 -1*H13_00 -3*H14_00
-1*H14_01 -2*H14_02 -3*H15_00 -2*H16_00 -1*H16_02 -1*H16_03 +1*H16_04
-1*H17_00 -2*H18_00 -1*H18_01 -4*H19_00 -2*H19_01 -3*H19_02 -3*H20_00
-4*H21_00 -2*H21_01 -3*H21_02 -1*H21_04 -2*H21_05 -2*H21_06 -1*H21_08
+2*H21_09 +1*H21_10 -3*H21_12 -1*H21_13 -2*H21_14 +1*H21_15 -1*H21_17
-4*H22_00 -2*H22_01 -3*H22_02 -1*H22_04 -2*H22_05 -2*H23_00 -1*H23_02
-4*H24_00 -2*H24_01 -3*H24_02 -2*H25_00 -4*H26_00  >= 0};
>
cnsts := {0 <= E02_00 + E04_02 + 2 E05_00 + E05_01 + E05_03 + E07_00 + E07_04
     + 3 E08_00 + 2 E08_01 + E08_02 + E08_03 - H01_00 - H02_00 - H03_00
     - 2 H04_00 - H04_01 - H04_02 + H04_03 - 2 H06_00 - H06_01 - H06_02
     - 2 H07_00 - 2 H08_00 - H08_01 - H08_02 - 2 H09_00 - H11_00 - 2 H12_00
     - H12_01 - H12_02 + H12_03 - 2 H14_00 - H14_01 - H14_02 - 2 H15_00
     - H16_00 - H16_03 + H16_04 - H18_00 - H18_01 - 3 H19_00 - 2 H19_01
     - 2 H19_02 - 2 H20_00 - 3 H21_00 - 2 H21_01 - 2 H21_02 - H21_04 - H21_05
     - 2 H21_06 - H21_08 + 2 H21_09 + H21_10 - 2 H21_12 - H21_13 - H21_14
     + H21_15 - 3 H22_00 - 2 H22_01 - 2 H22_02 - H22_04 - H22_05 - H23_00
     - 3 H24_00 - 2 H24_01 - 2 H24_02 - H25_00 - 3 H26_00, 0 <= E08_00 + H04_03
     + H12_03 + H16_04 - H19_00 - H21_00 + 2 H21_09 + H21_10 + H21_15 - H22_00
     - H24_00 - H26_00, 0 <= E05_00 + 2 E08_00 + E08_01 - H04_00 + H04_03
     - H06_00 - H07_00 - H08_00 - H09_00 - H12_00 + H12_03 - H14_00 - H15_00
     + H16_04 - 2 H19_00 - H19_01 - H19_02 - H20_00 - 2 H21_00 - H21_01
     - H21_02 - H21_06 + 2 H21_09 + H21_10 - H21_12 + H21_15 - 2 H22_00
     - H22_01 - H22_02 - 2 H24_00 - H24_01 - H24_02 - 2 H26_00, -E05_00
     - E08_00 - E08_01 + H04_00 - H04_03 + H06_00 + H07_00 + H08_00 + H09_00
     + H12_00 - H12_03 + H14_00 + H15_00 - H16_04 + H19_00 + H19_02 + H20_00
     + H21_00 + H21_02 - H21_09 - H21_10 + H21_12 - H21_15 + H22_00 + H22_02
     + H24_00 + H24_02 + H26_00 <= 0, -E02_00 - E04_02 - E05_00 - E05_01
     - E05_03 - E07_00 - E07_04 - 2 E08_00 - E08_01 - E08_02 - E08_03 + H01_00
     + H02_00 + H03_00 + H04_00 + H04_02 - H04_03 + H06_00 + H06_02 + H07_00
     + H08_00 + H08_02 + H09_00 + H11_00 + H12_00 + H12_02 - H12_03 + H14_00
     + H14_02 + H15_00 + H16_00 - H16_04 + H18_00 + 2 H19_00 + H19_01 + H19_02
     + H20_00 + 2 H21_00 + H21_01 + H21_02 + H21_05 + H21_06 - H21_09 - H21_10
     + H21_12 + H21_14 - H21_15 + 2 H22_00 + H22_01 + H22_02 + H22_05 + H23_00
     + 2 H24_00 + H24_01 + H24_02 + H25_00 + 2 H26_00 <= 0, -E08_00 - H04_03
     - H12_03 - H16_04 + H19_00 + H21_00 - H21_09 - H21_10 - H21_15 + H22_00
     + H24_00 + H26_00 <= 0, 0 <= E01_01 + E02_01 + 2 E02_00 + E04_00
     + 2 E04_02 + E04_03 + 3 E05_00 + 2 E05_01 + E05_02 + 2 E05_03 + E05_04
     + E06_01 + 2 E07_00 + E07_01 + E07_02 + 2 E07_04 + 4 E08_00 + 3 E08_01
     + 2 E08_02 + 2 E08_03 + E08_04 - 2 H01_00 - 2 H02_00 - H02_02 - 2 H03_00
     - 3 H04_00 - H04_01 - 2 H04_02 + H04_03 - H04_05 - H05_00 - 3 H06_00
     - H06_01 - 2 H06_02 - 3 H07_00 - 3 H08_00 - H08_01 - 2 H08_02 - 3 H09_00
     - H10_00 - 2 H11_00 - 3 H12_00 - H12_01 - 2 H12_02 + H12_03 - H12_05
     - H13_00 - 3 H14_00 - H14_01 - 2 H14_02 - 3 H15_00 - 2 H16_00 - H16_02
     - H16_03 + H16_04 - H17_00 - 2 H18_00 - H18_01 - 4 H19_00 - 2 H19_01
```

```
      - 3 H19_02 - 3 H20_00 - 4 H21_00 - 2 H21_01 - 3 H21_02 - H21_04 - 2 H21_05
      - 2 H21_06 - H21_08 + 2 H21_09 + H21_10 - 3 H21_12 - H21_13 - 2 H21_14
      + H21_15 - H21_17 - 4 H22_00 - 2 H22_01 - 3 H22_02 - H22_04 - 2 H22_05
      - 2 H23_00 - H23_02 - 4 H24_00 - 2 H24_01 - 3 H24_02 - 2 H25_00 - 4 H26_00
     , -1 <= -E01_01 - E02_01 - E04_00 - 2 E04_01 - 2 E04_02 - 3 E04_03 - E05_00
      - 2 E05_01 - 3 E05_02 - 2 E05_03 - 3 E05_04 - 2 E06_00 - 3 E06_01
      - 2 E07_00 - 3 E07_01 - 3 E07_02 - 4 E07_03 - 4 E07_04 - 2 E08_00
      - 3 E08_01 - 4 E08_02 - 4 E08_03 - 5 E08_04 - H02_02 - H04_00 - H04_01
      - 2 H04_02 - H04_03 - 2 H04_04 - 3 H04_05 - H05_00 - H06_00 - H06_01
      - 2 H06_02 - H07_00 - H08_00 - H08_01 - 2 H08_02 - H09_00 - H10_00
      - 2 H10_01 - 2 H11_00 - H12_00 - H12_01 - 2 H12_02 - H12_03 - 2 H12_04
      - 3 H12_05 - H13_00 - H14_00 - H14_01 - 2 H14_02 - H15_00 - 2 H16_00
      - 2 H16_01 - 3 H16_02 - 3 H16_03 - 3 H16_04 - 4 H16_05 - 3 H17_00
      - 2 H18_00 - 3 H18_01 - 3 H19_00 - 3 H19_01 - 4 H19_02 - 3 H20_00
      - 2 H21_00 - 2 H21_01 - 3 H21_02 - 2 H21_03 - 3 H21_04 - 4 H21_05
      - 2 H21_06 - 2 H21_07 - 3 H21_08 - 2 H21_09 - 3 H21_10 - 4 H21_11
      - 3 H21_12 - 3 H21_13 - 4 H21_14 - 3 H21_15 - 4 H21_16 - 5 H21_17
      - 2 H22_00 - 2 H22_01 - 3 H22_02 - 2 H22_03 - 3 H22_04 - 4 H22_05
      - 2 H23_00 - 2 H23_01 - 3 H23_02 - 2 H24_00 - 2 H24_01 - 3 H24_02
      - 2 H25_00 - 2 H26_00, 0 <= -2 E01_01 - 3 E01_00 - 2 E02_01 - 4 E02_00
      - 4 E03_00 - 3 E04_00 - E04_01 - 2 E04_02 - 4 E05_00 - 2 E05_01 - 2 E05_03
      - 2 E06_00 - E06_01 - 3 E07_00 - E07_01 - E07_02 + E07_03 - 4 E08_00
      - 2 E08_01 + 2 E08_04 - H01_00 - 2 H02_00 - 3 H02_01 - H02_02 - 3 H03_00
      - H04_00 - 2 H04_01 - 3 H04_03 - H04_04 + H04_05 - 3 H05_00 - 2 H06_00
      - 3 H06_01 - H06_02 - 3 H07_00 - H08_01 + H08_02 - H09_00 - 2 H10_00
      - H10_01 - H11_00 - 2 H12_00 - 3 H12_01 - H12_02 - 4 H12_03 - 2 H12_04
      - 3 H13_00 - 2 H14_00 - 3 H14_01 - H14_02 - 3 H15_00 - H16_00 - 2 H16_01
      - H16_04 + H16_05 - H17_00 - 2 H18_00 - H18_01 - H19_01 + H19_02 - H20_00
      - H21_01 + H21_02 - 2 H21_03 + 2 H21_05 - H21_06 - 2 H21_07 - 3 H21_09
      - H21_10 + H21_11 + H21_12 + 2 H21_14 - H21_15 + H21_16 + 3 H21_17
      - H22_00 - 2 H22_01 - 3 H22_03 - H22_04 + H22_05 - 2 H23_00 - 3 H23_01
      - H23_02 - 2 H24_00 - 3 H24_01 - H24_02 - 3 H25_00 - 3 H26_00}
>
> maximize(obj,cnsts,NONNEGATIVE);
{H21_15 = 0, H21_16 = 0, H22_00 = 0, H22_01 = 0, H22_02 = 0, H22_03 = 0,
    H22_04 = 0, H22_05 = 0, H23_00 = 0, H23_01 = 0, H23_02 = 0, H24_00 = 0,
    H24_01 = 0, H24_02 = 0, H25_00 = 0, H26_00 = 0, H20_00 = 0, H21_17 = 1/10,
    E07_02 = 0, E08_00 = 0, E08_01 = 0, E08_02 = 0, E08_03 = 0, H01_00 = 0,
    H02_00 = 0, H02_01 = 0, H02_02 = 0, H03_00 = 0, H04_00 = 0, H04_01 = 0,
    H04_02 = 0, H04_03 = 0, H04_04 = 0, H04_05 = 0, H05_00 = 0, H06_00 = 0,
    H06_01 = 0, H06_02 = 0, H07_00 = 0, H08_00 = 0, H08_01 = 0, H08_02 = 0,
    H09_00 = 0, H10_00 = 0, H10_01 = 0, H11_00 = 0, H12_00 = 0, H12_01 = 0,
    H12_02 = 0, H12_03 = 0, H12_04 = 0, H12_05 = 0, H13_00 = 0, H14_00 = 0,
    H14_01 = 0, H14_02 = 0, H15_00 = 0, H16_00 = 0, H16_01 = 0, H16_02 = 0,
    H16_03 = 0, H16_04 = 0, H16_05 = 0, H17_00 = 0, H18_00 = 0, H18_01 = 0,
    H19_00 = 0, H19_01 = 0, H19_02 = 0, H21_00 = 0, H21_01 = 0, H21_02 = 0,
    H21_03 = 0, H21_04 = 0, H21_05 = 0, H21_06 = 0, H21_07 = 0, H21_08 = 0,
    H21_09 = 0, H21_10 = 0, H21_11 = 0, H21_12 = 0, H21_13 = 0, H21_14 = 0,
    E08_04 = 1/10, E03_00 = 1/8, E01_01 = 0, E01_00 = 0, E02_01 = 0, E02_00 = 0,
    E04_00 = 0, E04_01 = 0, E04_02 = 0, E04_03 = 0, E05_00 = 0, E05_01 = 0,
    E05_02 = 0, E05_03 = 0, E05_04 = 0, E06_00 = 0, E06_01 = 0, E07_00 = 0,
    E07_01 = 0, E07_03 = 0, E07_04 = 0}
>
> subs(%,obj);
```



```
>
> (dualobj,dualcnsts):=dual(obj,cnsts,y);
dualobj, dualcnsts := y5, {1 <= 3 y5 - y7,
    1 <= -y1 - y2 + 4 y4 + y5 - 2 y6 - 3 y7 - y9, 1 <= -y2 + 4 y4 - y6 - 2 y7,
```

```
    2 <= 4 y4, 1 <= 3 y4 + y5 - y7, 1 <= y4 + 2 y5,
    1 <= -y2 + 2 y4 + 2 y5 - y6 - 2 y7, 1 <= 2 y4 + y5 - y7, 1 <= 3 y4,
    1 <= y2 - y4 + 2 y5 + y6 + 2 y7, 2 <= y2 + y4 + 2 y5 + y6 + 2 y7,
    3 <= y1 + y2 + 3 y4 + y5 + 2 y6 + 3 y7 + y9,
    1 <= y1 + y2 + y5 + 2 y6 + 3 y7 + y9, 1 <= y4 + y5 + y6 + y7,
    1 <= -y4 + 3 y5 + y7, 2 <= 3 y4 + y5 + y7,
    2 <= y1 + y2 + 2 y4 + y5 + 2 y6 + 3 y7 + y9, 2 <= 3 y4 + y5 + y6 + y7,
    1 <= 2 y4 + y5 + y6 + y7, 1 <= y2 + 2 y5 + y6 + 2 y7,
    1 <= -y1 - y2 - y3 + 3 y4 + y5 - y6 - y7 - y8 - y9, 1 <= y4 + y5 + y7,
    2 <= y2 + 3 y4 + y6 + 2 y7, 1 <= y1 + y2 + y4 + y5 + 2 y6 + 3 y7 + y9,
    1 <= -2 y4 + 5 y5 - y7, 1 <= y2 + y4 + y6 + 2 y7,
    1 <= y2 + 2 y4 + y6 + 2 y7, 1 <= -y1 - y2 + 2 y4 + 3 y5 - 2 y6 - 3 y7 - y9,
    1 <= -y1 - 2 y2 - y3 + 4 y4 + 2 y5 - 3 y6 - 4 y7 - y8 - 2 y9,
    1 <= -y4 + 4 y5, 1 <= -y2 + 4 y5 - y6 - 2 y7, 1 <= 2 y4 + 2 y5,
    1 <= y4 + 3 y5 - y7, 1 <= -y2 + 3 y4 + 2 y5 - y6 - 2 y7,
    1 <= y1 + y2 - y4 + 3 y5 + 2 y6 + 3 y7 + y9,
    1 <= y2 - 2 y4 + 4 y5 + y6 + 2 y7,
    2 <= y1 + y2 - y4 + 4 y5 + 2 y6 + 3 y7 + y9,
    3 <= y1 + y2 + y4 + 3 y5 + 2 y6 + 3 y7 + y9,
    1 <= y1 + 2 y2 + y3 + 2 y5 + 3 y6 + 4 y7 + y8 + 2 y9,
    1 <= y2 + y4 + 2 y5 + 2 y6 + 2 y7 + y9,
    2 <= y1 + 2 y2 + y3 + 3 y5 + 3 y6 + 4 y7 + y8 + 2 y9,
    2 <= y2 + y4 + 3 y5 + 2 y6 + 2 y7 + y9, 2 <= y4 + 3 y5 + y7,
    2 <= y2 + 2 y4 + 2 y5 + y6 + 2 y7, 2 <= y4 + 3 y5 + y6 + y7,
    1 <= 3 y5 + y6 + y7, 1 <= -y1 - y2 - y3 + y4 + 3 y5 - y6 - y7 - y8 - y9,
    1 <= 3 y5 + y7, 1 <= y2 + y4 + 2 y5 + y6 + 2 y7,
    1 <= -y1 - y2 - y3 + 4 y4 + y5 - y6 - y7 - y8 - y9,
    1 <= y1 + y2 + 2 y4 + y5 + 2 y6 + 3 y7 + y9, 1 <= 3 y4 + y5 + y6 + y7,
    2 <= y1 + y2 + y4 + y5 + 2 y6 + 3 y7 + y9, 1 <= 2 y4 + y5 + y7,
    3 <= y2 + 3 y4 + 2 y5 + y6 + 2 y7,
    4 <= y1 + 2 y2 + y3 + 3 y4 + 2 y5 + 3 y6 + 4 y7 + y8 + 2 y9,
    3 <= y1 + 2 y2 + y3 + 2 y4 + 2 y5 + 3 y6 + 4 y7 + y8 + 2 y9,
    3 <= y2 + 3 y4 + 2 y5 + 2 y6 + 2 y7 + y9, 2 <= 3 y4 + 2 y5,
    2 <= y2 - y4 + 4 y5 + y6 + 2 y7, 1 <= -3 y4 + 5 y5 + y7,
    2 <= y1 + 2 y2 + y3 + y4 + 2 y5 + 3 y6 + 4 y7 + y8 + 2 y9,
    2 <= y2 + 2 y4 + 2 y5 + 2 y6 + 2 y7 + y9,
    2 <= y1 + y2 + 3 y5 + 2 y6 + 3 y7 + y9,
    1 <= -y1 - y2 - y3 + 3 y4 + 2 y5 - 2 y6 - 2 y7 - 2 y8 - 2 y9}
>
> minimize(dualobj,dualcnsts,NONNEGATIVE);
bytes used=3001396, alloc=1179432, time=0.55
bytes used=4088808, alloc=1376004, time=0.82
bytes used=5089068, alloc=1376004, time=1.04
{y4 = 1/2, y3 = 0, y6 = 0, y8 = 0, y1 = 0, y7 = 1/4, y2 = 1/5, y9 = 1/5,

    y5 = 9/20}
>
> quit
```

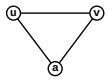
# MMM girth 5

In all operations considered, the edge $uv$ is the one that could possibly be selected by "some" algorithm for inclusion in the matching. The priorities are

- edges with an end-point that has a neighbour of degree 1 over
- edges with an end-point that has a neighbour of degree 2

In each case, choose the edge that, if added to the set, would give the smallest ratio of edges added to the set (which includes the edge $uv$ itself and all isolated edges created) to edges removed from the graph. Where there are two edges with the same ratio, choose the edge with the fewest vertices neighbouring its end-points, otherwise, ties are broken arbitrarily. Operations written in red (with "***" in the last column) are excluded from $OPS_2$.

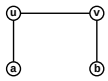
E01:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E01-00: | $V_2$ | | | | : | 0 | 3 | 0 | 0 | 0 | 0 | 1*** |
| E01-01: | $V_3$ | | | | : | 1 | 2 | 0 | 0 | 0 | 1 | 1*** |

Cycle of length smaller than 5.

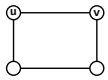
E02:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E02-00: | $V_2$ | $V_2$ | | | : | 0 | 4 | 0 | 0 | 0 | 2 | 1 |
| E02-01: | $V_2$ | $V_3$ | | | : | 1 | 3 | 0 | 0 | 1 | 1 | 1 |
| E02-02: | $V_3$ | $V_3$ | | | : | 2 | 2 | 0 | 0 | 2 | 0 | 1*** |

It is assumed here that the minimum degree is 2. It is also assumed that no isolated edges are generated. If the degree of both $a$ and $b$ is 3, there is always an edge that gives a ratio of less than 1/3.

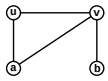
E03:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E03-00: | | | | | : | 2 | 2 | 0 | 0 | 2 | 0 | 2*** |

Cycle of length smaller than 5.

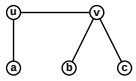
E04:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E04-00: | $V_2$ | $V_2$ | | | : | 1 | 3 | 0 | 0 | 0 | 1 | 1*** |
| E04-01: | $V_2$ | $V_3$ | | | : | 2 | 2 | 0 | 0 | 1 | 0 | 1*** |
| E04-02: | $V_3$ | $V_2$ | | | : | 2 | 2 | 0 | 0 | 0 | 2 | 1*** |
| E04-03: | $V_3$ | $V_3$ | | | : | 3 | 1 | 0 | 0 | 1 | 1 | 1*** |

Cycle of length smaller than 5.

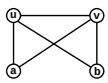
E05:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E05-00: | $V_2$ | $V_2$ | $V_2$ | | : | 1 | 4 | 0 | 0 | 0 | 3 | 1 |
| E05-01: | $V_2$ | $V_2$ | $V_3$ | | : | 2 | 3 | 0 | 0 | 1 | 2 | 1 |
| E05-02: | $V_2$ | $V_3$ | $V_3$ | | : | 3 | 2 | 0 | 0 | 2 | 1 | 1 |
| E05-03: | $V_3$ | $V_2$ | $V_2$ | | : | 2 | 3 | 0 | 0 | 1 | 2 | 1 |
| E05-04: | $V_3$ | $V_2$ | $V_3$ | | : | 3 | 2 | 0 | 0 | 2 | 1 | 1 |
| E05-05: | $V_3$ | $V_3$ | $V_3$ | | : | 4 | 1 | 0 | 0 | 3 | 0 | 1*** |

It is assumed here that the minimum degree is 2. If $a$, $b$ and $c$ have degree 3, there exists another edge with ratio at least 1/4, that has fewer vertices neighbouring its end-points. If any isolated edges were created, there would be another edge to select with smaller ratio.

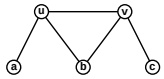
E06:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E06-00: | $V_2$ | $V_2$ | | | : | 2 | 2 | 0 | 0 | 0 | 0 | 1*** |
| E06-01: | $V_2$ | $V_3$ | | | : | 3 | 1 | 0 | 0 | 0 | 1 | 1*** |
| E06-02: | $V_3$ | $V_3$ | | | : | 4 | 0 | 0 | 0 | 0 | 2 | 1*** |

Cycle of length smaller than 5.

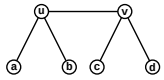
E07:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E07-00: | $V_2$ | $V_2$ | $V_2$ | | : | 2 | 3 | 0 | 0 | 0 | 2 | 1*** |
| E07-01: | $V_2$ | $V_2$ | $V_3$ | | : | 3 | 2 | 0 | 0 | 1 | 1 | 1*** |
| E07-02: | $V_2$ | $V_3$ | $V_2$ | | : | 3 | 2 | 0 | 0 | 1 | 1 | 1*** |
| E07-03: | $V_2$ | $V_3$ | $V_3$ | | : | 4 | 1 | 0 | 0 | 2 | 0 | 1*** |
| E07-04: | $V_3$ | $V_2$ | $V_3$ | | : | 4 | 1 | 0 | 0 | 1 | 2 | 1*** |
| E07-05: | $V_3$ | $V_3$ | $V_3$ | | : | 5 | 0 | 0 | 0 | 2 | 1 | 1*** |

Cycle of length smaller than 5.

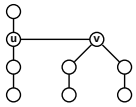
E08:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E08-00: | $V_2$ | $V_2$ | $V_2$ | $V_2$ | : | 2 | 4 | 0 | 0 | 0 | 4 | 1 |
| E08-01: | $V_2$ | $V_2$ | $V_2$ | $V_3$ | : | 3 | 3 | 0 | 0 | 1 | 3 | 1 |
| E08-02: | $V_2$ | $V_2$ | $V_3$ | $V_3$ | : | 4 | 2 | 0 | 0 | 2 | 2 | 1 |
| E08-03: | $V_2$ | $V_3$ | $V_2$ | $V_3$ | : | 4 | 2 | 0 | 0 | 2 | 2 | 1 |
| E08-04: | $V_2$ | $V_3$ | $V_3$ | $V_3$ | : | 5 | 1 | 0 | 0 | 3 | 1 | 1 |
| E08-05: | $V_3$ | $V_3$ | $V_3$ | $V_3$ | : | 6 | 0 | 0 | 0 | 4 | 0 | 1*** |

It is assumed here that the minimum degree is 2. The selected edge has to have an end-point that is incident with a vertex of current minimum degree as its end-points are both of degree 3. If any isolated edges were created, there would be another edge to select with smaller ratio.

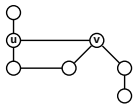
H26:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H26-00: | | | | | : | 6 | 0 | 0 | 0 | 4 | 0 | 4*** |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. Choose a different edge incident with $v$.

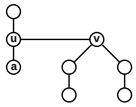
H25:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H25-00: | | | | | : | 6 | 0 | 0 | 0 | 4 | 0 | 3*** |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. Cycle of length smaller than 5.

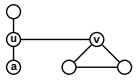
H24:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H24-00: | $V_1$ | | | | : | 2 | 2 | 4 | 0 | 0 | 0 | 3*** |
| H24-01: | $V_2$ | | | | : | 2 | 3 | 3 | 0 | 0 | 1 | 3*** |
| H24-02: | $V_3$ | | | | : | 3 | 2 | 3 | 0 | 1 | 0 | 3*** |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. Choose a different edge incident with $v$.

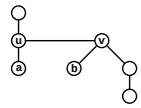
H23:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H23-00: | $V_1$ | | | | : | 2 | 2 | 2 | 0 | 0 | 0 | 2*** |
| H23-01: | $V_2$ | | | | : | 2 | 3 | 1 | 0 | 0 | 1 | 2*** |
| H23-02: | $V_3$ | | | | : | 3 | 2 | 1 | 0 | 1 | 0 | 2*** |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. Cycle of length smaller than 5.

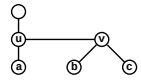
H22:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H22-00: | $V_1$ | $V_1$ | | | : | 2 | 1 | 4 | 0 | 0 | 0 | 2*** |
| H22-01: | $V_1$ | $V_2$ | | | : | 2 | 2 | 3 | 0 | 0 | 1 | 2*** |
| H22-02: | $V_1$ | $V_3$ | | | : | 3 | 1 | 3 | 0 | 1 | 0 | 2*** |
| H22-03: | $V_2$ | $V_2$ | | | : | 2 | 3 | 2 | 0 | 0 | 2 | 2*** |
| H22-04: | $V_2$ | $V_3$ | | | : | 3 | 2 | 2 | 0 | 1 | 1 | 2*** |
| H22-05: | $V_3$ | $V_3$ | | | : | 4 | 1 | 2 | 0 | 2 | 0 | 2*** |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. Choose the edge incident with $v$ and not incident with $b$.

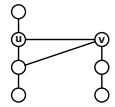
H21:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H21-00: | $V_1$ | $V_1$ | $V_1$ | | : | 2 | 0 | 4 | 0 | 0 | 0 | 1 |
| H21-01: | $V_1$ | $V_1$ | $V_2$ | | : | 2 | 1 | 3 | 0 | 0 | 1 | 1 |
| H21-02: | $V_1$ | $V_1$ | $V_3$ | | : | 3 | 0 | 3 | 0 | 1 | 0 | 1 |
| H21-03: | $V_1$ | $V_2$ | $V_2$ | | : | 2 | 2 | 2 | 0 | 0 | 2 | 1 |
| H21-04: | $V_1$ | $V_2$ | $V_3$ | | : | 3 | 1 | 2 | 0 | 1 | 1 | 1 |
| H21-05: | $V_1$ | $V_3$ | $V_3$ | | : | 4 | 0 | 2 | 0 | 2 | 0 | 1 |
| H21-06: | $V_2$ | $V_1$ | $V_1$ | | : | 2 | 1 | 3 | 0 | 0 | 1 | 1 |
| H21-07: | $V_2$ | $V_1$ | $V_2$ | | : | 2 | 2 | 2 | 0 | 0 | 2 | 1 |
| H21-08: | $V_2$ | $V_1$ | $V_3$ | | : | 3 | 1 | 2 | 0 | 1 | 1 | 1 |
| H21-09: | $V_2$ | $V_2$ | $V_2$ | | : | 2 | 3 | 1 | 0 | 0 | 3 | 1 |
| H21-10: | $V_2$ | $V_2$ | $V_3$ | | : | 3 | 2 | 1 | 0 | 1 | 2 | 1 |
| H21-11: | $V_2$ | $V_3$ | $V_3$ | | : | 4 | 1 | 1 | 0 | 2 | 1 | 1 |
| H21-12: | $V_3$ | $V_1$ | $V_1$ | | : | 3 | 0 | 3 | 0 | 1 | 0 | 1 |
| H21-13: | $V_3$ | $V_1$ | $V_2$ | | : | 3 | 1 | 2 | 0 | 1 | 1 | 1 |
| H21-14: | $V_3$ | $V_1$ | $V_3$ | | : | 4 | 0 | 2 | 0 | 2 | 0 | 1 |
| H21-15: | $V_3$ | $V_2$ | $V_2$ | | : | 3 | 2 | 1 | 0 | 1 | 2 | 1 |
| H21-16: | $V_3$ | $V_2$ | $V_3$ | | : | 4 | 1 | 1 | 0 | 2 | 1 | 1 |
| H21-17: | $V_3$ | $V_3$ | $V_3$ | | : | 5 | 0 | 1 | 0 | 3 | 0 | 1 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. It is also assumed that no isolated edges are generated.

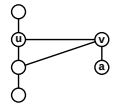
H20:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H20-00: | | | | | : | 5 | 0 | 1 | 0 | 3 | 0 | 3*** |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. Cycle of length smaller than 5.

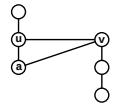
H19:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H19-00: | $V_1$ | | | | : | 3 | 0 | 4 | 0 | 0 | 0 | 2*** |
| H19-01: | $V_2$ | | | | : | 3 | 1 | 3 | 0 | 0 | 1 | 2*** |
| H19-02: | $V_3$ | | | | : | 4 | 0 | 3 | 0 | 1 | 0 | 2*** |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. Cycle of length smaller than 5.

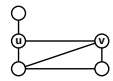
H18:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H18-00: | $V_2$ | | | | : | 2 | 2 | 2 | 0 | 0 | 0 | 2*** |
| H18-01: | $V_3$ | | | | : | 3 | 1 | 2 | 0 | 0 | 1 | 2*** |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. Cycle of length smaller than 5.

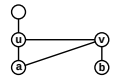
H17:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H17-00: | | | | | : | 3 | 1 | 2 | 0 | 0 | 1 | 2*** |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. Cycle of length smaller than 5.

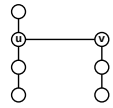
H16:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H16-00: | $V_2$ | $V_1$ | | | : | 2 | 1 | 2 | 0 | 0 | 0 | 1*** |
| H16-01: | $V_2$ | $V_2$ | | | : | 2 | 2 | 1 | 0 | 0 | 1 | 1*** |
| H16-02: | $V_2$ | $V_3$ | | | : | 3 | 1 | 1 | 0 | 1 | 0 | 1*** |
| H16-03: | $V_3$ | $V_1$ | | | : | 3 | 0 | 2 | 0 | 0 | 1 | 1*** |
| H16-04: | $V_3$ | $V_2$ | | | : | 3 | 1 | 1 | 0 | 0 | 2 | 1*** |
| H16-05: | $V_3$ | $V_3$ | | | : | 4 | 0 | 1 | 0 | 1 | 1 | 1*** |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. It is also assumed that no isolated edges are generated. Cycle of length smaller than 5.

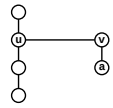
H15:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H15-00: | | | | | : | 4 | 0 | 1 | 0 | 1 | 1 | 3*** |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. Choose the other edge incident with $v$.

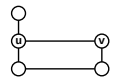
H14:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H14-00: | $V_1$ | | | | : | 1 | 2 | 3 | 0 | 0 | 0 | 2 |
| H14-01: | $V_2$ | | | | : | 1 | 3 | 2 | 0 | 0 | 1 | 2 |
| H14-02: | $V_3$ | | | | : | 2 | 2 | 2 | 0 | 1 | 0 | 2 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

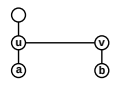
H13:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H13-00: | | | | | : | 2 | 2 | 2 | 0 | 1 | 0 | 2*** |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. Cycle of length smaller than 5.

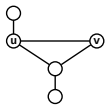
H12:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H12-00: | $V_1$ | $V_1$ | | | : | 1 | 2 | 3 | 0 | 0 | 0 | 1 |
| H12-01: | $V_1$ | $V_2$ | | | : | 1 | 3 | 2 | 0 | 0 | 1 | 1 |
| H12-02: | $V_1$ | $V_3$ | | | : | 2 | 2 | 2 | 0 | 1 | 0 | 1 |
| H12-03: | $V_2$ | $V_2$ | | | : | 1 | 4 | 1 | 0 | 0 | 2 | 1 |
| H12-04: | $V_2$ | $V_3$ | | | : | 2 | 3 | 1 | 0 | 1 | 1 | 1 |
| H12-05: | $V_3$ | $V_3$ | | | : | 3 | 2 | 1 | 0 | 2 | 0 | 1 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. It is also assumed that no isolated edges are generated.

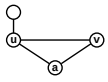
H11:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H11-00: | | | | | : | 3 | 2 | 1 | 0 | 2 | 0 | 2*** |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. Cycle of length smaller than 5.

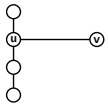
H10:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H10-00: | $V_2$ | | | | : | 1 | 2 | 1 | 0 | 0 | 0 | 1*** |
| H10-01: | $V_3$ | | | | : | 2 | 1 | 1 | 0 | 0 | 1 | 1*** |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. It is also assumed that no isolated edges are generated. Cycle of length smaller than 5.

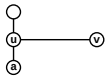
H09:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H09-00: | | | | | : | 1 | 1 | 3 | 0 | 0 | 0 | 2 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

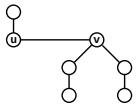
H08:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H08-00: | $V_1$ | | | | : | 1 | 0 | 3 | 0 | 0 | 0 | 1 |
| H08-01: | $V_2$ | | | | : | 1 | 1 | 2 | 0 | 0 | 1 | 1 |
| H08-02: | $V_3$ | | | | : | 2 | 0 | 2 | 0 | 1 | 0 | 1 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. It is also assumed that no isolated edges are generated.

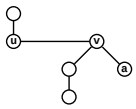
H07:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H07-00: | | | | | : | 1 | 3 | 3 | 0 | 0 | 0 | 3 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

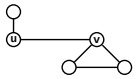
H06:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H06-00: | $V_1$ | | | | : | 1 | 2 | 3 | 0 | 0 | 0 | 2 |
| H06-01: | $V_2$ | | | | : | 1 | 3 | 2 | 0 | 0 | 1 | 2 |
| H06-02: | $V_3$ | | | | : | 2 | 2 | 2 | 0 | 1 | 0 | 2 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

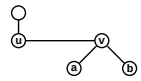
H05:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H05-00: | | | | | : | 2 | 2 | 2 | 0 | 1 | 0 | 2*** |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. Cycle of length smaller than 5.

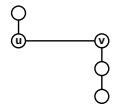
H04:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H04-00: | $V_1$ | $V_1$ | | | : | 1 | 1 | 3 | 0 | 0 | 0 | 1 |
| H04-01: | $V_1$ | $V_2$ | | | : | 1 | 2 | 2 | 0 | 0 | 1 | 1 |
| H04-02: | $V_1$ | $V_3$ | | | : | 2 | 1 | 2 | 0 | 1 | 0 | 1 |
| H04-03: | $V_2$ | $V_2$ | | | : | 1 | 3 | 1 | 0 | 0 | 2 | 1 |
| H04-04: | $V_2$ | $V_3$ | | | : | 2 | 2 | 1 | 0 | 1 | 1 | 1 |
| H04-05: | $V_3$ | $V_3$ | | | : | 3 | 1 | 1 | 0 | 2 | 0 | 1 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. It is also assumed that no isolated edges are generated.

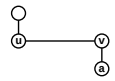
H03:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H03-00: | | | | | : | 0 | 3 | 2 | 0 | 0 | 0 | 2 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1.

H02:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H02-00: | $V_1$ | | | | : | 0 | 2 | 2 | 0 | 0 | 0 | 1 |
| H02-01: | $V_2$ | | | | : | 0 | 3 | 1 | 0 | 0 | 1 | 1 |
| H02-02: | $V_3$ | | | | : | 1 | 2 | 1 | 0 | 1 | 0 | 1 |

It is assumed here that an edge is chosen that has an end-point with a neighbour of degree 1. It is also assumed that no isolated edges are generated.

H01:


| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H01-00: | | | | | : | 0 | 1 | 2 | 0 | 0 | 0 | 1 |

```
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 \  MAPLE  /  reserved. Maple and Maple V are registered trademarks of
 <____ ____>  Waterloo Maple Inc.
      |       Type ? for help.
> with(simplex):
Warning, new definition for maximize
Warning, new definition for minimize
>
> obj:= 1*E02_00 +1*E02_01 +1*E05_00 +1*E05_01 +1*E05_02 +1*E05_03
+1*E05_04 +1*E08_00 +1*E08_01 +1*E08_02 +1*E08_03 +1*E08_04 +1*H21_00
+1*H21_01 +1*H21_02 +1*H21_03 +1*H21_04 +1*H21_05 +1*H21_06 +1*H21_07
+1*H21_08 +1*H21_09 +1*H21_10 +1*H21_11 +1*H21_12 +1*H21_13 +1*H21_14
+1*H21_15 +1*H21_16 +1*H21_17 +2*H14_00 +2*H14_01 +2*H14_02 +1*H12_00
+1*H12_01 +1*H12_02 +1*H12_03 +1*H12_04 +1*H12_05 +2*H09_00 +1*H08_00
+1*H08_01 +1*H08_02 +3*H07_00 +2*H06_00 +2*H06_01 +2*H06_02 +1*H04_00
+1*H04_01 +1*H04_02 +1*H04_03 +1*H04_04 +1*H04_05 +2*H03_00 +1*H02_00
+1*H02_01 +1*H02_02 +1*H01_00;

obj := E02_01 + E02_00 + E05_01 + E05_00 + E05_02 + E05_03 + E05_04 + E08_00

     + E08_01 + E08_02 + E08_03 + E08_04 + H21_00 + H21_01 + H21_02 + H21_03

     + H21_04 + H21_05 + H21_06 + H21_07 + H21_08 + H21_09 + H21_10 + H21_11

     + H21_12 + H21_13 + H21_14 + H21_15 + H21_16 + H21_17 + 2 H14_00

     + 2 H14_01 + 2 H14_02 + H12_00 + H12_01 + H12_02 + H12_03 + H12_04

     + H12_05 + 2 H09_00 + H08_00 + H08_01 + H08_02 + 3 H07_00 + 2 H06_00

     + 2 H06_01 + 2 H06_02 + H04_00 + H04_01 + H04_02 + H04_03 + H04_04

     + H04_05 + 2 H03_00 + H02_00 + H02_01 + H02_02 + H01_00

> cnsts:={
>
>  +1*E02_00 +2*E05_00 +1*E05_01 +1*E05_03 +3*E08_00 +2*E08_01
> +1*E08_02 +1*E08_03 -3*H21_00 -2*H21_01 -2*H21_02 -1*H21_04
> -1*H21_05 -2*H21_06 -1*H21_08 +2*H21_09 +1*H21_10 -2*H21_12
> -1*H21_13 -1*H21_14 +1*H21_15 -2*H14_00 -1*H14_01 -1*H14_02
> -2*H12_00 -1*H12_01 -1*H12_02 +1*H12_03 -2*H09_00 -2*H08_00
> -1*H08_01 -1*H08_02 -2*H07_00 -2*H06_00 -1*H06_01 -1*H06_02
> -2*H04_00 -1*H04_01 -1*H04_02 +1*H04_03 -1*H03_00 -1*H02_00
> -1*H01_00 >= 0,
>
>  +1*E05_00 +2*E08_00 +1*E08_01 -2*H21_00 -1*H21_01 -1*H21_02
> -1*H21_06 +2*H21_09 +1*H21_10 -1*H21_12 +1*H21_15 -1*H14_00
> -1*H12_00 +1*H12_03 -1*H09_00 -1*H08_00 -1*H07_00 -1*H06_00
> -1*H04_00 +1*H04_03 >= 0,
>
>  +1*E08_00 -1*H21_00 +2*H21_09 +1*H21_10 +1*H21_15 +1*H12_03
> +1*H04_03 >= 0,
>
>  +2*E02_00 +1*E02_01 +3*E05_00 +2*E05_01 +1*E05_02 +2*E05_03
> +1*E05_04 +4*E08_00 +3*E08_01 +2*E08_02 +2*E08_03 +1*E08_04
> -4*H21_00 -2*H21_01 -3*H21_02 -1*H21_04 -2*H21_05 -2*H21_06
> -1*H21_08 +2*H21_09 +1*H21_10 -3*H21_12 -1*H21_13 -2*H21_14
> +1*H21_15 -1*H21_17 -3*H14_00 -1*H14_01 -2*H14_02 -3*H12_00
```

```
> -1*H12_01 -2*H12_02 +1*H12_03 -1*H12_05 -3*H09_00 -3*H08_00
> -1*H08_01 -2*H08_02 -3*H07_00 -3*H06_00 -1*H06_01 -2*H06_02
> -3*H04_00 -1*H04_01 -2*H04_02 +1*H04_03 -1*H04_05 -2*H03_00
> -2*H02_00 -1*H02_02 -2*H01_00 >= 0,
>
> -1*E02_00 -1*E05_00 -1*E05_01 -1*E05_03 -2*E08_00 -1*E08_01
> -1*E08_02 -1*E08_03 +2*H21_00 +1*H21_01 +1*H21_02 +1*H21_05
> +1*H21_06 -1*H21_09 -1*H21_10 +1*H21_12 +1*H21_14 -1*H21_15
> +1*H14_00 +1*H14_02 +1*H12_00 +1*H12_02 -1*H12_03 +1*H09_00
> +1*H08_00 +1*H08_02 +1*H07_00 +1*H06_00 +1*H06_02 +1*H04_00
> +1*H04_02 -1*H04_03 +1*H03_00 +1*H02_00 +1*H01_00  <= 0,
>
> -1*E05_00 -1*E08_00 -1*E08_01 +1*H21_00 +1*H21_02 -1*H21_09
> -1*H21_10 +1*H21_12 -1*H21_15 +1*H14_00 +1*H12_00 -1*H12_03
> +1*H09_00 +1*H08_00 +1*H07_00 +1*H06_00 +1*H04_00 -1*H04_03  <= 0,
>
> -1*E08_00 +1*H21_00 -1*H21_09 -1*H21_10 -1*H21_15 -1*H12_03
> -1*H04_03  <= 0,
>
> -1*E02_01 -1*E05_00 -2*E05_01 -3*E05_02 -2*E05_03 -3*E05_04
> -2*E08_00 -3*E08_01 -4*E08_02 -4*E08_03 -5*E08_04 -2*H21_00
> -2*H21_01 -3*H21_02 -2*H21_03 -3*H21_04 -4*H21_05 -2*H21_06
> -2*H21_07 -3*H21_08 -2*H21_09 -3*H21_10 -4*H21_11 -3*H21_12
> -3*H21_13 -4*H21_14 -3*H21_15 -4*H21_16 -5*H21_17 -1*H14_00
> -1*H14_01 -2*H14_02 -1*H12_00 -1*H12_01 -2*H12_02 -1*H12_03
> -2*H12_04 -3*H12_05 -1*H09_00 -1*H08_00 -1*H08_01 -2*H08_02
> -1*H07_00 -1*H06_00 -1*H06_01 -2*H06_02 -1*H04_00 -1*H04_01
> -2*H04_02 -1*H04_03 -2*H04_04 -3*H04_05 -1*H02_02  >= -1,
>
> -4*E02_00 -2*E02_01 -4*E05_00 -2*E05_01 -2*E05_03 -4*E08_00
> -2*E08_01 +2*E08_04 -1*H21_01 +1*H21_02 -2*H21_03 +2*H21_05
> -1*H21_06 -2*H21_07 -3*H21_09 -1*H21_10 +1*H21_11 +1*H21_12
> +2*H21_14 -1*H21_15 +1*H21_16 +3*H21_17 -2*H14_00 -3*H14_01
> -1*H14_02 -2*H12_00 -3*H12_01 -1*H12_02 -4*H12_03 -2*H12_04
> -1*H09_00 -1*H08_01 +1*H08_02 -3*H07_00 -2*H06_00 -3*H06_01
> -1*H06_02 -1*H04_00 -2*H04_01 -3*H04_03 -1*H04_04 +1*H04_05
> -3*H03_00 -2*H02_00 -3*H02_01 -1*H02_02 -1*H01_00  >= 0,
>
> +2*E02_00 +1*E02_01 +3*E05_00 +2*E05_01 +1*E05_02 +2*E05_03
> +1*E05_04 +4*E08_00 +3*E08_01 +2*E08_02 +2*E08_03 +1*E08_04
> -4*H21_00 -2*H21_01 -3*H21_02 -1*H21_04 -2*H21_05 -2*H21_06
> -1*H21_08 +2*H21_09 +1*H21_10 -3*H21_12 -1*H21_13 -2*H21_14
> +1*H21_15 -1*H21_17 -3*H14_00 -1*H14_01 -2*H14_02 -3*H12_00
> -1*H12_01 -2*H12_02 +1*H12_03 -1*H12_05 -3*H09_00 -3*H08_00
> -1*H08_01 -2*H08_02 -3*H07_00 -3*H06_00 -1*H06_01 -2*H06_02
> -3*H04_00 -1*H04_01 -2*H04_02 +1*H04_03 -1*H04_05 -2*H03_00
> -2*H02_00 -1*H02_02 -2*H01_00  >= 0};

cnsts := {0 <= E02_00 + E05_01 + 2 E05_00 + E05_03 + 3 E08_00 + 2 E08_01

     + E08_02 + E08_03 - 3 H21_00 - 2 H21_01 - 2 H21_02 - H21_04 - H21_05

     - 2 H21_06 - H21_08 + 2 H21_09 + H21_10 - 2 H21_12 - H21_13 - H21_14

     + H21_15 - 2 H14_00 - H14_01 - H14_02 - 2 H12_00 - H12_01 - H12_02

     + H12_03 - 2 H09_00 - 2 H08_00 - H08_01 - H08_02 - 2 H07_00 - 2 H06_00

     - H06_01 - H06_02 - 2 H04_00 - H04_01 - H04_02 + H04_03 - H03_00 - H02_00
```

```
 - H01_00,
0 <= E08_00 - H21_00 + 2 H21_09 + H21_10 + H21_15 + H12_03 + H04_03,
-E08_00 + H21_00 - H21_09 - H21_10 - H21_15 - H12_03 - H04_03 <= 0, -1 <=
-E02_01 - 2 E05_01 - E05_00 - 3 E05_02 - 2 E05_03 - 3 E05_04 - 2 E08_00
 - 3 E08_01 - 4 E08_02 - 4 E08_03 - 5 E08_04 - 2 H21_00 - 2 H21_01
 - 3 H21_02 - 2 H21_03 - 3 H21_04 - 4 H21_05 - 2 H21_06 - 2 H21_07
 - 3 H21_08 - 2 H21_09 - 3 H21_10 - 4 H21_11 - 3 H21_12 - 3 H21_13
 - 4 H21_14 - 3 H21_15 - 4 H21_16 - 5 H21_17 - H14_00 - H14_01 - 2 H14_02
 - H12_00 - H12_01 - 2 H12_02 - H12_03 - 2 H12_04 - 3 H12_05 - H09_00
 - H08_00 - H08_01 - 2 H08_02 - H07_00 - H06_00 - H06_01 - 2 H06_02
 - H04_00 - H04_01 - 2 H04_02 - H04_03 - 2 H04_04 - 3 H04_05 - H02_02,
-E05_00 - E08_00 - E08_01 + H21_00 + H21_02 - H21_09 - H21_10 + H21_12
 - H21_15 + H14_00 + H12_00 - H12_03 + H09_00 + H08_00 + H07_00 + H06_00
 + H04_00 - H04_03 <= 0, -E02_00 - E05_01 - E05_00 - E05_03 - 2 E08_00
 - E08_01 - E08_02 - E08_03 + 2 H21_00 + H21_01 + H21_02 + H21_05 + H21_06
 - H21_09 - H21_10 + H21_12 + H21_14 - H21_15 + H14_00 + H14_02 + H12_00
 + H12_02 - H12_03 + H09_00 + H08_00 + H08_02 + H07_00 + H06_00 + H06_02
 + H04_00 + H04_02 - H04_03 + H03_00 + H02_00 + H01_00 <= 0, 0 <= -2 E02_01
 - 4 E02_00 - 2 E05_01 - 4 E05_00 - 2 E05_03 - 4 E08_00 - 2 E08_01
 + 2 E08_04 - H21_01 + H21_02 - 2 H21_03 + 2 H21_05 - H21_06 - 2 H21_07
 - 3 H21_09 - H21_10 + H21_11 + H21_12 + 2 H21_14 - H21_15 + H21_16
 + 3 H21_17 - 2 H14_00 - 3 H14_01 - H14_02 - 2 H12_00 - 3 H12_01 - H12_02
 - 4 H12_03 - 2 H12_04 - H09_00 - H08_01 + H08_02 - 3 H07_00 - 2 H06_00
 - 3 H06_01 - H06_02 - H04_00 - 2 H04_01 - 3 H04_03 - H04_04 + H04_05
 - 3 H03_00 - 2 H02_00 - 3 H02_01 - H02_02 - H01_00, 0 <= E02_01 + 2 E02_00
 + 2 E05_01 + 3 E05_00 + E05_02 + 2 E05_03 + E05_04 + 4 E08_00 + 3 E08_01
 + 2 E08_02 + 2 E08_03 + E08_04 - 4 H21_00 - 2 H21_01 - 3 H21_02 - H21_04
 - 2 H21_05 - 2 H21_06 - H21_08 + 2 H21_09 + H21_10 - 3 H21_12 - H21_13
 - 2 H21_14 + H21_15 - H21_17 - 3 H14_00 - H14_01 - 2 H14_02 - 3 H12_00
```

```
      - H12_01 - 2 H12_02 + H12_03 - H12_05 - 3 H09_00 - 3 H08_00 - H08_01

      - 2 H08_02 - 3 H07_00 - 3 H06_00 - H06_01 - 2 H06_02 - 3 H04_00 - H04_01

      - 2 H04_02 + H04_03 - H04_05 - 2 H03_00 - 2 H02_00 - H02_02 - 2 H01_00, 0

      <= E05_00 + 2 E08_00 + E08_01 - 2 H21_00 - H21_01 - H21_02 - H21_06

      + 2 H21_09 + H21_10 - H21_12 + H21_15 - H14_00 - H12_00 + H12_03 - H09_00

      - H08_00 - H07_00 - H06_00 - H04_00 + H04_03}

>
> maximize(obj,cnsts,NONNEGATIVE);

{H21_13 = 0, H21_14 = 0, H21_15 = 0, H21_16 = 0, H14_00 = 0, H14_01 = 0,

    H12_00 = 0, H12_01 = 0, H12_02 = 0, H12_03 = 0, H12_04 = 0, H12_05 = 0,

    H09_00 = 0, H08_00 = 0, H08_01 = 0, H08_02 = 0, H06_00 = 0, H06_01 = 0,

    H06_02 = 0, H04_00 = 0, H04_01 = 0, H04_02 = 0, H04_03 = 0, H04_04 = 0,

    H04_05 = 0, H03_00 = 0, H02_00 = 0, H02_01 = 0, H02_02 = 0, H01_00 = 0,

    E08_01 = 1/14, H07_00 = 1/14, H21_17 = 1/14, E08_04 = 1/14, H14_02 = 0,

    H21_05 = 0, E05_03 = 0, E05_04 = 0, E02_01 = 0, E02_00 = 0, E05_01 = 0,

    E05_00 = 0, E05_02 = 0, E08_00 = 0, E08_02 = 0, E08_03 = 0, H21_00 = 0,

    H21_01 = 0, H21_02 = 0, H21_03 = 0, H21_04 = 0, H21_06 = 0, H21_07 = 0,

    H21_08 = 0, H21_09 = 0, H21_10 = 0, H21_11 = 0, H21_12 = 0}

> subs(%,obj);
```



```
> (dualobj,dualcnsts):=dual(obj,cnsts,y);
dualobj, dualcnsts := y9, {

    1 <= -y1 - y2 - y3 + 3 y4 - 2 y5 - 2 y6 - 2 y7 - 2 y8 + 2 y9,

    1 <= y6 + y7 + 3 y9, 1 <= y3 - 2 y4 + 2 y6 + y7 + 4 y9,

    1 <= y1 + y2 + 2 y3 + 2 y5 + 4 y6 + 3 y7 + y8 + 2 y9,

    1 <= y3 + y4 + y5 + 2 y6 + 2 y7 + 2 y9,

    1 <= y2 + y3 - y4 + y5 + 3 y6 + 2 y7 + 3 y9, 1 <= 2 y4 + 2 y9,

    1 <= -y2 - y3 + 2 y4 - y5 - 3 y6 - 2 y7 + 3 y9, 1 <= -y3 - 2 y6 -
    y7 + 4 y9,

    1 <= -2 y4 - y6 + 5 y9,

    1 <= -y1 - y2 - 2 y3 + 4 y4 - 2 y5 - 4 y6 - 3 y7 - y8 + 2 y9,

    1 <= -y6 + 3 y9, 1 <= -y3 + 2 y4 - 2 y6 - y7 + 2 y9,
```

```
    1 <= -y2 - y3 + 4 y4 - y5 - 3 y6 - 2 y7 + y9, 1 <= 2 y4 - y6 + y9,

    1 <= -y3 + 4 y4 - 2 y6 - y7, 1 <= y4 + 2 y9, 1 <= -y4 + y6 + 3 y9,

    2 <= y3 + 3 y4 + 2 y6 + y7, 1 <= y3 + 2 y4 + 2 y6 + y7, 1 <= 3 y4,

    1 <= y4 + y6 + y9, 1 <= y3 + y4 + 2 y6 + y7,

    1 <= y2 + y3 + y4 + y5 + 3 y6 + 2 y7 + y9, 1 <= 2 y4 + y6 + y7 + y9,

    1 <= y3 + 2 y6 + y7 + 2 y9,

    1 <= -y1 - y2 - y3 + 3 y4 - y5 - y6 - y7 - y8 + y9,

    2 <= y2 + y3 + y4 + y5 + 3 y6 + 2 y7 + y9,

    1 <= y2 + y3 + y5 + 3 y6 + 2 y7 + y9, 1 <= y4 + y6 + y7 + y9,

    1 <= y3 - y4 + 2 y6 + y7 + 2 y9,

    3 <= y2 + y3 + 3 y4 + y5 + 3 y6 + 2 y7 + y9,

    1 <= y3 + y4 + 2 y6 + y7 + 2 y9,

    1 <= -y1 - y2 - y3 + 4 y4 - y5 - y6 - y7 - y8 + y9, 1 <= y6 + 3 y9,

    2 <= y3 + y4 + 2 y6 + y7 + 2 y9,

    1 <= y2 + y3 + 2 y4 + y5 + 3 y6 + 2 y7 + y9, 1 <= 3 y4 + y6 + y7 + y9,

    2 <= 3 y4 + y6 + y7 + y9, 1 <= -3 y4 + y6 + 5 y9,

    2 <= y2 + y3 + 2 y4 + y5 + 3 y6 + 2 y7 + y9, 1 <= -y4 + 4 y9,

    1 <= -y1 - y2 - y3 + y4 - y5 - y6 - y7 - y8 + 3 y9}

>
> minimize(dualobj,dualcnsts,NONNEGATIVE);
                                                        16
{y1 = 0, y5 = 0, y8 = 0, y6 = 8/35, y9 = 3/7, y4 = --, y2 = 2/7, y3 = 8/35,
                                                        35

    y7 = 0}

>
> quit
```

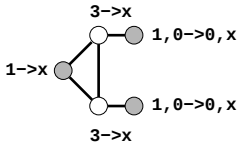
# MCDS

In all operations considered, the vertex at the “left” of the figure is the one that could possibly be selected by “some” algorithm for inclusion in the connected dominating set. The priorities are

- vertices of degree 2 over
- vertices of degree 1

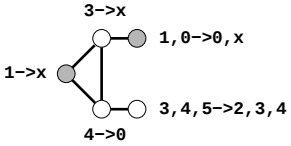
All operations are in $OPS_2$.

Q04:



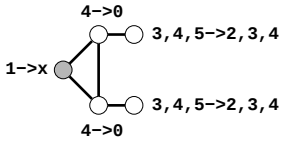

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q04-00: | $V_0$ | $V_0$: | -2 | -1 | | -2 | | | | | | | | 1 |
| Q04-01: | $V_0$ | $V_1$: | -1 | -2 | | -2 | | | 1 | | | | | 1 |
| Q04-02: | $V_1$ | $V_1$: | | -3 | | -2 | | | 2 | | | | | 1 |

Q05:



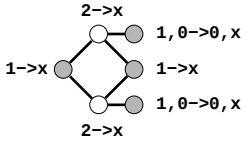

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q05-00: | $V_0$ | $V_3$: | -1 | -1 | | -2 | -1 | | 1 | | 1 | | | 1 |
| Q05-01: | $V_0$ | $V_4$: | -1 | -1 | | -1 | -2 | | 1 | | | 1 | | 1 |
| Q05-02: | $V_0$ | $V_5$: | -1 | -1 | | -1 | -1 | -1 | 1 | | | | 1 | 1 |
| Q05-03: | $V_1$ | $V_3$: | | -2 | | -2 | -1 | | 2 | | 1 | | | 1 |
| Q05-04: | $V_1$ | $V_4$: | | -2 | | -1 | -2 | | 2 | | | 1 | | 1 |
| Q05-05: | $V_1$ | $V_5$: | | -2 | | -1 | -1 | -1 | 2 | | | | 1 | 1 |

Q06:



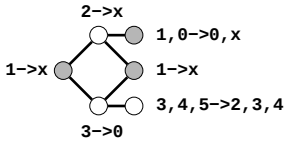

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q06-00: | $V_3$ | $V_3$: | | -1 | | -2 | -2 | | 2 | | 2 | | | 1 |
| Q06-01: | $V_3$ | $V_4$: | | -1 | | -1 | -3 | | 2 | | 1 | 1 | | 1 |
| Q06-02: | $V_3$ | $V_5$: | | -1 | | -1 | -2 | -1 | 2 | | 1 | | 1 | 1 |
| Q06-03: | $V_4$ | $V_4$: | | -1 | | | -4 | | 2 | | | 2 | | 1 |
| Q06-04: | $V_4$ | $V_5$: | | -1 | | | -3 | -1 | 2 | | | 1 | 1 | 1 |
| Q06-05: | $V_5$ | $V_5$: | | -1 | | | -2 | -2 | 2 | | | | 2 | 1 |

Q13:



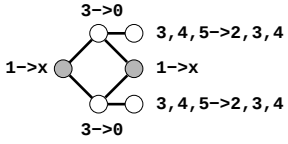

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q13-00: | $V_0$ | $V_0$: | -2 | -2 | -2 | | | | | | | | | 1 |
| Q13-01: | $V_0$ | $V_1$: | -1 | -3 | -2 | | | | 1 | | | | | 1 |
| Q13-02: | $V_1$ | $V_1$: | | -4 | -2 | | | | 2 | | | | | 1 |

Q14:



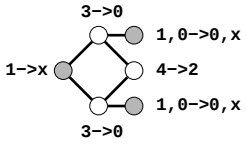

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q14-00: | $V_0$ | $V_3$: | -1 | -2 | -1 | -2 | | | 1 | | 1 | | | 1 |
| Q14-01: | $V_0$ | $V_4$: | -1 | -2 | -1 | -1 | -1 | | 1 | | | 1 | | 1 |
| Q14-02: | $V_0$ | $V_5$: | -1 | -2 | -1 | -1 | | -1 | 1 | | | | 1 | 1 |
| Q14-03: | $V_1$ | $V_3$: | | -3 | -1 | -2 | | | 2 | | 1 | | | 1 |
| Q14-04: | $V_1$ | $V_4$: | | -3 | -1 | -1 | -1 | | 2 | | | 1 | | 1 |
| Q14-05: | $V_1$ | $V_5$: | | -3 | -1 | -1 | | -1 | 2 | | | | 1 | 1 |

Q15:



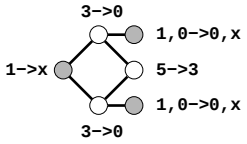

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q15-00: | $V_3$ | $V_3$: | | -2 | | -4 | | | 2 | | 2 | | | 1 |
| Q15-01: | $V_3$ | $V_4$: | | -2 | | -3 | -1 | | 2 | | 1 | 1 | | 1 |
| Q15-02: | $V_3$ | $V_5$: | | -2 | | -3 | | -1 | 2 | | 1 | | 1 | 1 |
| Q15-03: | $V_4$ | $V_4$: | | -2 | | -2 | -2 | | 2 | | | 2 | | 1 |
| Q15-04: | $V_4$ | $V_5$: | | -2 | | -2 | -1 | -1 | 2 | | | 1 | 1 | 1 |
| Q15-05: | $V_5$ | $V_5$: | | -2 | | -2 | | -2 | 2 | | | | 2 | 1 |

Q16:



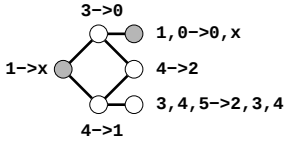

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q16-00: | $V_0$ | $V_0$: | -2 | -1 | | -2 | -1 | | 2 | | 1 | | | 1 |
| Q16-01: | $V_0$ | $V_1$: | -1 | -2 | | -2 | -1 | | 3 | | 1 | | | 1 |
| Q16-02: | $V_1$ | $V_1$: | | -3 | | -2 | -1 | | 4 | | 1 | | | 1 |

Q17:



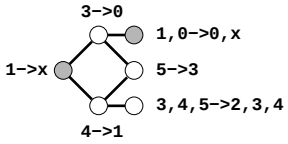

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q17-00: | $V_0$ | $V_0$: | -2 | -1 | | -2 | | -1 | 2 | | | 1 | | 1 |
| Q17-01: | $V_0$ | $V_1$: | -1 | -2 | | -2 | | -1 | 3 | | | 1 | | 1 |
| Q17-02: | $V_1$ | $V_1$: | | -3 | | -2 | | -1 | 4 | | | 1 | | 1 |

Q18:



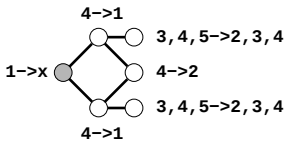

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q18-00: | $V_0$ | $V_3$: | -1 | -1 | | -2 | -2 | | 1 | 1 | 2 | | | 1 |
| Q18-01: | $V_0$ | $V_4$: | -1 | -1 | | -1 | -3 | | 1 | 1 | 1 | 1 | | 1 |
| Q18-02: | $V_0$ | $V_5$: | -1 | -1 | | -1 | -2 | -1 | 1 | 1 | 1 | | 1 | 1 |
| Q18-03: | $V_1$ | $V_3$: | | -2 | | -2 | -2 | | 2 | 1 | 2 | | | 1 |
| Q18-04: | $V_1$ | $V_4$: | | -2 | | -1 | -3 | | 2 | 1 | 1 | 1 | | 1 |
| Q18-05: | $V_1$ | $V_5$: | | -2 | | -1 | -2 | -1 | 2 | 1 | 1 | | 1 | 1 |

Q19:



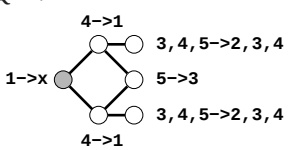

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q19-00: | $V_0$ | $V_3$: | -1 | -1 | | -2 | -1 | -1 | 1 | 1 | 1 | 1 | | 1 |
| Q19-01: | $V_0$ | $V_4$: | -1 | -1 | | -1 | -2 | -1 | 1 | 1 | | 2 | | 1 |
| Q19-02: | $V_0$ | $V_5$: | -1 | -1 | | -1 | -1 | -2 | 1 | 1 | | 1 | 1 | 1 |
| Q19-03: | $V_1$ | $V_3$: | | -2 | | -2 | -1 | -1 | 2 | 1 | 1 | 1 | | 1 |
| Q19-04: | $V_1$ | $V_4$: | | -2 | | -1 | -2 | -1 | 2 | 1 | | 2 | | 1 |
| Q19-05: | $V_1$ | $V_5$: | | -2 | | -1 | -1 | -2 | 2 | 1 | | 1 | 1 | 1 |

Q20:



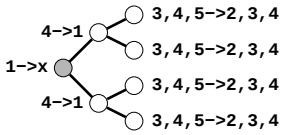

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q20-00: | $V_3$ | $V_3$: | | -1 | | -2 | -3 | | | 2 | 3 | | | 1 |
| Q20-01: | $V_3$ | $V_4$: | | -1 | | -1 | -4 | | | 2 | 2 | 1 | | 1 |
| Q20-02: | $V_3$ | $V_5$: | | -1 | | -1 | -3 | -1 | | 2 | 2 | | 1 | 1 |
| Q20-03: | $V_4$ | $V_4$: | | -1 | | | -5 | | | 2 | 1 | 2 | | 1 |
| Q20-04: | $V_4$ | $V_5$: | | -1 | | | -4 | -1 | | 2 | 1 | 1 | 1 | 1 |
| Q20-05: | $V_5$ | $V_5$: | | -1 | | | -3 | -2 | | 2 | 1 | | 2 | 1 |

Q21:



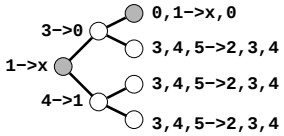

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q21-00: | $V_3$ | $V_3$: | | -1 | | -2 | -2 | -1 | | 2 | 2 | 1 | | 1 |
| Q21-01: | $V_3$ | $V_4$: | | -1 | | -1 | -3 | -1 | | 2 | 1 | 2 | | 1 |
| Q21-02: | $V_3$ | $V_5$: | | -1 | | -1 | -2 | -2 | | 2 | 1 | 1 | 1 | 1 |
| Q21-03: | $V_4$ | $V_4$: | | -1 | | | -4 | -1 | | 2 | | 3 | | 1 |
| Q21-04: | $V_4$ | $V_5$: | | -1 | | | -3 | -2 | | 2 | | 2 | 1 | 1 |
| Q21-05: | $V_5$ | $V_5$: | | -1 | | | -2 | -3 | | 2 | | 1 | 2 | 1 |

Q22:



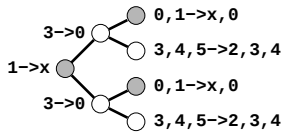

| variable | end-points | | | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q22-00: | $V_3$ | $V_3$ | $V_3$ | $V_3$: | | -1 | | -4 | -2 | | | 2 | 4 | | | 1 |
| Q22-01: | $V_3$ | $V_3$ | $V_3$ | $V_4$: | | -1 | | -3 | -3 | | | 2 | 3 | 1 | | 1 |
| Q22-02: | $V_3$ | $V_3$ | $V_3$ | $V_5$: | | -1 | | -3 | -2 | -1 | | 2 | 3 | | 1 | 1 |
| Q22-03: | $V_3$ | $V_3$ | $V_4$ | $V_4$: | | -1 | | -2 | -4 | | | 2 | 2 | 2 | | 1 |
| Q22-04: | $V_3$ | $V_3$ | $V_4$ | $V_5$: | | -1 | | -2 | -3 | -1 | | 2 | 2 | 1 | 1 | 1 |
| Q22-05: | $V_3$ | $V_3$ | $V_5$ | $V_5$: | | -1 | | -2 | -2 | -2 | | 2 | 2 | | 2 | 1 |
| Q22-06: | $V_3$ | $V_4$ | $V_4$ | $V_4$: | | -1 | | -1 | -5 | | | 2 | 1 | 3 | | 1 |
| Q22-07: | $V_3$ | $V_4$ | $V_4$ | $V_5$: | | -1 | | -1 | -4 | -1 | | 2 | 1 | 2 | 1 | 1 |
| Q22-08: | $V_3$ | $V_4$ | $V_5$ | $V_5$: | | -1 | | -1 | -3 | -2 | | 2 | 1 | 1 | 2 | 1 |
| Q22-09: | $V_3$ | $V_5$ | $V_5$ | $V_5$: | | -1 | | -1 | -2 | -3 | | 2 | 1 | | 3 | 1 |
| Q22-10: | $V_4$ | $V_4$ | $V_4$ | $V_4$: | | -1 | | | -6 | | | 2 | | 4 | | 1 |
| Q22-11: | $V_4$ | $V_4$ | $V_4$ | $V_5$: | | -1 | | | -5 | -1 | | 2 | | 3 | 1 | 1 |
| Q22-12: | $V_4$ | $V_4$ | $V_5$ | $V_5$: | | -1 | | | -4 | -2 | | 2 | | 2 | 2 | 1 |
| Q22-13: | $V_4$ | $V_5$ | $V_5$ | $V_5$: | | -1 | | | -3 | -3 | | 2 | | 1 | 3 | 1 |
| Q22-14: | $V_5$ | $V_5$ | $V_5$ | $V_5$: | | -1 | | | -2 | -4 | | 2 | | | 4 | 1 |

Q23:



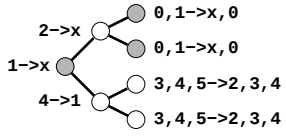

| variable | end-points | | | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q23-00: | $V_0$ | $V_3$ | $V_3$ | $V_3$: | -1 | -1 | | -4 | -1 | | 1 | 1 | 3 | | | 1 |
| Q23-01: | $V_0$ | $V_3$ | $V_3$ | $V_4$: | -1 | -1 | | -3 | -2 | | 1 | 1 | 2 | 1 | | 1 |
| Q23-02: | $V_0$ | $V_3$ | $V_3$ | $V_5$: | -1 | -1 | | -3 | -1 | -1 | 1 | 1 | 2 | | 1 | 1 |
| Q23-03: | $V_0$ | $V_3$ | $V_4$ | $V_4$: | -1 | -1 | | -2 | -3 | | 1 | 1 | 1 | 2 | | 1 |
| Q23-04: | $V_0$ | $V_3$ | $V_4$ | $V_5$: | -1 | -1 | | -2 | -2 | -1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Q23-05: | $V_0$ | $V_3$ | $V_5$ | $V_5$: | -1 | -1 | | -2 | -1 | -2 | 1 | 1 | 1 | | 2 | 1 |
| Q23-06: | $V_0$ | $V_4$ | $V_4$ | $V_4$: | -1 | -1 | | -1 | -4 | | 1 | 1 | | 3 | | 1 |
| Q23-07: | $V_0$ | $V_4$ | $V_4$ | $V_5$: | -1 | -1 | | -1 | -3 | -1 | 1 | 1 | | 2 | 1 | 1 |
| Q23-08: | $V_0$ | $V_4$ | $V_5$ | $V_5$: | -1 | -1 | | -1 | -2 | -2 | 1 | 1 | | 1 | 2 | 1 |
| Q23-09: | $V_0$ | $V_5$ | $V_5$ | $V_5$: | -1 | -1 | | -1 | -1 | -3 | 1 | 1 | | | 3 | 1 |
| Q23-10: | $V_1$ | $V_3$ | $V_3$ | $V_3$: | | -2 | | -4 | -1 | | 2 | 1 | 3 | | | 1 |
| Q23-11: | $V_1$ | $V_3$ | $V_3$ | $V_4$: | | -2 | | -3 | -2 | | 2 | 1 | 2 | 1 | | 1 |
| Q23-12: | $V_1$ | $V_3$ | $V_3$ | $V_5$: | | -2 | | -3 | -1 | -1 | 2 | 1 | 2 | | 1 | 1 |
| Q23-13: | $V_1$ | $V_3$ | $V_4$ | $V_4$: | | -2 | | -2 | -3 | | 2 | 1 | 1 | 2 | | 1 |
| Q23-14: | $V_1$ | $V_3$ | $V_4$ | $V_5$: | | -2 | | -2 | -2 | -1 | 2 | 1 | 1 | 1 | 1 | 1 |
| Q23-15: | $V_1$ | $V_3$ | $V_5$ | $V_5$: | | -2 | | -2 | -1 | -2 | 2 | 1 | 1 | | 2 | 1 |
| Q23-16: | $V_1$ | $V_4$ | $V_4$ | $V_4$: | | -2 | | -1 | -4 | | 2 | 1 | | 3 | | 1 |
| Q23-17: | $V_1$ | $V_4$ | $V_4$ | $V_5$: | | -2 | | -1 | -3 | -1 | 2 | 1 | | 2 | 1 | 1 |
| Q23-18: | $V_1$ | $V_4$ | $V_5$ | $V_5$: | | -2 | | -1 | -2 | -2 | 2 | 1 | | 1 | 2 | 1 |
| Q23-19: | $V_1$ | $V_5$ | $V_5$ | $V_5$: | | -2 | | -1 | -1 | -3 | 2 | 1 | | | 3 | 1 |

Q24:



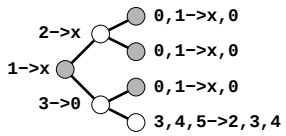

| variable | end-points | | | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q24-00: | $V_0$ | $V_0$ | $V_3$ | $V_3$: | -2 | -1 | | -4 | | | 2 | | 2 | | | 1 |
| Q24-01: | $V_0$ | $V_0$ | $V_3$ | $V_4$: | -2 | -1 | | -3 | -1 | | 2 | | 1 | 1 | | 1 |
| Q24-02: | $V_0$ | $V_0$ | $V_3$ | $V_5$: | -2 | -1 | | -3 | | -1 | 2 | | 1 | | 1 | 1 |
| Q24-03: | $V_0$ | $V_0$ | $V_4$ | $V_4$: | -2 | -1 | | -2 | -2 | | 2 | | | 2 | | 1 |
| Q24-04: | $V_0$ | $V_0$ | $V_4$ | $V_5$: | -2 | -1 | | -2 | -1 | -1 | 2 | | | 1 | 1 | 1 |
| Q24-05: | $V_0$ | $V_0$ | $V_5$ | $V_5$: | -2 | -1 | | -2 | | -2 | 2 | | | | 2 | 1 |
| Q24-06: | $V_0$ | $V_1$ | $V_3$ | $V_3$: | -1 | -2 | | -4 | | | 3 | | 2 | | | 1 |
| Q24-07: | $V_0$ | $V_1$ | $V_3$ | $V_4$: | -1 | -2 | | -3 | -1 | | 3 | | 1 | 1 | | 1 |
| Q24-08: | $V_0$ | $V_1$ | $V_3$ | $V_5$: | -1 | -2 | | -3 | | -1 | 3 | | 1 | | 1 | 1 |
| Q24-09: | $V_0$ | $V_1$ | $V_4$ | $V_4$: | -1 | -2 | | -2 | -2 | | 3 | | | 2 | | 1 |
| Q24-10: | $V_0$ | $V_1$ | $V_4$ | $V_5$: | -1 | -2 | | -2 | -1 | -1 | 3 | | | 1 | 1 | 1 |
| Q24-11: | $V_0$ | $V_1$ | $V_5$ | $V_5$: | -1 | -2 | | -2 | | -2 | 3 | | | | 2 | 1 |
| Q24-12: | $V_1$ | $V_1$ | $V_3$ | $V_3$: | | -3 | | -4 | | | 4 | | 2 | | | 1 |
| Q24-13: | $V_1$ | $V_1$ | $V_3$ | $V_4$: | | -3 | | -3 | -1 | | 4 | | 1 | 1 | | 1 |
| Q24-14: | $V_1$ | $V_1$ | $V_3$ | $V_5$: | | -3 | | -3 | | -1 | 4 | | 1 | | 1 | 1 |
| Q24-15: | $V_1$ | $V_1$ | $V_4$ | $V_4$: | | -3 | | -2 | -2 | | 4 | | | 2 | | 1 |
| Q24-16: | $V_1$ | $V_1$ | $V_4$ | $V_5$: | | -3 | | -2 | -1 | -1 | 4 | | | 1 | 1 | 1 |
| Q24-17: | $V_1$ | $V_1$ | $V_5$ | $V_5$: | | -3 | | -2 | | -2 | 4 | | | | 2 | 1 |

Q25:



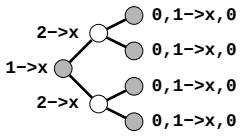

| variable | end-points | | | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q25-00: | $V_0$ | $V_0$ | $V_3$ | $V_3$: | -2 | -1 | -1 | -2 | -1 | | | 1 | 2 | | | 1 |
| Q25-01: | $V_0$ | $V_0$ | $V_3$ | $V_4$: | -2 | -1 | -1 | -1 | -2 | | | 1 | 1 | 1 | | 1 |
| Q25-02: | $V_0$ | $V_0$ | $V_3$ | $V_5$: | -2 | -1 | -1 | -1 | -1 | -1 | | 1 | 1 | | 1 | 1 |
| Q25-03: | $V_0$ | $V_0$ | $V_4$ | $V_4$: | -2 | -1 | -1 | | -3 | | | 1 | | 2 | | 1 |
| Q25-04: | $V_0$ | $V_0$ | $V_4$ | $V_5$: | -2 | -1 | -1 | | -2 | -1 | | 1 | | 1 | 1 | 1 |
| Q25-05: | $V_0$ | $V_0$ | $V_5$ | $V_5$: | -2 | -1 | -1 | | -1 | -2 | | 1 | | | 2 | 1 |
| Q25-06: | $V_0$ | $V_1$ | $V_3$ | $V_3$: | -1 | -2 | -1 | -2 | -1 | | 1 | 1 | 2 | | | 1 |
| Q25-07: | $V_0$ | $V_1$ | $V_3$ | $V_4$: | -1 | -2 | -1 | -1 | -2 | | 1 | 1 | 1 | 1 | | 1 |
| Q25-08: | $V_0$ | $V_1$ | $V_3$ | $V_5$: | -1 | -2 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | | 1 | 1 |
| Q25-09: | $V_0$ | $V_1$ | $V_4$ | $V_4$: | -1 | -2 | -1 | | -3 | | 1 | 1 | | 2 | | 1 |
| Q25-10: | $V_0$ | $V_1$ | $V_4$ | $V_5$: | -1 | -2 | -1 | | -2 | -1 | 1 | 1 | | 1 | 1 | 1 |
| Q25-11: | $V_0$ | $V_1$ | $V_5$ | $V_5$: | -1 | -2 | -1 | | -1 | -2 | 1 | 1 | | | 2 | 1 |
| Q25-12: | $V_1$ | $V_1$ | $V_3$ | $V_3$: | | -3 | -1 | -2 | -1 | | 2 | 1 | 2 | | | 1 |
| Q25-13: | $V_1$ | $V_1$ | $V_3$ | $V_4$: | | -3 | -1 | -1 | -2 | | 2 | 1 | 1 | 1 | | 1 |
| Q25-14: | $V_1$ | $V_1$ | $V_3$ | $V_5$: | | -3 | -1 | -1 | -1 | -1 | 2 | 1 | 1 | | 1 | 1 |
| Q25-15: | $V_1$ | $V_1$ | $V_4$ | $V_4$: | | -3 | -1 | | -3 | | 2 | 1 | | 2 | | 1 |
| Q25-16: | $V_1$ | $V_1$ | $V_4$ | $V_5$: | | -3 | -1 | | -2 | -1 | 2 | 1 | | 1 | 1 | 1 |
| Q25-17: | $V_1$ | $V_1$ | $V_5$ | $V_5$: | | -3 | -1 | | -1 | -2 | 2 | 1 | | | 2 | 1 |

Q26:



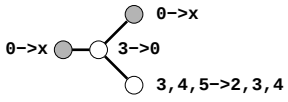

| variable | end-points | | | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q26-00: | $V_0$ | $V_0$ | $V_0$ | $V_3$: | -3 | -1 | -1 | -2 | | | 1 | | 1 | | | 1 |
| Q26-01: | $V_0$ | $V_0$ | $V_0$ | $V_4$: | -3 | -1 | -1 | -1 | -1 | | 1 | | | 1 | | 1 |
| Q26-02: | $V_0$ | $V_0$ | $V_0$ | $V_5$: | -3 | -1 | -1 | -1 | | -1 | 1 | | | | 1 | 1 |
| Q26-03: | $V_0$ | $V_0$ | $V_1$ | $V_3$: | -2 | -2 | -1 | -2 | | | 2 | | 1 | | | 1 |
| Q26-04: | $V_0$ | $V_0$ | $V_1$ | $V_4$: | -2 | -2 | -1 | -1 | -1 | | 2 | | | 1 | | 1 |
| Q26-05: | $V_0$ | $V_0$ | $V_1$ | $V_5$: | -2 | -2 | -1 | -1 | | -1 | 2 | | | | 1 | 1 |
| Q26-06: | $V_0$ | $V_1$ | $V_1$ | $V_3$: | -1 | -3 | -1 | -2 | | | 3 | | 1 | | | 1 |
| Q26-07: | $V_0$ | $V_1$ | $V_1$ | $V_4$: | -1 | -3 | -1 | -1 | -1 | | 3 | | | 1 | | 1 |
| Q26-08: | $V_0$ | $V_1$ | $V_1$ | $V_5$: | -1 | -3 | -1 | -1 | | -1 | 3 | | | | 1 | 1 |
| Q26-09: | $V_1$ | $V_1$ | $V_1$ | $V_3$: | | -4 | -1 | -2 | | | 4 | | 1 | | | 1 |
| Q26-10: | $V_1$ | $V_1$ | $V_1$ | $V_4$: | | -4 | -1 | -1 | -1 | | 4 | | | 1 | | 1 |
| Q26-11: | $V_1$ | $V_1$ | $V_1$ | $V_5$: | | -4 | -1 | -1 | | -1 | 4 | | | | 1 | 1 |

Q27:



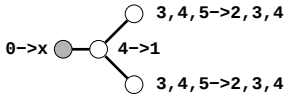

| variable | end-points | | | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q27-00: | $V_0$ | $V_0$ | $V_0$ | $V_0$: | -4 | -1 | -2 | | | | | | | | | 1 |
| Q27-01: | $V_0$ | $V_0$ | $V_0$ | $V_1$: | -3 | -2 | -2 | | | | 1 | | | | | 1 |
| Q27-02: | $V_0$ | $V_0$ | $V_1$ | $V_1$: | -2 | -3 | -2 | | | | 2 | | | | | 1 |
| Q27-03: | $V_0$ | $V_1$ | $V_1$ | $V_1$: | -1 | -4 | -2 | | | | 3 | | | | | 1 |
| Q27-04: | $V_1$ | $V_1$ | $V_1$ | $V_1$: | | -5 | -2 | | | | 4 | | | | | 1 |

Q28:



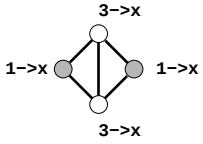

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q28-00: | | $V_3$: | -2 | | | -2 | | | 1 | | 1 | | | 1 |
| Q28-01: | | $V_4$: | -2 | | | -1 | -1 | | 1 | | | 1 | | 1 |
| Q28-02: | | $V_5$: | -2 | | | -1 | | -1 | 1 | | | | 1 | 1 |

Q29:



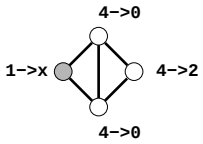

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q29-00: | $V_3$ | $V_3$: | -1 | | | -2 | -1 | | | 1 | 2 | | | 1 |
| Q29-01: | $V_3$ | $V_4$: | -1 | | | -1 | -2 | | | 1 | 1 | 1 | | 1 |
| Q29-02: | $V_3$ | $V_5$: | -1 | | | -1 | -1 | -1 | | 1 | 1 | | 1 | 1 |
| Q29-03: | $V_4$ | $V_4$: | -1 | | | | -3 | | | 1 | | 2 | | 1 |
| Q29-04: | $V_4$ | $V_5$: | -1 | | | | -2 | -1 | | 1 | | 1 | 1 | 1 |
| Q29-05: | $V_5$ | $V_5$: | -1 | | | | -1 | -2 | | 1 | | | 2 | 1 |

Q01:



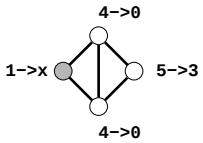

| variable | end-points | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q01-00: | | | -2 | | -2 | | | | | | | | 1 |

Q02:



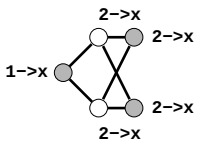

| variable | end-points | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q02-00: | | | -1 | | | -3 | | 2 | | 1 | | | 1 |

Q03:



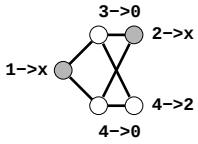

| variable | end-points | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q03-00: | | | -1 | | | -2 | -1 | 2 | | | 1 | | 1 |

Q07:



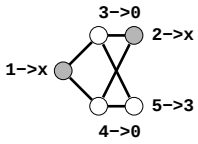

| variable | end-points | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q07-00: | | | -3 | -2 | | | | | | | | | 1 |

Q08:



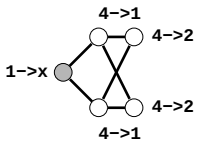

| variable | end-points | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q08-00: | | | -2 | | -2 | -1 | | | 2 | 1 | | | 1 |

Q09:



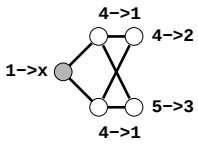

| variable | end-points | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q09-00: | | | -2 | | -2 | | -1 | | 2 | | 1 | | 1 |

Q10:



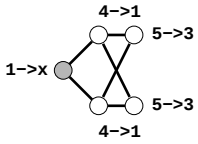

| variable | end-points | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q10-00: | | | -1 | | | -4 | | | 2 | 2 | | | 1 |

Q11:



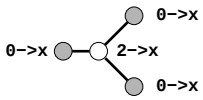

| variable | end-points | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q11-00: | | | -1 | | | -3 | -1 | | 2 | 1 | 1 | | 1 |

Q12:




| variable | end-points | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q12-00: | | | -1 | | | -2 | -2 | | 2 | | 2 | | 1 |

Q30:




| variable | end-points | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q30-00: | | -3 | | -1 | | | | | | | | | 1 |

```
    |\^/|     Maple V Release 5.1 (Macquarie University)
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 \  MAPLE  /  reserved. Maple and Maple V are registered trademarks of
 <____ ____>  Waterloo Maple Inc.
      |       Type ? for help.
> with(simplex):
Warning, new definition for maximize
Warning, new definition for minimize
>
> obj:=
1*Q04_00 +1*Q04_01 +1*Q04_02 +1*Q05_00 +1*Q05_01 +1*Q05_02 +1*Q05_03
+1*Q05_04 +1*Q05_05 +1*Q06_00 +1*Q06_01 +1*Q06_02 +1*Q06_03 +1*Q06_04
+1*Q06_05 +1*Q13_00 +1*Q13_01 +1*Q13_02 +1*Q14_00 +1*Q14_01 +1*Q14_02
+1*Q14_03 +1*Q14_04 +1*Q14_05 +1*Q15_00 +1*Q15_01 +1*Q15_02 +1*Q15_03
+1*Q15_04 +1*Q15_05 +1*Q16_00 +1*Q16_01 +1*Q16_02 +1*Q17_00 +1*Q17_01
+1*Q17_02 +1*Q18_00 +1*Q18_01 +1*Q18_02 +1*Q18_03 +1*Q18_04 +1*Q18_05
+1*Q19_00 +1*Q19_01 +1*Q19_02 +1*Q19_03 +1*Q19_04 +1*Q19_05 +1*Q20_00
+1*Q20_01 +1*Q20_02 +1*Q20_03 +1*Q20_04 +1*Q20_05 +1*Q21_00 +1*Q21_01
+1*Q21_02 +1*Q21_03 +1*Q21_04 +1*Q21_05 +1*Q22_00 +1*Q22_01 +1*Q22_02
+1*Q22_03 +1*Q22_04 +1*Q22_05 +1*Q22_06 +1*Q22_07 +1*Q22_08 +1*Q22_09
+1*Q22_10 +1*Q22_11 +1*Q22_12 +1*Q22_13 +1*Q22_14 +1*Q23_00 +1*Q23_01
+1*Q23_02 +1*Q23_03 +1*Q23_04 +1*Q23_05 +1*Q23_06 +1*Q23_07 +1*Q23_08
+1*Q23_09 +1*Q23_10 +1*Q23_11 +1*Q23_12 +1*Q23_13 +1*Q23_14 +1*Q23_15
+1*Q23_16 +1*Q23_17 +1*Q23_18 +1*Q23_19 +1*Q24_00 +1*Q24_01 +1*Q24_02
+1*Q24_03 +1*Q24_04 +1*Q24_05 +1*Q24_06 +1*Q24_07 +1*Q24_08 +1*Q24_09
+1*Q24_10 +1*Q24_11 +1*Q24_12 +1*Q24_13 +1*Q24_14 +1*Q24_15 +1*Q24_16
+1*Q24_17 +1*Q25_00 +1*Q25_01 +1*Q25_02 +1*Q25_03 +1*Q25_04 +1*Q25_05
+1*Q25_06 +1*Q25_07 +1*Q25_08 +1*Q25_09 +1*Q25_10 +1*Q25_11 +1*Q25_12
+1*Q25_13 +1*Q25_14 +1*Q25_15 +1*Q25_16 +1*Q25_17 +1*Q26_00 +1*Q26_01
+1*Q26_02 +1*Q26_03 +1*Q26_04 +1*Q26_05 +1*Q26_06 +1*Q26_07 +1*Q26_08
+1*Q26_09 +1*Q26_10 +1*Q26_11 +1*Q27_00 +1*Q27_01 +1*Q27_02 +1*Q27_03
+1*Q27_04 +1*Q28_00 +1*Q28_01 +1*Q28_02 +1*Q29_00 +1*Q29_01 +1*Q29_02
+1*Q29_03 +1*Q29_04 +1*Q29_05 +1*Q01_00 +1*Q02_00 +1*Q03_00 +1*Q07_00
+1*Q08_00 +1*Q09_00 +1*Q10_00 +1*Q11_00 +1*Q12_00 +1*Q30_00;

obj := Q04_01 + Q04_00 + Q05_00 + Q04_02 + Q05_01 + Q05_02 + Q05_03 + Q05_04

     + Q05_05 + Q06_00 + Q06_01 + Q06_02 + Q06_03 + Q06_04 + Q06_05 + Q13_00

     + Q13_01 + Q13_02 + Q14_00 + Q14_01 + Q14_02 + Q14_03 + Q14_04 + Q14_05

     + Q15_00 + Q15_01 + Q15_02 + Q15_03 + Q15_04 + Q15_05 + Q16_00 + Q16_01

     + Q16_02 + Q17_00 + Q17_01 + Q17_02 + Q18_00 + Q18_01 + Q18_02 + Q18_03

     + Q18_04 + Q18_05 + Q19_00 + Q19_01 + Q19_02 + Q19_03 + Q19_04 + Q19_05

     + Q20_00 + Q20_01 + Q20_02 + Q20_03 + Q20_04 + Q20_05 + Q21_00 + Q21_01

     + Q21_02 + Q21_03 + Q21_04 + Q21_05 + Q22_00 + Q22_01 + Q22_02 + Q22_03

     + Q22_04 + Q22_05 + Q22_06 + Q22_07 + Q22_08 + Q22_09 + Q22_10 + Q22_11

     + Q22_12 + Q22_13 + Q22_14 + Q23_00 + Q23_01 + Q23_02 + Q23_03 + Q23_04

     + Q23_05 + Q23_06 + Q23_07 + Q23_08 + Q23_09 + Q23_10 + Q23_11 + Q23_12

     + Q23_13 + Q23_14 + Q23_15 + Q23_16 + Q23_17 + Q23_18 + Q23_19 + Q24_00
```

```
     + Q24_01 + Q24_02 + Q24_03 + Q24_04 + Q24_05 + Q24_06 + Q24_07 + Q24_08

     + Q24_09 + Q24_10 + Q24_11 + Q24_12 + Q24_13 + Q24_14 + Q24_15 + Q24_16

     + Q24_17 + Q25_00 + Q25_01 + Q25_02 + Q25_03 + Q25_04 + Q25_05 + Q25_06

     + Q25_07 + Q25_08 + Q25_09 + Q25_10 + Q25_11 + Q25_12 + Q25_13 + Q25_14

     + Q25_15 + Q25_16 + Q25_17 + Q26_00 + Q26_01 + Q26_02 + Q26_03 + Q26_04

     + Q26_05 + Q26_06 + Q26_07 + Q26_08 + Q26_09 + Q26_10 + Q26_11 + Q27_00

     + Q27_01 + Q27_02 + Q27_03 + Q27_04 + Q28_00 + Q28_01 + Q28_02 + Q29_00

     + Q29_01 + Q29_02 + Q29_03 + Q29_04 + Q29_05 + Q01_00 + Q02_00 + Q03_00

     + Q07_00 + Q08_00 + Q09_00 + Q10_00 + Q11_00 + Q12_00 + Q30_00
>
> cnsts:={
>
-1*Q04_02 -1*Q13_01 -2*Q13_02 -1*Q14_03 -1*Q14_04 -1*Q14_05 -1*Q16_02
-1*Q17_02 +1*Q20_00 +1*Q20_01 +1*Q20_02 +1*Q20_03 +1*Q20_04 +1*Q20_05
+1*Q21_00 +1*Q21_01 +1*Q21_02 +1*Q21_03 +1*Q21_04 +1*Q21_05 +1*Q22_00
+1*Q22_01 +1*Q22_02 +1*Q22_03 +1*Q22_04 +1*Q22_05 +1*Q22_06 +1*Q22_07
+1*Q22_08 +1*Q22_09 +1*Q22_10 +1*Q22_11 +1*Q22_12 +1*Q22_13 +1*Q22_14
-1*Q24_12 -1*Q24_13 -1*Q24_14 -1*Q24_15 -1*Q24_16 -1*Q24_17 -1*Q25_12
-1*Q25_13 -1*Q25_14 -1*Q25_15 -1*Q25_16 -1*Q25_17 -1*Q26_06 -1*Q26_07
-1*Q26_08 -2*Q26_09 -2*Q26_10 -2*Q26_11 -1*Q27_02 -2*Q27_03 -3*Q27_04
-1*Q07_00 +1*Q10_00 +1*Q11_00 +1*Q12_00 >= 0,
>
+2*Q02_00 +2*Q03_00 -2*Q04_00 +2*Q04_02 +2*Q05_03 +2*Q05_04 +2*Q05_05
+2*Q06_00 +2*Q06_01 +2*Q06_02 +2*Q06_03 +2*Q06_04 +2*Q06_05 -2*Q13_00
+2*Q13_02 +2*Q14_03 +2*Q14_04 +2*Q14_05 +2*Q15_00 +2*Q15_01 +2*Q15_02
+2*Q15_03 +2*Q15_04 +2*Q15_05 +2*Q16_01 +4*Q16_02 +2*Q17_01 +4*Q17_02
+2*Q18_03 +2*Q18_04 +2*Q18_05 +2*Q19_03 +2*Q19_04 +2*Q19_05 +2*Q23_10
+2*Q23_11 +2*Q23_12 +2*Q23_13 +2*Q23_14 +2*Q23_15 +2*Q23_16 +2*Q23_17
+2*Q23_18 +2*Q23_19 +2*Q24_06 +2*Q24_07 +2*Q24_08 +2*Q24_09 +2*Q24_10
+2*Q24_11 +4*Q24_12 +4*Q24_13 +4*Q24_14 +4*Q24_15 +4*Q24_16 +4*Q24_17
-2*Q25_00 -2*Q25_01 -2*Q25_02 -2*Q25_03 -2*Q25_04 -2*Q25_05 +2*Q25_12
+2*Q25_13 +2*Q25_14 +2*Q25_15 +2*Q25_16 +2*Q25_17 -2*Q26_00 -2*Q26_01
-2*Q26_02 +2*Q26_06 +2*Q26_07 +2*Q26_08 +4*Q26_09 +4*Q26_10 +4*Q26_11
-4*Q27_00 -2*Q27_01 +2*Q27_03 +4*Q27_04 -1*Q28_00 -1*Q28_01 -1*Q28_02
-1*Q29_00 -1*Q29_01 -1*Q29_02 -1*Q29_03 -1*Q29_04 -1*Q29_05 -3*Q30_00
>= 0,
>
-2*Q01_00 -1*Q02_00 -1*Q03_00 -1*Q04_00 -2*Q04_01 -3*Q04_02 -1*Q05_00
-1*Q05_01 -1*Q05_02 -2*Q05_03 -2*Q05_04 -2*Q05_05 -1*Q06_00 -1*Q06_01
-1*Q06_02 -1*Q06_03 -1*Q06_04 -1*Q06_05 -3*Q07_00 +1*Q10_00 +1*Q11_00
+1*Q12_00 -2*Q13_00 -3*Q13_01 -4*Q13_02 -2*Q14_00 -2*Q14_01 -2*Q14_02
-3*Q14_03 -3*Q14_04 -3*Q14_05 -2*Q15_00 -2*Q15_01 -2*Q15_02 -2*Q15_03
-2*Q15_04 -2*Q15_05 -1*Q16_00 -2*Q16_01 -3*Q16_02 -1*Q17_00 -2*Q17_01
-3*Q17_02 -1*Q18_03 -1*Q18_04 -1*Q18_05 -1*Q19_03 -1*Q19_04 -1*Q19_05
+1*Q20_00 +1*Q20_01 +1*Q20_02 +1*Q20_03 +1*Q20_04 +1*Q20_05 +1*Q21_00
+1*Q21_01 +1*Q21_02 +1*Q21_03 +1*Q21_04 +1*Q21_05 +1*Q22_00 +1*Q22_01
+1*Q22_02 +1*Q22_03 +1*Q22_04 +1*Q22_05 +1*Q22_06 +1*Q22_07 +1*Q22_08
+1*Q22_09 +1*Q22_10 +1*Q22_11 +1*Q22_12 +1*Q22_13 +1*Q22_14 -1*Q23_10
-1*Q23_11 -1*Q23_12 -1*Q23_13 -1*Q23_14 -1*Q23_15 -1*Q23_16 -1*Q23_17
-1*Q23_18 -1*Q23_19 -1*Q24_00 -1*Q24_01 -1*Q24_02 -1*Q24_03 -1*Q24_04
-1*Q24_05 -2*Q24_06 -2*Q24_07 -2*Q24_08 -2*Q24_09 -2*Q24_10 -2*Q24_11
```

```
-3*Q24_12 -3*Q24_13 -3*Q24_14 -3*Q24_15 -3*Q24_16 -3*Q24_17 -1*Q25_06
-1*Q25_07 -1*Q25_08 -1*Q25_09 -1*Q25_10 -1*Q25_11 -2*Q25_12 -2*Q25_13
-2*Q25_14 -2*Q25_15 -2*Q25_16 -2*Q25_17 -1*Q26_00 -1*Q26_01 -1*Q26_02
-2*Q26_03 -2*Q26_04 -2*Q26_05 -3*Q26_06 -3*Q26_07 -3*Q26_08 -4*Q26_09
-4*Q26_10 -4*Q26_11 -1*Q27_00 -2*Q27_01 -3*Q27_02 -4*Q27_03 -5*Q27_04
+1*Q29_00 +1*Q29_01 +1*Q29_02 +1*Q29_03 +1*Q29_04 +1*Q29_05  >= 0,
>
+1*Q02_00 +1*Q05_00 +1*Q05_03 +2*Q06_00 +1*Q06_01 +1*Q06_02 -2*Q07_00
+1*Q08_00 +2*Q10_00 +1*Q11_00 -2*Q13_00 -2*Q13_01 -2*Q13_02 -1*Q14_01
-1*Q14_02 -1*Q14_04 -1*Q14_05 +2*Q15_00 +1*Q15_01 +1*Q15_02 +1*Q16_00
+1*Q16_01 +1*Q16_02 +2*Q18_00 +1*Q18_01 +1*Q18_02 +2*Q18_03 +1*Q18_04
+1*Q18_05 +1*Q19_00 +1*Q19_03 +3*Q20_00 +2*Q20_01 +2*Q20_02 +1*Q20_03
+1*Q20_04 +1*Q20_05 +2*Q21_00 +1*Q21_01 +1*Q21_02 +4*Q22_00 +3*Q22_01
+3*Q22_02 +2*Q22_03 +2*Q22_04 +2*Q22_05 +1*Q22_06 +1*Q22_07 +1*Q22_08
+1*Q22_09 +3*Q23_00 +2*Q23_01 +2*Q23_02 +1*Q23_03 +1*Q23_04 +1*Q23_05
+3*Q23_10 +2*Q23_11 +2*Q23_12 +1*Q23_13 +1*Q23_14 +1*Q23_15 +2*Q24_00
+1*Q24_01 +1*Q24_02 +2*Q24_06 +1*Q24_07 +1*Q24_08 +2*Q24_12 +1*Q24_13
+1*Q24_14 +1*Q25_00 -1*Q25_03 -1*Q25_04 -1*Q25_05 +1*Q25_06 -1*Q25_09
-1*Q25_10 -1*Q25_11 +1*Q25_12 -1*Q25_15 -1*Q25_16 -1*Q25_17 -1*Q26_01
-1*Q26_02 -1*Q26_04 -1*Q26_05 -1*Q26_07 -1*Q26_08 -1*Q26_10 -1*Q26_11
-2*Q27_00 -2*Q27_01 -2*Q27_02 -2*Q27_03 -2*Q27_04 +1*Q28_00 +2*Q29_00
+1*Q29_01 +1*Q29_02 -1*Q30_00  >= 0,
>
-2*Q01_00 +1*Q03_00 -2*Q04_00 -2*Q04_01 -2*Q04_02 -2*Q05_00 -1*Q05_02
-2*Q05_03 -1*Q05_05 -2*Q06_00 -1*Q06_02 +2*Q06_03 +1*Q06_04 -2*Q08_00
-1*Q09_00 +1*Q11_00 +2*Q12_00 -2*Q14_00 -1*Q14_02 -2*Q14_03 -1*Q14_05
-4*Q15_00 -2*Q15_01 -3*Q15_02 -1*Q15_04 -2*Q15_05 -2*Q16_00 -2*Q16_01
-2*Q16_02 -1*Q17_00 -1*Q17_01 -1*Q17_02 -2*Q18_00 -1*Q18_02 -2*Q18_03
-1*Q18_05 -1*Q19_00 +1*Q19_01 -1*Q19_03 +1*Q19_04 -2*Q20_00 -1*Q20_02
+2*Q20_03 +1*Q20_04 -1*Q21_00 +1*Q21_01 +3*Q21_03 +2*Q21_04 +1*Q21_05
-4*Q22_00 -2*Q22_01 -3*Q22_02 -1*Q22_04 -2*Q22_05 +2*Q22_06 +1*Q22_07
-1*Q22_09 +4*Q22_10 +3*Q22_11 +2*Q22_12 +1*Q22_13 -4*Q23_00 -2*Q23_01
-3*Q23_02 -1*Q23_04 -2*Q23_05 +2*Q23_06 +1*Q23_07 -1*Q23_09 -4*Q23_10
-2*Q23_11 -3*Q23_12 -1*Q23_14 -2*Q23_15 +2*Q23_16 +1*Q23_17 -1*Q23_19
-4*Q24_00 -2*Q24_01 -3*Q24_02 -1*Q24_04 -2*Q24_05 -4*Q24_06 -2*Q24_07
-3*Q24_08 -1*Q24_10 -2*Q24_11 -4*Q24_12 -2*Q24_13 -3*Q24_14 -1*Q24_16
-2*Q24_17 -2*Q25_00 -1*Q25_02 +2*Q25_03 +1*Q25_04 -2*Q25_06 -1*Q25_08
+2*Q25_09 +1*Q25_10 -2*Q25_12 -1*Q25_14 +2*Q25_15 +1*Q25_16 -2*Q26_00
-1*Q26_02 -2*Q26_03 -1*Q26_05 -2*Q26_06 -1*Q26_08 -2*Q26_09 -1*Q26_11
-2*Q28_00 -1*Q28_02 -2*Q29_00 -1*Q29_02 +2*Q29_03 +1*Q29_04  >= 0,
>
-3*Q02_00 -2*Q03_00 -1*Q05_00 -2*Q05_01 -1*Q05_03 -2*Q05_04 -2*Q06_00
-3*Q06_01 -1*Q06_02 -4*Q06_03 -2*Q06_04 -1*Q08_00 -4*Q10_00 -3*Q11_00
-2*Q12_00 -1*Q14_01 +1*Q14_02 -1*Q14_04 +1*Q14_05 -1*Q15_01 +1*Q15_02
-2*Q15_03 +2*Q15_05 -1*Q16_00 -1*Q16_01 -1*Q16_02 -2*Q18_00 -3*Q18_01
-1*Q18_02 -2*Q18_03 -3*Q18_04 -1*Q18_05 -1*Q19_00 -2*Q19_01 -1*Q19_03
-2*Q19_04 -3*Q20_00 -4*Q20_01 -2*Q20_02 -5*Q20_03 -3*Q20_04 -1*Q20_05
-2*Q21_00 -3*Q21_01 -1*Q21_02 -4*Q21_03 -2*Q21_04 -2*Q22_00 -3*Q22_01
-1*Q22_02 -4*Q22_03 -2*Q22_04 -5*Q22_06 -3*Q22_07 -1*Q22_08 +1*Q22_09
-6*Q22_10 -4*Q22_11 -2*Q22_12 +2*Q22_14 -1*Q23_00 -2*Q23_01 -3*Q23_03
-1*Q23_04 +1*Q23_05 -4*Q23_06 -2*Q23_07 +2*Q23_09 -1*Q23_10 -2*Q23_11
-3*Q23_13 -1*Q23_14 +1*Q23_15 -4*Q23_16 -2*Q23_17 +2*Q23_19 -1*Q24_01
+1*Q24_02 -2*Q24_03 +2*Q24_05 -1*Q24_07 +1*Q24_08 -2*Q24_09 +2*Q24_11
-1*Q24_13 +1*Q24_14 -2*Q24_15 +2*Q24_17 -1*Q25_00 -2*Q25_01 -3*Q25_03
-1*Q25_04 +1*Q25_05 -1*Q25_06 -2*Q25_07 -3*Q25_09 -1*Q25_10 +1*Q25_11
-1*Q25_12 -2*Q25_13 -3*Q25_15 -1*Q25_16 +1*Q25_17 -1*Q26_01 +1*Q26_02
-1*Q26_04 +1*Q26_05 -1*Q26_07 +1*Q26_08 -1*Q26_10 +1*Q26_11 -1*Q28_01
+1*Q28_02 -1*Q29_00 -2*Q29_01 -3*Q29_03 -1*Q29_04 +1*Q29_05  >= 0,
>
```

```
-1*Q03_00 -1*Q05_02 -1*Q05_05 -1*Q06_02 -1*Q06_04 -2*Q06_05 -1*Q09_00
-1*Q11_00 -2*Q12_00 -1*Q14_02 -1*Q14_05 -1*Q15_02 -1*Q15_04 -2*Q15_05
-1*Q17_00 -1*Q17_01 -1*Q17_02 -1*Q18_02 -1*Q18_05 -1*Q19_00 -1*Q19_01
-2*Q19_02 -1*Q19_03 -1*Q19_04 -2*Q19_05 -1*Q20_02 -1*Q20_04 -2*Q20_05
-1*Q21_00 -1*Q21_01 -2*Q21_02 -1*Q21_03 -2*Q21_04 -3*Q21_05 -1*Q22_02
-1*Q22_04 -2*Q22_05 -1*Q22_07 -2*Q22_08 -3*Q22_09 -1*Q22_11 -2*Q22_12
-3*Q22_13 -4*Q22_14 -1*Q23_02 -1*Q23_04 -2*Q23_05 -1*Q23_07 -2*Q23_08
-3*Q23_09 -1*Q23_12 -1*Q23_14 -2*Q23_15 -1*Q23_17 -2*Q23_18 -3*Q23_19
-1*Q24_02 -1*Q24_04 -2*Q24_05 -1*Q24_08 -1*Q24_10 -2*Q24_11 -1*Q24_14
-1*Q24_16 -2*Q24_17 -1*Q25_02 -1*Q25_04 -2*Q25_05 -1*Q25_08 -1*Q25_10
-2*Q25_11 -1*Q25_14 -1*Q25_16 -2*Q25_17 -1*Q26_02 -1*Q26_05 -1*Q26_08
-1*Q26_11 -1*Q28_02 -1*Q29_02 -1*Q29_04 -2*Q29_05  >= -1};

cnsts := {0 <= -Q04_02 - Q13_01 - 2 Q13_02 - Q14_03 - Q14_04 - Q14_05 - Q16_02

     - Q17_02 + Q20_00 + Q20_01 + Q20_02 + Q20_03 + Q20_04 + Q20_05 + Q21_00

     + Q21_01 + Q21_02 + Q21_03 + Q21_04 + Q21_05 + Q22_00 + Q22_01 + Q22_02

     + Q22_03 + Q22_04 + Q22_05 + Q22_06 + Q22_07 + Q22_08 + Q22_09 + Q22_10

     + Q22_11 + Q22_12 + Q22_13 + Q22_14 - Q24_12 - Q24_13 - Q24_14 - Q24_15

     - Q24_16 - Q24_17 - Q25_12 - Q25_13 - Q25_14 - Q25_15 - Q25_16 - Q25_17

     - Q26_06 - Q26_07 - Q26_08 - 2 Q26_09 - 2 Q26_10 - 2 Q26_11 - Q27_02

     - 2 Q27_03 - 3 Q27_04 - Q07_00 + Q10_00 + Q11_00 + Q12_00, 0 <= -2 Q04_00

     + 2 Q04_02 + 2 Q05_03 + 2 Q05_04 + 2 Q05_05 + 2 Q06_00 + 2 Q06_01

     + 2 Q06_02 + 2 Q06_03 + 2 Q06_04 + 2 Q06_05 - 2 Q13_00 + 2 Q13_02

     + 2 Q14_03 + 2 Q14_04 + 2 Q14_05 + 2 Q15_00 + 2 Q15_01 + 2 Q15_02

     + 2 Q15_03 + 2 Q15_04 + 2 Q15_05 + 2 Q16_01 + 4 Q16_02 + 2 Q17_01

     + 4 Q17_02 + 2 Q18_03 + 2 Q18_04 + 2 Q18_05 + 2 Q19_03 + 2 Q19_04

     + 2 Q19_05 + 2 Q23_10 + 2 Q23_11 + 2 Q23_12 + 2 Q23_13 + 2 Q23_14

     + 2 Q23_15 + 2 Q23_16 + 2 Q23_17 + 2 Q23_18 + 2 Q23_19 + 2 Q24_06

     + 2 Q24_07 + 2 Q24_08 + 2 Q24_09 + 2 Q24_10 + 2 Q24_11 + 4 Q24_12

     + 4 Q24_13 + 4 Q24_14 + 4 Q24_15 + 4 Q24_16 + 4 Q24_17 - 2 Q25_00

     - 2 Q25_01 - 2 Q25_02 - 2 Q25_03 - 2 Q25_04 - 2 Q25_05 + 2 Q25_12

     + 2 Q25_13 + 2 Q25_14 + 2 Q25_15 + 2 Q25_16 + 2 Q25_17 - 2 Q26_00

     - 2 Q26_01 - 2 Q26_02 + 2 Q26_06 + 2 Q26_07 + 2 Q26_08 + 4 Q26_09

     + 4 Q26_10 + 4 Q26_11 - 4 Q27_00 - 2 Q27_01 + 2 Q27_03 + 4 Q27_04 - Q28_00

     - Q28_01 - Q28_02 - Q29_00 - Q29_01 - Q29_02 - Q29_03 - Q29_04 - Q29_05

     + 2 Q02_00 + 2 Q03_00 - 3 Q30_00, 0 <= -2 Q04_01 - Q04_00 - Q05_00
```

- 3 Q04_02 - Q05_01 - Q05_02 - 2 Q05_03 - 2 Q05_04 - 2 Q05_05 - Q06_00
- Q06_01 - Q06_02 - Q06_03 - Q06_04 - Q06_05 - 2 Q13_00 - 3 Q13_01
- 4 Q13_02 - 2 Q14_00 - 2 Q14_01 - 2 Q14_02 - 3 Q14_03 - 3 Q14_04
- 3 Q14_05 - 2 Q15_00 - 2 Q15_01 - 2 Q15_02 - 2 Q15_03 - 2 Q15_04
- 2 Q15_05 - Q16_00 - 2 Q16_01 - 3 Q16_02 - Q17_00 - 2 Q17_01 - 3 Q17_02
- Q18_03 - Q18_04 - Q18_05 - Q19_03 - Q19_04 - Q19_05 + Q20_00 + Q20_01
+ Q20_02 + Q20_03 + Q20_04 + Q20_05 + Q21_00 + Q21_01 + Q21_02 + Q21_03
+ Q21_04 + Q21_05 + Q22_00 + Q22_01 + Q22_02 + Q22_03 + Q22_04 + Q22_05
+ Q22_06 + Q22_07 + Q22_08 + Q22_09 + Q22_10 + Q22_11 + Q22_12 + Q22_13
+ Q22_14 - Q23_10 - Q23_11 - Q23_12 - Q23_13 - Q23_14 - Q23_15 - Q23_16
- Q23_17 - Q23_18 - Q23_19 - Q24_00 - Q24_01 - Q24_02 - Q24_03 - Q24_04
- Q24_05 - 2 Q24_06 - 2 Q24_07 - 2 Q24_08 - 2 Q24_09 - 2 Q24_10 - 2 Q24_11
- 3 Q24_12 - 3 Q24_13 - 3 Q24_14 - 3 Q24_15 - 3 Q24_16 - 3 Q24_17 - Q25_06
- Q25_07 - Q25_08 - Q25_09 - Q25_10 - Q25_11 - 2 Q25_12 - 2 Q25_13
- 2 Q25_14 - 2 Q25_15 - 2 Q25_16 - 2 Q25_17 - Q26_00 - Q26_01 - Q26_02
- 2 Q26_03 - 2 Q26_04 - 2 Q26_05 - 3 Q26_06 - 3 Q26_07 - 3 Q26_08
- 4 Q26_09 - 4 Q26_10 - 4 Q26_11 - Q27_00 - 2 Q27_01 - 3 Q27_02 - 4 Q27_03
- 5 Q27_04 + Q29_00 + Q29_01 + Q29_02 + Q29_03 + Q29_04 + Q29_05
- 2 Q01_00 - Q02_00 - Q03_00 - 3 Q07_00 + Q10_00 + Q11_00 + Q12_00, 0 <=
Q05_00 + Q05_03 + 2 Q06_00 + Q06_01 + Q06_02 - 2 Q13_00 - 2 Q13_01
- 2 Q13_02 - Q14_01 - Q14_02 - Q14_04 - Q14_05 + 2 Q15_00 + Q15_01
+ Q15_02 + Q16_00 + Q16_01 + Q16_02 + 2 Q18_00 + Q18_01 + Q18_02
+ 2 Q18_03 + Q18_04 + Q18_05 + Q19_00 + Q19_03 + 3 Q20_00 + 2 Q20_01
+ 2 Q20_02 + Q20_03 + Q20_04 + Q20_05 + 2 Q21_00 + Q21_01 + Q21_02
+ 4 Q22_00 + 3 Q22_01 + 3 Q22_02 + 2 Q22_03 + 2 Q22_04 + 2 Q22_05 + Q22_06
+ Q22_07 + Q22_08 + Q22_09 + 3 Q23_00 + 2 Q23_01 + 2 Q23_02 + Q23_03
+ Q23_04 + Q23_05 + 3 Q23_10 + 2 Q23_11 + 2 Q23_12 + Q23_13 + Q23_14
+ Q23_15 + 2 Q24_00 + Q24_01 + Q24_02 + 2 Q24_06 + Q24_07 + Q24_08
+ 2 Q24_12 + Q24_13 + Q24_14 + Q25_00 - Q25_03 - Q25_04 - Q25_05 + Q25_06
- Q25_09 - Q25_10 - Q25_11 + Q25_12 - Q25_15 - Q25_16 - Q25_17 - Q26_01

- Q26_02 - Q26_04 - Q26_05 - Q26_07 - Q26_08 - Q26_10 - Q26_11 - 2 Q27_00
- 2 Q27_01 - 2 Q27_02 - 2 Q27_03 - 2 Q27_04 + Q28_00 + 2 Q29_00 + Q29_01
+ Q29_02 + Q02_00 - 2 Q07_00 + Q08_00 + 2 Q10_00 + Q11_00 - Q30_00, 0 <=
-2 Q04_01 - 2 Q04_00 - 2 Q05_00 - 2 Q04_02 - Q05_02 - 2 Q05_03 - Q05_05
- 2 Q06_00 - Q06_02 + 2 Q06_03 + Q06_04 - 2 Q14_00 - Q14_02 - 2 Q14_03
- Q14_05 - 4 Q15_00 - 2 Q15_01 - 3 Q15_02 - Q15_04 - 2 Q15_05 - 2 Q16_00
- 2 Q16_01 - 2 Q16_02 - Q17_00 - Q17_01 - Q17_02 - 2 Q18_00 - Q18_02
- 2 Q18_03 - Q18_05 - Q19_00 + Q19_01 - Q19_03 + Q19_04 - 2 Q20_00
- Q20_02 + 2 Q20_03 + Q20_04 - Q21_00 + Q21_01 + 3 Q21_03 + 2 Q21_04
+ Q21_05 - 4 Q22_00 - 2 Q22_01 - 3 Q22_02 - Q22_04 - 2 Q22_05 + 2 Q22_06
+ Q22_07 - Q22_09 + 4 Q22_10 + 3 Q22_11 + 2 Q22_12 + Q22_13 - 4 Q23_00
- 2 Q23_01 - 3 Q23_02 - Q23_04 - 2 Q23_05 + 2 Q23_06 + Q23_07 - Q23_09
- 4 Q23_10 - 2 Q23_11 - 3 Q23_12 - Q23_14 - 2 Q23_15 + 2 Q23_16 + Q23_17
- Q23_19 - 4 Q24_00 - 2 Q24_01 - 3 Q24_02 - Q24_04 - 2 Q24_05 - 4 Q24_06
- 2 Q24_07 - 3 Q24_08 - Q24_10 - 2 Q24_11 - 4 Q24_12 - 2 Q24_13 - 3 Q24_14
- Q24_16 - 2 Q24_17 - 2 Q25_00 - Q25_02 + 2 Q25_03 + Q25_04 - 2 Q25_06
- Q25_08 + 2 Q25_09 + Q25_10 - 2 Q25_12 - Q25_14 + 2 Q25_15 + Q25_16
- 2 Q26_00 - Q26_02 - 2 Q26_03 - Q26_05 - 2 Q26_06 - Q26_08 - 2 Q26_09
- Q26_11 - 2 Q28_00 - Q28_02 - 2 Q29_00 - Q29_02 + 2 Q29_03 + Q29_04
- 2 Q01_00 + Q03_00 - 2 Q08_00 - Q09_00 + Q11_00 + 2 Q12_00, 0 <= -Q05_00
- 2 Q05_01 - Q05_03 - 2 Q05_04 - 2 Q06_00 - 3 Q06_01 - Q06_02 - 4 Q06_03
- 2 Q06_04 - Q14_01 + Q14_02 - Q14_04 + Q14_05 - Q15_01 + Q15_02
- 2 Q15_03 + 2 Q15_05 - Q16_00 - Q16_01 - Q16_02 - 2 Q18_00 - 3 Q18_01
- Q18_02 - 2 Q18_03 - 3 Q18_04 - Q18_05 - Q19_00 - 2 Q19_01 - Q19_03
- 2 Q19_04 - 3 Q20_00 - 4 Q20_01 - 2 Q20_02 - 5 Q20_03 - 3 Q20_04 - Q20_05
- 2 Q21_00 - 3 Q21_01 - Q21_02 - 4 Q21_03 - 2 Q21_04 - 2 Q22_00 - 3 Q22_01
- Q22_02 - 4 Q22_03 - 2 Q22_04 - 5 Q22_06 - 3 Q22_07 - Q22_08 + Q22_09
- 6 Q22_10 - 4 Q22_11 - 2 Q22_12 + 2 Q22_14 - Q23_00 - 2 Q23_01 - 3 Q23_03
- Q23_04 + Q23_05 - 4 Q23_06 - 2 Q23_07 + 2 Q23_09 - Q23_10 - 2 Q23_11

```
      - 3 Q23_13 - Q23_14 + Q23_15 - 4 Q23_16 - 2 Q23_17 + 2 Q23_19 - Q24_01
      + Q24_02 - 2 Q24_03 + 2 Q24_05 - Q24_07 + Q24_08 - 2 Q24_09 + 2 Q24_11
      - Q24_13 + Q24_14 - 2 Q24_15 + 2 Q24_17 - Q25_00 - 2 Q25_01 - 3 Q25_03
      - Q25_04 + Q25_05 - Q25_06 - 2 Q25_07 - 3 Q25_09 - Q25_10 + Q25_11
      - Q25_12 - 2 Q25_13 - 3 Q25_15 - Q25_16 + Q25_17 - Q26_01 + Q26_02
      - Q26_04 + Q26_05 - Q26_07 + Q26_08 - Q26_10 + Q26_11 - Q28_01 + Q28_02
      - Q29_00 - 2 Q29_01 - 3 Q29_03 - Q29_04 + Q29_05 - 3 Q02_00 - 2 Q03_00
      - Q08_00 - 4 Q10_00 - 3 Q11_00 - 2 Q12_00, -1 <= -Q05_02 - Q05_05 - Q06_02
      - Q06_04 - 2 Q06_05 - Q14_02 - Q14_05 - Q15_02 - Q15_04 - 2 Q15_05
      - Q17_00 - Q17_01 - Q17_02 - Q18_02 - Q18_05 - Q19_00 - Q19_01 - 2 Q19_02
      - Q19_03 - Q19_04 - 2 Q19_05 - Q20_02 - Q20_04 - 2 Q20_05 - Q21_00
      - Q21_01 - 2 Q21_02 - Q21_03 - 2 Q21_04 - 3 Q21_05 - Q22_02 - Q22_04
      - 2 Q22_05 - Q22_07 - 2 Q22_08 - 3 Q22_09 - Q22_11 - 2 Q22_12 - 3 Q22_13
      - 4 Q22_14 - Q23_02 - Q23_04 - 2 Q23_05 - Q23_07 - 2 Q23_08 - 3 Q23_09
      - Q23_12 - Q23_14 - 2 Q23_15 - Q23_17 - 2 Q23_18 - 3 Q23_19 - Q24_02
      - Q24_04 - 2 Q24_05 - Q24_08 - Q24_10 - 2 Q24_11 - Q24_14 - Q24_16
      - 2 Q24_17 - Q25_02 - Q25_04 - 2 Q25_05 - Q25_08 - Q25_10 - 2 Q25_11
      - Q25_14 - Q25_16 - 2 Q25_17 - Q26_02 - Q26_05 - Q26_08 - Q26_11 - Q28_02
      - Q29_02 - Q29_04 - 2 Q29_05 - Q03_00 - Q09_00 - Q11_00 - 2 Q12_00}
> maximize(obj,cnsts,NONNEGATIVE);
{Q14_05 = 0, Q15_00 = 0, Q15_01 = 0, Q15_02 = 0, Q15_03 = 0, Q14_01 = 0,
    Q14_02 = 0, Q14_03 = 0, Q14_04 = 0, Q13_00 = 0, Q13_01 = 0, Q13_02 = 0,
    Q14_00 = 0, Q06_04 = 0, Q06_05 = 0, Q24_17 = 0, Q29_02 = 0, Q23_19 = 1/4,
    Q06_01 = 0, Q06_02 = 0, Q06_03 = 0, Q06_00 = 0, Q05_03 = 0, Q05_04 = 0,
    Q05_05 = 0, Q05_02 = 0, Q05_00 = 0, Q04_02 = 0, Q05_01 = 0, Q04_00 = 0,
    Q04_01 = 0, Q18_02 = 0, Q18_03 = 0, Q18_04 = 0, Q18_05 = 0, Q19_00 = 0,
    Q17_01 = 0, Q17_02 = 0, Q18_00 = 0, Q18_01 = 0, Q16_02 = 0, Q17_00 = 0,
    Q16_01 = 0, Q15_04 = 0, Q15_05 = 0, Q16_00 = 0, Q22_03 = 0, Q22_04 = 0,
    Q22_05 = 0, Q22_02 = 0, Q21_04 = 0, Q21_05 = 0, Q22_00 = 0, Q22_01 = 0,
    Q21_02 = 0, Q21_03 = 0, Q20_03 = 0, Q20_04 = 0, Q20_05 = 0, Q21_00 = 0,
```

```
    Q21_01 = 0, Q19_05 = 0, Q20_00 = 0, Q20_01 = 0, Q20_02 = 0, Q19_01 = 0,
    Q19_02 = 0, Q19_03 = 0, Q19_04 = 0, Q23_09 = 0, Q23_10 = 0, Q23_11 = 0,
    Q23_12 = 0, Q23_06 = 0, Q23_07 = 0, Q23_08 = 0, Q23_04 = 0, Q23_05 = 0,
    Q22_07 = 0, Q22_08 = 0, Q22_06 = 0, Q23_00 = 0, Q23_01 = 0, Q23_02 = 0,
    Q23_03 = 0, Q22_09 = 0, Q22_10 = 0, Q22_11 = 0, Q22_12 = 0, Q22_13 = 0,
    Q22_14 = 0, Q24_11 = 0, Q24_12 = 0, Q24_14 = 0, Q24_15 = 0, Q24_08 = 0,
    Q24_09 = 0, Q24_10 = 0, Q24_07 = 0, Q23_13 = 0, Q23_14 = 0, Q23_15 = 0,
    Q24_02 = 0, Q24_03 = 0, Q24_04 = 0, Q24_01 = 0, Q23_16 = 0, Q23_17 = 0,
    Q23_18 = 0, Q24_00 = 0, Q25_01 = 0, Q24_16 = 0, Q25_00 = 0, Q25_02 = 0,
    Q25_03 = 0, Q24_06 = 0, Q24_05 = 0, Q24_13 = 0, Q25_05 = 0, Q25_04 = 0,
    Q25_06 = 0, Q25_07 = 0, Q25_08 = 0, Q25_09 = 0, Q25_10 = 0, Q25_11 = 0,
    Q25_12 = 0, Q26_02 = 0, Q26_03 = 0, Q26_04 = 0, Q26_05 = 0, Q26_06 = 0,
    Q25_16 = 0, Q25_17 = 0, Q26_00 = 0, Q26_01 = 0, Q25_15 = 0, Q25_13 = 0,
    Q25_14 = 0, Q26_10 = 0, Q26_11 = 0, Q27_00 = 0, Q27_01 = 0, Q12_00 = 0,
    Q30_00 = 0, Q08_00 = 0, Q09_00 = 0, Q10_00 = 0, Q11_00 = 0, Q01_00 = 0,
    Q02_00 = 0, Q03_00 = 0, Q07_00 = 0, Q29_01 = 0, Q29_05 = 0, Q29_00 = 0,
    Q27_02 = 0, Q27_03 = 0, Q27_04 = 0, Q28_00 = 0, Q28_01 = 0, Q29_04 = 0,
    Q28_02 = 1/4, Q29_03 = 1/4, Q26_08 = 0, Q26_09 = 0, Q26_07 = 0}
> subs(%,obj);
                                    3/4
> (dualobj,dualcnsts):=dual(obj,cnsts,y);
dualobj, dualcnsts := y5, {1 <= -y1 - y3 + 3 y4 + y5 + y6 - 3 y7,
    1 <= y1 - 4 y2 + 3 y3 + 4 y4 - 2 y7,
    1 <= y1 - 4 y2 + 3 y3 + 3 y4 + y5 - y6 - y7, 1 <= y1 - 4 y2 + 3 y3 + 2 y6,
    1 <= 2 y2 + y3 + y6 + y7, 1 <= 2 y2 + y3 + y4 + y5 - y6 + y7,
    1 <= y1 - 2 y2 + 2 y3 + 2 y5 - y6 + y7,
    1 <= y1 - 2 y2 + 2 y3 - 2 y4 + 3 y6 + y7,
    1 <= y1 - 2 y2 + 2 y3 - y4 + y5 + y6 + y7, 1 <= y3 + 2 y5 - y6 + y7,
    1 <= y1 - 2 y2 + 2 y3 + 2 y4 + y6 - y7, 1 <= y1 - 2 y2 + 2 y3 + 2 y6,
    1 <= y1 - 2 y2 + 2 y3 + y4 + y5, 1 <= y3 - 2 y4 + 3 y6 + y7,
```

1 <= y3 - y4 + y5 + y6 + y7, 1 <= 2 y2 - y4 + y5 + y6 + y7,

1 <= 2 y2 + 2 y5 - y6 + y7, 1 <= 2 y2 + y4 + y5,

1 <= 2 y2 - 2 y4 + 3 y6 + y7, 1 <= y1 - 4 y2 + 3 y3 + 2 y4 + 2 y5 - 2 y6,

1 <= 2 y2 + 2 y4 + y6 - y7, 1 <= 2 y2 + 2 y6, 1 <= y2 - y3 - 2 y4 + 3 y6,

1 <= y2 - y3 - y4 + y5 + y6, 1 <= y2 - y3 + 2 y4 + y6 - 2 y7,

1 <= y2 - y3 + 2 y6 - y7, 1 <= y2 - y3 + y4 + y5 - y7, 1 <= y2 + 2 y4 - y7,

1 <= y2 + y6, 1 <= y2 + y4 + y5 - y6, 1 <= 4 y2 + y3 + 2 y7,

1 <= 3 y1 - 4 y2 + 5 y3 + 2 y7, 1 <= 2 y1 - 4 y2 + 4 y3 + y4 + y5 - y6 + y7,

1 <= 2 y1 - 4 y2 + 4 y3 + 2 y4, 1 <= 2 y1 - 4 y2 + 4 y3 + y6 + y7,

1 <= 3 y2 + y7, 1 <= -y1 - y3 + 4 y5 - 2 y6, 1 <= 4 y4 + y6 - 3 y7,

1 <= 3 y4 + y5 - 2 y7, 1 <= 2 y4 + y6 - y7, 1 <= y4 + y5,

1 <= y2 - y3 + 2 y5 - y6, 1 <= -2 y2 + y3 + 2 y4 + 2 y6 - 2 y7,

1 <= -2 y2 + 2 y3 + y4 + y5, 1 <= y1 - 2 y2 + 3 y3 + 2 y4, 1 <= y3 + 2 y6,

1 <= y3 + y4 + y5, 1 <= -2 y2 + 2 y3 + 2 y4 + y6 - y7,

1 <= 2 y4 + 2 y5 - y6 - y7, 1 <= -2 y4 + 4 y6, 1 <= y4 + 3 y5 - 2 y6,

1 <= 2 y3 + 2 y4, 1 <= 2 y2 + y3 + 2 y4,

1 <= y1 - 2 y2 + 3 y3 + y4 + y5 - y6 + y7, 1 <= y1 - 2 y2 + 3 y3 + y6 + y7,

1 <= 2 y3 + y4 + y5 - y6 + y7, 1 <= 2 y3 + y6 + y7,

1 <= 2 y1 - 2 y2 + 4 y3 + 2 y7, 1 <= y1 + 3 y3 + 2 y7,

1 <= -2 y2 + y3 + 2 y5, 1 <= 2 y2 + 2 y3 + 2 y7,

1 <= -2 y2 + y3 - y4 + y5 + 2 y6, 1 <= -2 y2 + y3 - 2 y4 + 4 y6,

1 <= -2 y2 + y3 + y4 + y5 + y6 - y7, 1 <= -2 y2 + y3 + 3 y6 - y7,

1 <= -2 y2 + 2 y3 + 2 y6, 1 <= y3 + 2 y4 + y6 - y7,

1 <= -2 y2 + 2 y3 + 2 y4 + 2 y5 - 2 y6,

1 <= -2 y2 + 2 y3 + 3 y4 + y5 - y6 - y7, 1 <= -2 y2 + 2 y3 + 4 y4 - 2 y7,

1 <= 2 y4 + 2 y6 - 2 y7, 1 <= -2 y2 + y3 + 4 y4 + y6 - 3 y7,

1 <= -2 y2 + y3 + 3 y4 + y5 - 2 y7, 1 <= y1 - 4 y2 + 3 y3 + y4 + y5,

1 <= y3 + 2 y4 + 2 y5 - 2 y6, 1 <= y1 - 4 y2 + 3 y3 + 2 y4 + y6 - y7,

1 <= -y4 + y5 + 2 y6, 1 <= y4 + y5 + y6 - y7, 1 <= 3 y6 - y7,

```
    1 <= -y1 - y3 - 2 y4 + 5 y6 - y7, 1 <= -y1 - y3 + y4 + y5 + 2 y6 - 2 y7,

    1 <= -y1 - y3 + 4 y6 - 2 y7, 1 <= -y1 - y3 + 2 y4 + 3 y6 - 3 y7, 1 <= 2 y5,

    1 <= -y1 - y3 - 2 y4 + 2 y5 + 2 y6, 1 <= -y1 - y3 - 3 y4 + y5 + 4 y6,

    1 <= -y1 - y3 + 2 y5 + y6 - y7, 1 <= -y1 - y3 + 2 y4 + 2 y5 - 2 y7,

    1 <= -y1 - y3 - y4 + y5 + 3 y6 - y7, 1 <= -y1 - y3 + 4 y4 + 2 y6 - 4 y7,

    1 <= -y1 - y3 - 4 y4 + 6 y6, 1 <= -y1 - y3 + y4 + 3 y5 - y6 - y7,

    1 <= -2 y2 + y3 + y4 + 3 y5 - 2 y6, 1 <= -y1 - y3 - y4 + 3 y5,

    1 <= -2 y2 + y3 + 2 y4 + 2 y5 - y6 - y7, 1 <= y3 + 4 y4 - 2 y7,

    1 <= y3 + 3 y4 + y5 - y6 - y7}
> minimize(dualobj,dualcnsts,NONNEGATIVE);

      {y4 = 3/4, y2 = 1/4, y7 = 3/4, y3 = 0, y1 = 1/2, y6 = 3/4, y5 = 3/4}
> quit
```

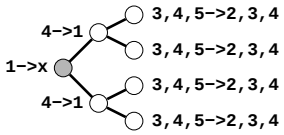
# MCDS girth 5

In all operations considered, the vertices at the “left” of the figure are the ones that could possibly be selected by “some” algorithm for inclusion in the connected dominating set. The priorities are

- vertices of degree 2 over
- vertices of degree 1

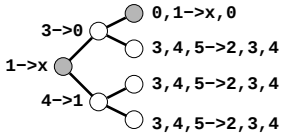
All operations are in $OPS_2$.

Q22:



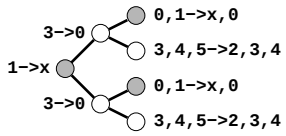

| variable | end-points | | | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q22-00: | $V_3$ | $V_3$ | $V_3$ | $V_3$: | | -1 | | -4 | -2 | | | 2 | 4 | | | 1 |
| Q22-01: | $V_3$ | $V_3$ | $V_3$ | $V_4$: | | -1 | | -3 | -3 | | | 2 | 3 | 1 | | 1 |
| Q22-02: | $V_3$ | $V_3$ | $V_3$ | $V_5$: | | -1 | | -3 | -2 | -1 | | 2 | 3 | | 1 | 1 |
| Q22-03: | $V_3$ | $V_3$ | $V_4$ | $V_4$: | | -1 | | -2 | -4 | | | 2 | 2 | 2 | | 1 |
| Q22-04: | $V_3$ | $V_3$ | $V_4$ | $V_5$: | | -1 | | -2 | -3 | -1 | | 2 | 2 | 1 | 1 | 1 |
| Q22-05: | $V_3$ | $V_3$ | $V_5$ | $V_5$: | | -1 | | -2 | -2 | -2 | | 2 | 2 | | 2 | 1 |
| Q22-06: | $V_3$ | $V_4$ | $V_4$ | $V_4$: | | -1 | | -1 | -5 | | | 2 | 1 | 3 | | 1 |
| Q22-07: | $V_3$ | $V_4$ | $V_4$ | $V_5$: | | -1 | | -1 | -4 | -1 | | 2 | 1 | 2 | 1 | 1 |
| Q22-08: | $V_3$ | $V_4$ | $V_5$ | $V_5$: | | -1 | | -1 | -3 | -2 | | 2 | 1 | 1 | 2 | 1 |
| Q22-09: | $V_3$ | $V_5$ | $V_5$ | $V_5$: | | -1 | | -1 | -2 | -3 | | 2 | 1 | | 3 | 1 |
| Q22-10: | $V_4$ | $V_4$ | $V_4$ | $V_4$: | | -1 | | | -6 | | | 2 | | 4 | | 1 |
| Q22-11: | $V_4$ | $V_4$ | $V_4$ | $V_5$: | | -1 | | | -5 | -1 | | 2 | | 3 | 1 | 1 |
| Q22-12: | $V_4$ | $V_4$ | $V_5$ | $V_5$: | | -1 | | | -4 | -2 | | 2 | | 2 | 2 | 1 |
| Q22-13: | $V_4$ | $V_5$ | $V_5$ | $V_5$: | | -1 | | | -3 | -3 | | 2 | | 1 | 3 | 1 |
| Q22-14: | $V_5$ | $V_5$ | $V_5$ | $V_5$: | | -1 | | | -2 | -4 | | 2 | | | 4 | 1 |

Q23:



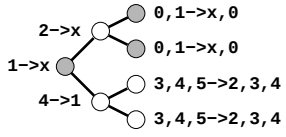

| variable | end-points | | | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q23-00: | $V_0$ | $V_3$ | $V_3$ | $V_3$: | -1 | -1 | | -4 | -1 | | 1 | 1 | 3 | | | 1 |
| Q23-01: | $V_0$ | $V_3$ | $V_3$ | $V_4$: | -1 | -1 | | -3 | -2 | | 1 | 1 | 2 | 1 | | 1 |
| Q23-02: | $V_0$ | $V_3$ | $V_3$ | $V_5$: | -1 | -1 | | -3 | -1 | -1 | 1 | 1 | 2 | | 1 | 1 |
| Q23-03: | $V_0$ | $V_3$ | $V_4$ | $V_4$: | -1 | -1 | | -2 | -3 | | 1 | 1 | 1 | 2 | | 1 |
| Q23-04: | $V_0$ | $V_3$ | $V_4$ | $V_5$: | -1 | -1 | | -2 | -2 | -1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Q23-05: | $V_0$ | $V_3$ | $V_5$ | $V_5$: | -1 | -1 | | -2 | -1 | -2 | 1 | 1 | 1 | | 2 | 1 |
| Q23-06: | $V_0$ | $V_4$ | $V_4$ | $V_4$: | -1 | -1 | | -1 | -4 | | 1 | 1 | | 3 | | 1 |
| Q23-07: | $V_0$ | $V_4$ | $V_4$ | $V_5$: | -1 | -1 | | -1 | -3 | -1 | 1 | 1 | | 2 | 1 | 1 |
| Q23-08: | $V_0$ | $V_4$ | $V_5$ | $V_5$: | -1 | -1 | | -1 | -2 | -2 | 1 | 1 | | 1 | 2 | 1 |
| Q23-09: | $V_0$ | $V_5$ | $V_5$ | $V_5$: | -1 | -1 | | -1 | -1 | -3 | 1 | 1 | | | 3 | 1 |
| Q23-10: | $V_1$ | $V_3$ | $V_3$ | $V_3$: | | -2 | | -4 | -1 | | 2 | 1 | 3 | | | 1 |
| Q23-11: | $V_1$ | $V_3$ | $V_3$ | $V_4$: | | -2 | | -3 | -2 | | 2 | 1 | 2 | 1 | | 1 |
| Q23-12: | $V_1$ | $V_3$ | $V_3$ | $V_5$: | | -2 | | -3 | -1 | -1 | 2 | 1 | 2 | | 1 | 1 |
| Q23-13: | $V_1$ | $V_3$ | $V_4$ | $V_4$: | | -2 | | -2 | -3 | | 2 | 1 | 1 | 2 | | 1 |
| Q23-14: | $V_1$ | $V_3$ | $V_4$ | $V_5$: | | -2 | | -2 | -2 | -1 | 2 | 1 | 1 | 1 | 1 | 1 |
| Q23-15: | $V_1$ | $V_3$ | $V_5$ | $V_5$: | | -2 | | -2 | -1 | -2 | 2 | 1 | 1 | | 2 | 1 |
| Q23-16: | $V_1$ | $V_4$ | $V_4$ | $V_4$: | | -2 | | -1 | -4 | | 2 | 1 | | 3 | | 1 |
| Q23-17: | $V_1$ | $V_4$ | $V_4$ | $V_5$: | | -2 | | -1 | -3 | -1 | 2 | 1 | | 2 | 1 | 1 |
| Q23-18: | $V_1$ | $V_4$ | $V_5$ | $V_5$: | | -2 | | -1 | -2 | -2 | 2 | 1 | | 1 | 2 | 1 |
| Q23-19: | $V_1$ | $V_5$ | $V_5$ | $V_5$: | | -2 | | -1 | -1 | -3 | 2 | 1 | | | 3 | 1 |

Q24:



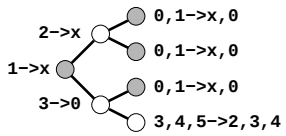

| variable | end-points | | | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q24-00: | $V_0$ | $V_0$ | $V_3$ | $V_3$: | -2 | -1 | | -4 | | | 2 | | 2 | | | 1 |
| Q24-01: | $V_0$ | $V_0$ | $V_3$ | $V_4$: | -2 | -1 | | -3 | -1 | | 2 | | 1 | 1 | | 1 |
| Q24-02: | $V_0$ | $V_0$ | $V_3$ | $V_5$: | -2 | -1 | | -3 | | -1 | 2 | | 1 | | 1 | 1 |
| Q24-03: | $V_0$ | $V_0$ | $V_4$ | $V_4$: | -2 | -1 | | -2 | -2 | | 2 | | | 2 | | 1 |
| Q24-04: | $V_0$ | $V_0$ | $V_4$ | $V_5$: | -2 | -1 | | -2 | -1 | -1 | 2 | | | 1 | 1 | 1 |
| Q24-05: | $V_0$ | $V_0$ | $V_5$ | $V_5$: | -2 | -1 | | -2 | | -2 | 2 | | | | 2 | 1 |
| Q24-06: | $V_0$ | $V_1$ | $V_3$ | $V_3$: | -1 | -2 | | -4 | | | 3 | | 2 | | | 1 |
| Q24-07: | $V_0$ | $V_1$ | $V_3$ | $V_4$: | -1 | -2 | | -3 | -1 | | 3 | | 1 | 1 | | 1 |
| Q24-08: | $V_0$ | $V_1$ | $V_3$ | $V_5$: | -1 | -2 | | -3 | | -1 | 3 | | 1 | | 1 | 1 |
| Q24-09: | $V_0$ | $V_1$ | $V_4$ | $V_4$: | -1 | -2 | | -2 | -2 | | 3 | | | 2 | | 1 |
| Q24-10: | $V_0$ | $V_1$ | $V_4$ | $V_5$: | -1 | -2 | | -2 | -1 | -1 | 3 | | | 1 | 1 | 1 |
| Q24-11: | $V_0$ | $V_1$ | $V_5$ | $V_5$: | -1 | -2 | | -2 | | -2 | 3 | | | | 2 | 1 |
| Q24-12: | $V_1$ | $V_1$ | $V_3$ | $V_3$: | | -3 | | -4 | | | 4 | | 2 | | | 1 |
| Q24-13: | $V_1$ | $V_1$ | $V_3$ | $V_4$: | | -3 | | -3 | -1 | | 4 | | 1 | 1 | | 1 |
| Q24-14: | $V_1$ | $V_1$ | $V_3$ | $V_5$: | | -3 | | -3 | | -1 | 4 | | 1 | | 1 | 1 |
| Q24-15: | $V_1$ | $V_1$ | $V_4$ | $V_4$: | | -3 | | -2 | -2 | | 4 | | | 2 | | 1 |
| Q24-16: | $V_1$ | $V_1$ | $V_4$ | $V_5$: | | -3 | | -2 | -1 | -1 | 4 | | | 1 | 1 | 1 |
| Q24-17: | $V_1$ | $V_1$ | $V_5$ | $V_5$: | | -3 | | -2 | | -2 | 4 | | | | 2 | 1 |

Q25:



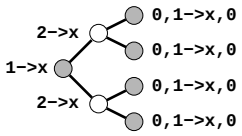

| variable | end-points | | | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q25-00: | $V_0$ | $V_0$ | $V_3$ | $V_3$: | -2 | -1 | -1 | -2 | -1 | | | 1 | 2 | | | 1 |
| Q25-01: | $V_0$ | $V_0$ | $V_3$ | $V_4$: | -2 | -1 | -1 | -1 | -2 | | | 1 | 1 | 1 | | 1 |
| Q25-02: | $V_0$ | $V_0$ | $V_3$ | $V_5$: | -2 | -1 | -1 | -1 | -1 | -1 | | 1 | 1 | | 1 | 1 |
| Q25-03: | $V_0$ | $V_0$ | $V_4$ | $V_4$: | -2 | -1 | -1 | | -3 | | | 1 | | 2 | | 1 |
| Q25-04: | $V_0$ | $V_0$ | $V_4$ | $V_5$: | -2 | -1 | -1 | | -2 | -1 | | 1 | | 1 | 1 | 1 |
| Q25-05: | $V_0$ | $V_0$ | $V_5$ | $V_5$: | -2 | -1 | -1 | | -1 | -2 | | 1 | | | 2 | 1 |
| Q25-06: | $V_0$ | $V_1$ | $V_3$ | $V_3$: | -1 | -2 | -1 | -2 | -1 | | 1 | 1 | 2 | | | 1 |
| Q25-07: | $V_0$ | $V_1$ | $V_3$ | $V_4$: | -1 | -2 | -1 | -1 | -2 | | 1 | 1 | 1 | 1 | | 1 |
| Q25-08: | $V_0$ | $V_1$ | $V_3$ | $V_5$: | -1 | -2 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | | 1 | 1 |
| Q25-09: | $V_0$ | $V_1$ | $V_4$ | $V_4$: | -1 | -2 | -1 | | -3 | | 1 | 1 | | 2 | | 1 |
| Q25-10: | $V_0$ | $V_1$ | $V_4$ | $V_5$: | -1 | -2 | -1 | | -2 | -1 | 1 | 1 | | 1 | 1 | 1 |
| Q25-11: | $V_0$ | $V_1$ | $V_5$ | $V_5$: | -1 | -2 | -1 | | -1 | -2 | 1 | 1 | | | 2 | 1 |
| Q25-12: | $V_1$ | $V_1$ | $V_3$ | $V_3$: | | -3 | -1 | -2 | -1 | | 2 | 1 | 2 | | | 1 |
| Q25-13: | $V_1$ | $V_1$ | $V_3$ | $V_4$: | | -3 | -1 | -1 | -2 | | 2 | 1 | 1 | 1 | | 1 |
| Q25-14: | $V_1$ | $V_1$ | $V_3$ | $V_5$: | | -3 | -1 | -1 | -1 | -1 | 2 | 1 | 1 | | 1 | 1 |
| Q25-15: | $V_1$ | $V_1$ | $V_4$ | $V_4$: | | -3 | -1 | | -3 | | 2 | 1 | | 2 | | 1 |
| Q25-16: | $V_1$ | $V_1$ | $V_4$ | $V_5$: | | -3 | -1 | | -2 | -1 | 2 | 1 | | 1 | 1 | 1 |
| Q25-17: | $V_1$ | $V_1$ | $V_5$ | $V_5$: | | -3 | -1 | | -1 | -2 | 2 | 1 | | | 2 | 1 |

Q26:



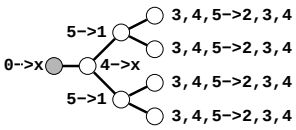

| variable | end-points | | | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q26-00: | $V_0$ | $V_0$ | $V_0$ | $V_3$: | -3 | -1 | -1 | -2 | | | 1 | | 1 | | | 1 |
| Q26-01: | $V_0$ | $V_0$ | $V_0$ | $V_4$: | -3 | -1 | -1 | -1 | -1 | | 1 | | | 1 | | 1 |
| Q26-02: | $V_0$ | $V_0$ | $V_0$ | $V_5$: | -3 | -1 | -1 | -1 | | -1 | 1 | | | | 1 | 1 |
| Q26-03: | $V_0$ | $V_0$ | $V_1$ | $V_3$: | -2 | -2 | -1 | -2 | | | 2 | | 1 | | | 1 |
| Q26-04: | $V_0$ | $V_0$ | $V_1$ | $V_4$: | -2 | -2 | -1 | -1 | -1 | | 2 | | | 1 | | 1 |
| Q26-05: | $V_0$ | $V_0$ | $V_1$ | $V_5$: | -2 | -2 | -1 | -1 | | -1 | 2 | | | | 1 | 1 |
| Q26-06: | $V_0$ | $V_1$ | $V_1$ | $V_3$: | -1 | -3 | -1 | -2 | | | 3 | | 1 | | | 1 |
| Q26-07: | $V_0$ | $V_1$ | $V_1$ | $V_4$: | -1 | -3 | -1 | -1 | -1 | | 3 | | | 1 | | 1 |
| Q26-08: | $V_0$ | $V_1$ | $V_1$ | $V_5$: | -1 | -3 | -1 | -1 | | -1 | 3 | | | | 1 | 1 |
| Q26-09: | $V_1$ | $V_1$ | $V_1$ | $V_3$: | | -4 | -1 | -2 | | | 4 | | 1 | | | 1 |
| Q26-10: | $V_1$ | $V_1$ | $V_1$ | $V_4$: | | -4 | -1 | -1 | -1 | | 4 | | | 1 | | 1 |
| Q26-11: | $V_1$ | $V_1$ | $V_1$ | $V_5$: | | -4 | -1 | -1 | | -1 | 4 | | | | 1 | 1 |

Q27:



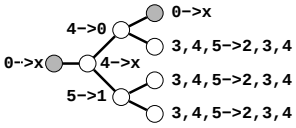

| variable | end-points | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q27-00: | $V_0$ $V_0$ $V_0$ $V_0$: | -4 | -1 | -2 | | | | | | | | | 1 |
| Q27-01: | $V_0$ $V_0$ $V_0$ $V_1$: | -3 | -2 | -2 | | | | 1 | | | | | 1 |
| Q27-02: | $V_0$ $V_0$ $V_1$ $V_1$: | -2 | -3 | -2 | | | | 2 | | | | | 1 |
| Q27-03: | $V_0$ $V_1$ $V_1$ $V_1$: | -1 | -4 | -2 | | | | 3 | | | | | 1 |
| Q27-04: | $V_1$ $V_1$ $V_1$ $V_1$: | | -5 | -2 | | | | 4 | | | | | 1 |

Q34:



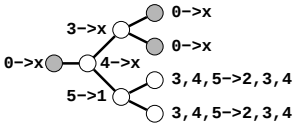

| variable | end-points | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q34-00: | $V_3$ $V_3$ $V_3$ $V_3$: | -1 | | | -4 | -1 | -2 | | 2 | 4 | | | 2 |
| Q34-01: | $V_3$ $V_3$ $V_3$ $V_4$: | -1 | | | -3 | -2 | -2 | | 2 | 3 | 1 | | 2 |
| Q34-02: | $V_3$ $V_3$ $V_3$ $V_5$: | -1 | | | -3 | -1 | -3 | | 2 | 3 | | 1 | 2 |
| Q34-03: | $V_3$ $V_3$ $V_4$ $V_4$: | -1 | | | -2 | -3 | -2 | | 2 | 2 | 2 | | 2 |
| Q34-04: | $V_3$ $V_3$ $V_4$ $V_5$: | -1 | | | -2 | -2 | -3 | | 2 | 2 | 1 | 1 | 2 |
| Q34-05: | $V_3$ $V_3$ $V_5$ $V_5$: | -1 | | | -2 | -1 | -4 | | 2 | 2 | | 2 | 2 |
| Q34-06: | $V_3$ $V_4$ $V_4$ $V_4$: | -1 | | | -1 | -4 | -2 | | 2 | 1 | 3 | | 2 |
| Q34-07: | $V_3$ $V_4$ $V_4$ $V_5$: | -1 | | | -1 | -3 | -3 | | 2 | 1 | 2 | 1 | 2 |
| Q34-08: | $V_3$ $V_4$ $V_5$ $V_5$: | -1 | | | -1 | -2 | -4 | | 2 | 1 | 1 | 2 | 2 |
| Q34-09: | $V_3$ $V_5$ $V_5$ $V_5$: | -1 | | | -1 | -1 | -5 | | 2 | 1 | | 3 | 2 |
| Q34-10: | $V_4$ $V_4$ $V_4$ $V_4$: | -1 | | | | -5 | -2 | | 2 | | 4 | | 2 |
| Q34-11: | $V_4$ $V_4$ $V_4$ $V_5$: | -1 | | | | -4 | -3 | | 2 | | 3 | 1 | 2 |
| Q34-12: | $V_4$ $V_4$ $V_5$ $V_5$: | -1 | | | | -3 | -4 | | 2 | | 2 | 2 | 2 |
| Q34-13: | $V_4$ $V_5$ $V_5$ $V_5$: | -1 | | | | -2 | -5 | | 2 | | 1 | 3 | 2 |
| Q34-14: | $V_5$ $V_5$ $V_5$ $V_5$: | -1 | | | | -1 | -6 | | 2 | | | 4 | 2 |

Q35:



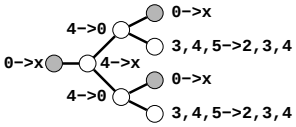

| variable | end-points | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q35-00: | $V_3$ $V_3$ $V_3$: | -2 | | | -3 | -2 | -1 | 1 | 1 | 3 | | | 2 |
| Q35-01: | $V_3$ $V_3$ $V_4$: | -2 | | | -2 | -3 | -1 | 1 | 1 | 2 | 1 | | 2 |
| Q35-02: | $V_3$ $V_3$ $V_5$: | -2 | | | -2 | -2 | -2 | 1 | 1 | 2 | | 1 | 2 |
| Q35-03: | $V_3$ $V_4$ $V_4$: | -2 | | | -1 | -4 | -1 | 1 | 1 | 1 | 2 | | 2 |
| Q35-04: | $V_3$ $V_4$ $V_5$: | -2 | | | -1 | -3 | -2 | 1 | 1 | 1 | 1 | 1 | 2 |
| Q35-05: | $V_3$ $V_5$ $V_5$: | -2 | | | -1 | -2 | -3 | 1 | 1 | 1 | | 2 | 2 |
| Q35-06: | $V_4$ $V_4$ $V_4$: | -2 | | | | -5 | -1 | 1 | 1 | | 3 | | 2 |
| Q35-07: | $V_4$ $V_4$ $V_5$: | -2 | | | | -4 | -2 | 1 | 1 | | 2 | 1 | 2 |
| Q35-08: | $V_4$ $V_5$ $V_5$: | -2 | | | | -3 | -3 | 1 | 1 | | 1 | 2 | 2 |
| Q35-09: | $V_5$ $V_5$ $V_5$: | -2 | | | | -2 | -4 | 1 | 1 | | | 3 | 2 |

Q36:



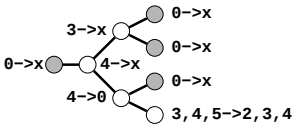

| variable | end-points | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q36-00: | $V_3$ $V_3$: | -3 | | | -3 | -1 | -1 | | 1 | 2 | | | 2 |
| Q36-01: | $V_3$ $V_4$: | -3 | | | -2 | -2 | -1 | | 1 | 1 | 1 | | 2 |
| Q36-02: | $V_3$ $V_5$: | -3 | | | -2 | -1 | -2 | | 1 | 1 | | 1 | 2 |
| Q36-03: | $V_4$ $V_4$: | -3 | | | -1 | -3 | -1 | | 1 | | 2 | | 2 |
| Q36-04: | $V_4$ $V_5$: | -3 | | | -1 | -2 | -2 | | 1 | | 1 | 1 | 2 |
| Q36-05: | $V_5$ $V_5$: | -3 | | | -1 | -1 | -3 | | 1 | | | 2 | 2 |

Q37:



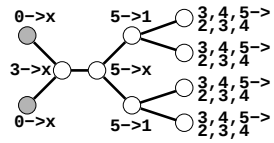

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q37-00: | $V_3$ | $V_3$: | -3 | | | -2 | -3 | | 2 | | 2 | | | 2 |
| Q37-01: | $V_3$ | $V_4$: | -3 | | | -1 | -4 | | 2 | | 1 | 1 | | 2 |
| Q37-02: | $V_3$ | $V_5$: | -3 | | | -1 | -3 | -1 | 2 | | 1 | | 1 | 2 |
| Q37-03: | $V_4$ | $V_4$: | -3 | | | | -5 | | 2 | | | 2 | | 2 |
| Q37-04: | $V_4$ | $V_5$: | -3 | | | | -4 | -1 | 2 | | | 1 | 1 | 2 |
| Q37-05: | $V_5$ | $V_5$: | -3 | | | | -3 | -2 | 2 | | | | 2 | 2 |

Q38:



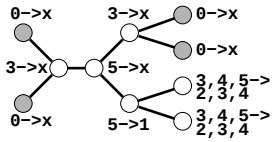

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q38-00: | | $V_3$: | -4 | | | -2 | -2 | | 1 | | 1 | | | 2 |
| Q38-01: | | $V_4$: | -4 | | | -1 | -3 | | 1 | | | 1 | | 2 |
| Q38-02: | | $V_5$: | -4 | | | -1 | -2 | -1 | 1 | | | | 1 | 2 |

Q40:



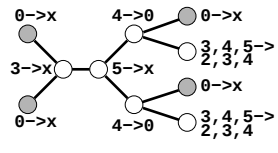

| variable | end-points | | | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q40-00: | $V_3$ | $V_3$ | $V_3$ | $V_3$: | -2 | | | -5 | | -3 | | 2 | 4 | | | 3 |
| Q40-01: | $V_3$ | $V_3$ | $V_3$ | $V_4$: | -2 | | | -4 | -1 | -3 | | 2 | 3 | 1 | | 3 |
| Q40-02: | $V_3$ | $V_3$ | $V_3$ | $V_5$: | -2 | | | -4 | | -4 | | 2 | 3 | | 1 | 3 |
| Q40-03: | $V_3$ | $V_3$ | $V_4$ | $V_4$: | -2 | | | -3 | -2 | -3 | | 2 | 2 | 2 | | 3 |
| Q40-04: | $V_3$ | $V_3$ | $V_4$ | $V_5$: | -2 | | | -3 | -1 | -4 | | 2 | 2 | 1 | 1 | 3 |
| Q40-05: | $V_3$ | $V_3$ | $V_5$ | $V_5$: | -2 | | | -3 | | -5 | | 2 | 2 | | 2 | 3 |
| Q40-06: | $V_3$ | $V_4$ | $V_4$ | $V_4$: | -2 | | | -2 | -3 | -3 | | 2 | 1 | 3 | | 3 |
| Q40-07: | $V_3$ | $V_4$ | $V_4$ | $V_5$: | -2 | | | -2 | -2 | -4 | | 2 | 1 | 2 | 1 | 3 |
| Q40-08: | $V_3$ | $V_4$ | $V_5$ | $V_5$: | -2 | | | -2 | -1 | -5 | | 2 | 1 | 1 | 2 | 3 |
| Q40-09: | $V_3$ | $V_5$ | $V_5$ | $V_5$: | -2 | | | -2 | | -6 | | 2 | 1 | | 3 | 3 |
| Q40-10: | $V_4$ | $V_4$ | $V_4$ | $V_4$: | -2 | | | -1 | -4 | -3 | | 2 | | 4 | | 3 |
| Q40-11: | $V_4$ | $V_4$ | $V_4$ | $V_5$: | -2 | | | -1 | -3 | -4 | | 2 | | 3 | 1 | 3 |
| Q40-12: | $V_4$ | $V_4$ | $V_5$ | $V_5$: | -2 | | | -1 | -2 | -5 | | 2 | | 2 | 2 | 3 |
| Q40-13: | $V_4$ | $V_5$ | $V_5$ | $V_5$: | -2 | | | -1 | -1 | -6 | | 2 | | 1 | 3 | 3 |
| Q40-14: | $V_5$ | $V_5$ | $V_5$ | $V_5$: | -2 | | | -1 | | -7 | | 2 | | | 4 | 3 |

Q41:



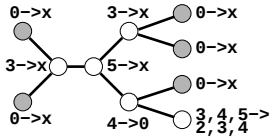

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q41-00: | $V_3$ | $V_3$: | -4 | | | -4 | | -2 | | 1 | 2 | | | 3 |
| Q41-01: | $V_3$ | $V_4$: | -4 | | | -3 | -1 | -2 | | 1 | 1 | 1 | | 3 |
| Q41-02: | $V_3$ | $V_5$: | -4 | | | -3 | | -3 | | 1 | 1 | | 1 | 3 |
| Q41-03: | $V_4$ | $V_4$: | -4 | | | -2 | -2 | -2 | | 1 | | 2 | | 3 |
| Q41-04: | $V_4$ | $V_5$: | -4 | | | -2 | -1 | -3 | | 1 | | 1 | 1 | 3 |
| Q41-05: | $V_5$ | $V_5$: | -4 | | | -2 | | -4 | | 1 | | | 2 | 3 |

Q42:



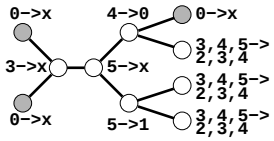

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q42-00: | $V_3$ | $V_3$: | -4 | | | -3 | -2 | -1 | 2 | | 2 | | | 3 |
| Q42-01: | $V_3$ | $V_4$: | -4 | | | -2 | -3 | -1 | 2 | | 1 | 1 | | 3 |
| Q42-02: | $V_3$ | $V_5$: | -4 | | | -2 | -2 | -2 | 2 | | 1 | | 1 | 3 |
| Q42-03: | $V_4$ | $V_4$: | -4 | | | -1 | -4 | -1 | 2 | | | 2 | | 3 |
| Q42-04: | $V_4$ | $V_5$: | -4 | | | -1 | -3 | -2 | 2 | | | 1 | 1 | 3 |
| Q42-05: | $V_5$ | $V_5$: | -4 | | | -1 | -2 | -3 | 2 | | | | 2 | 3 |

Q43:



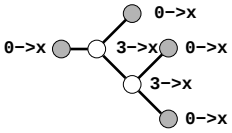

| variable | end-points | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q43-00: | | $V_3$: | -5 | | | -3 | -1 | -1 | 1 | | 1 | | | 3 |
| Q43-01: | | $V_4$: | -5 | | | -2 | -2 | -1 | 1 | | | 1 | | 3 |
| Q43-02: | | $V_5$: | -5 | | | -2 | -1 | -2 | 1 | | | | 1 | 3 |

Q45:



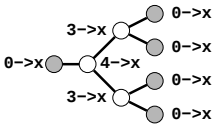

| variable | end-points | | | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q45-00: | $V_3$ | $V_3$ | $V_3$: | -3 | | | -4 | -1 | -2 | 1 | 1 | 3 | | | 3 |
| Q45-01: | $V_3$ | $V_3$ | $V_4$: | -3 | | | -3 | -2 | -2 | 1 | 1 | 2 | 1 | | 3 |
| Q45-02: | $V_3$ | $V_3$ | $V_5$: | -3 | | | -3 | -1 | -3 | 1 | 1 | 2 | | 1 | 3 |
| Q45-03: | $V_3$ | $V_4$ | $V_4$: | -3 | | | -2 | -3 | -2 | 1 | 1 | 1 | 2 | | 3 |
| Q45-04: | $V_3$ | $V_4$ | $V_5$: | -3 | | | -2 | -2 | -3 | 1 | 1 | 1 | 1 | 1 | 3 |
| Q45-05: | $V_3$ | $V_5$ | $V_5$: | -3 | | | -2 | -1 | -4 | 1 | 1 | 1 | | 2 | 3 |
| Q45-06: | $V_4$ | $V_4$ | $V_4$: | -3 | | | -1 | -4 | -2 | 1 | 1 | | 3 | | 3 |
| Q45-07: | $V_4$ | $V_4$ | $V_5$: | -3 | | | -1 | -3 | -3 | 1 | 1 | | 2 | 1 | 3 |
| Q45-08: | $V_4$ | $V_5$ | $V_5$: | -3 | | | -1 | -2 | -4 | 1 | 1 | | 1 | 2 | 3 |
| Q45-09: | $V_5$ | $V_5$ | $V_5$: | -3 | | | -1 | -1 | -5 | 1 | 1 | | | 3 | 3 |

Q31:



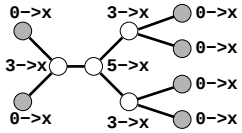

| variable | end-points | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q31-00: | | -4 | | | -2 | | | | | | | | 2 |

Q39:



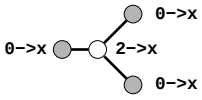

| variable | end-points | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q39-00: | | -5 | | | -2 | -1 | | | | | | | 2 |

Q44:




| variable | end-points | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q44-00: | | -6 | | | -3 | | -1 | | | | | | 3 |

Q30:




| variable | end-points | $Y_0^-$ | $Y_1^-$ | $Y_2^-$ | $Y_3^-$ | $Y_4^-$ | $Y_5^-$ | $Y_0^+$ | $Y_1^+$ | $Y_2^+$ | $Y_3^+$ | $Y_4^+$ | $\Delta\mathcal{C}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Q30-00: | | -3 | | -1 | | | | | | | | | 1 |

```
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 <____ ____>  Waterloo Maple Inc.
      |       Type ? for help.
> with(simplex):
Warning, new definition for maximize
Warning, new definition for minimize
>
> obj:=
1*Q22_00 +1*Q22_01 +1*Q22_02 +1*Q22_03 +1*Q22_04 +1*Q22_05 +1*Q22_06
+1*Q22_07 +1*Q22_08 +1*Q22_09 +1*Q22_10 +1*Q22_11 +1*Q22_12 +1*Q22_13
+1*Q22_14 +1*Q23_00 +1*Q23_01 +1*Q23_02 +1*Q23_03 +1*Q23_04 +1*Q23_05
+1*Q23_06 +1*Q23_07 +1*Q23_08 +1*Q23_09 +1*Q23_10 +1*Q23_11 +1*Q23_12
+1*Q23_13 +1*Q23_14 +1*Q23_15 +1*Q23_16 +1*Q23_17 +1*Q23_18 +1*Q23_19
+1*Q24_00 +1*Q24_01 +1*Q24_02 +1*Q24_03 +1*Q24_04 +1*Q24_05 +1*Q24_06
+1*Q24_07 +1*Q24_08 +1*Q24_09 +1*Q24_10 +1*Q24_11 +1*Q24_12 +1*Q24_13
+1*Q24_14 +1*Q24_15 +1*Q24_16 +1*Q24_17 +1*Q25_00 +1*Q25_01 +1*Q25_02
+1*Q25_03 +1*Q25_04 +1*Q25_05 +1*Q25_06 +1*Q25_07 +1*Q25_08 +1*Q25_09
+1*Q25_10 +1*Q25_11 +1*Q25_12 +1*Q25_13 +1*Q25_14 +1*Q25_15 +1*Q25_16
+1*Q25_17 +1*Q26_00 +1*Q26_01 +1*Q26_02 +1*Q26_03 +1*Q26_04 +1*Q26_05
+1*Q26_06 +1*Q26_07 +1*Q26_08 +1*Q26_09 +1*Q26_10 +1*Q26_11 +1*Q27_00
+1*Q27_01 +1*Q27_02 +1*Q27_03 +1*Q27_04 +2*Q34_00 +2*Q34_01 +2*Q34_02
+2*Q34_03 +2*Q34_04 +2*Q34_05 +2*Q34_06 +2*Q34_07 +2*Q34_08 +2*Q34_09
+2*Q34_10 +2*Q34_11 +2*Q34_12 +2*Q34_13 +2*Q34_14 +2*Q35_00 +2*Q35_01
+2*Q35_02 +2*Q35_03 +2*Q35_04 +2*Q35_05 +2*Q35_06 +2*Q35_07 +2*Q35_08
+2*Q35_09 +1*Q36_00 +1*Q36_01 +1*Q36_02 +1*Q36_03 +1*Q36_04 +1*Q36_05
+1*Q37_00 +1*Q37_01 +1*Q37_02 +1*Q37_03 +1*Q37_04 +1*Q37_05 +2*Q38_00
+2*Q38_01 +2*Q38_02 +3*Q40_00 +3*Q40_01 +3*Q40_02 +3*Q40_03 +3*Q40_04
+3*Q40_05 +3*Q40_06 +3*Q40_07 +3*Q40_08 +3*Q40_09 +3*Q40_10 +3*Q40_11
+3*Q40_12 +3*Q40_13 +3*Q40_14 +3*Q41_00 +3*Q41_01 +3*Q41_02 +3*Q41_03
+3*Q41_04 +3*Q41_05 +3*Q42_00 +3*Q42_01 +3*Q42_02 +3*Q42_03 +3*Q42_04
+3*Q42_05 +3*Q43_00 +3*Q43_01 +3*Q43_02 +3*Q45_00 +3*Q45_01 +3*Q45_02
+3*Q45_03 +3*Q45_04 +3*Q45_05 +3*Q45_06 +3*Q45_07 +3*Q45_08 +3*Q45_09
+2*Q31_00 +2*Q39_00 +3*Q44_00 +1*Q30_00;

obj := Q22_01 + Q22_00 + Q22_03 + Q22_02 + Q22_04 + Q22_05 + Q22_06 + Q22_07

     + Q22_08 + Q22_09 + Q22_10 + Q22_11 + Q22_12 + Q22_13 + Q22_14 + Q23_00

     + Q23_01 + Q23_02 + Q23_03 + Q23_04 + Q23_05 + Q23_06 + Q23_07 + Q23_08

     + Q23_09 + Q23_10 + Q23_11 + Q23_12 + Q23_13 + Q23_14 + Q23_15 + Q23_16

     + Q23_17 + Q23_18 + Q23_19 + Q24_00 + Q24_01 + Q24_02 + Q24_03 + Q24_04

     + Q24_05 + Q24_06 + Q24_07 + Q24_08 + Q24_09 + Q24_10 + Q24_11 + Q24_12

     + Q24_13 + Q24_14 + Q24_15 + Q24_16 + Q24_17 + Q25_00 + Q25_01 + Q25_02

     + Q25_03 + Q25_04 + Q25_05 + Q25_06 + Q25_07 + Q25_08 + Q25_09 + Q25_10

     + Q25_11 + Q25_12 + Q25_13 + Q25_14 + Q25_15 + Q25_16 + Q25_17 + Q26_00

     + Q26_01 + Q26_02 + Q26_03 + Q26_04 + Q26_05 + Q26_06 + Q26_07 + Q26_08

     + Q26_09 + Q26_10 + Q26_11 + Q27_00 + Q27_01 + Q27_02 + Q27_03 + Q27_04

     + 2 Q34_00 + 2 Q34_01 + 2 Q34_02 + 2 Q34_03 + 2 Q34_04 + 2 Q34_05
```

```
      + 2 Q34_06 + 2 Q34_07 + 2 Q34_08 + 2 Q34_09 + 2 Q34_10 + 2 Q34_11

      + 2 Q34_12 + 2 Q34_13 + 2 Q34_14 + 2 Q35_00 + 2 Q35_01 + 2 Q35_02

      + 2 Q35_03 + 2 Q35_04 + 2 Q35_05 + 2 Q35_06 + 2 Q35_07 + 2 Q35_08

      + 2 Q35_09 + Q36_00 + Q36_01 + Q36_02 + Q36_03 + Q36_04 + Q36_05 + Q37_00

      + Q37_01 + Q37_02 + Q37_03 + Q37_04 + Q37_05 + 2 Q38_00 + 2 Q38_01

      + 2 Q38_02 + 3 Q40_00 + 3 Q40_01 + 3 Q40_02 + 3 Q40_03 + 3 Q40_04

      + 3 Q40_05 + 3 Q40_06 + 3 Q40_07 + 3 Q40_08 + 3 Q40_09 + 3 Q40_10

      + 3 Q40_11 + 3 Q40_12 + 3 Q40_13 + 3 Q40_14 + 3 Q41_00 + 3 Q41_01

      + 3 Q41_02 + 3 Q41_03 + 3 Q41_04 + 3 Q41_05 + 3 Q42_00 + 3 Q42_01

      + 3 Q42_02 + 3 Q42_03 + 3 Q42_04 + 3 Q42_05 + 3 Q43_00 + 3 Q43_01

      + 3 Q43_02 + 3 Q45_00 + 3 Q45_01 + 3 Q45_02 + 3 Q45_03 + 3 Q45_04

      + 3 Q45_05 + 3 Q45_06 + 3 Q45_07 + 3 Q45_08 + 3 Q45_09 + 2 Q31_00

      + 2 Q39_00 + 3 Q44_00 + Q30_00

>
> cnsts:={
> +1*Q22_00 +1*Q22_01 +1*Q22_02 +1*Q22_03 +1*Q22_04 +1*Q22_05
> +1*Q22_06 +1*Q22_07 +1*Q22_08 +1*Q22_09 +1*Q22_10 +1*Q22_11
> +1*Q22_12 +1*Q22_13 +1*Q22_14 -1*Q24_12 -1*Q24_13 -1*Q24_14
> -1*Q24_15 -1*Q24_16 -1*Q24_17 -1*Q25_12 -1*Q25_13 -1*Q25_14
> -1*Q25_15 -1*Q25_16 -1*Q25_17 -1*Q26_06 -1*Q26_07 -1*Q26_08
> -2*Q26_09 -2*Q26_10 -2*Q26_11 -1*Q27_02 -2*Q27_03 -3*Q27_04 >= 0,
>
> +2*Q23_10 +2*Q23_11 +2*Q23_12 +2*Q23_13 +2*Q23_14 +2*Q23_15
> +2*Q23_16 +2*Q23_17 +2*Q23_18 +2*Q23_19 +2*Q24_06 +2*Q24_07
> +2*Q24_08 +2*Q24_09 +2*Q24_10 +2*Q24_11 +4*Q24_12 +4*Q24_13
> +4*Q24_14 +4*Q24_15 +4*Q24_16 +4*Q24_17 -2*Q25_00 -2*Q25_01
> -2*Q25_02 -2*Q25_03 -2*Q25_04 -2*Q25_05 +2*Q25_12 +2*Q25_13
> +2*Q25_14 +2*Q25_15 +2*Q25_16 +2*Q25_17 -2*Q26_00 -2*Q26_01
> -2*Q26_02 +2*Q26_06 +2*Q26_07 +2*Q26_08 +4*Q26_09 +4*Q26_10
> +4*Q26_11 -4*Q27_00 -2*Q27_01 +2*Q27_03 +4*Q27_04 -4*Q31_00
> -1*Q34_00 -1*Q34_01 -1*Q34_02 -1*Q34_03 -1*Q34_04 -1*Q34_05
> -1*Q34_06 -1*Q34_07 -1*Q34_08 -1*Q34_09 -1*Q34_10 -1*Q34_11
> -1*Q34_12 -1*Q34_13 -1*Q34_14 -1*Q35_00 -1*Q35_01 -1*Q35_02
> -1*Q35_03 -1*Q35_04 -1*Q35_05 -1*Q35_06 -1*Q35_07 -1*Q35_08
> -1*Q35_09 -3*Q36_00 -3*Q36_01 -3*Q36_02 -3*Q36_03 -3*Q36_04
> -3*Q36_05 -1*Q37_00 -1*Q37_01 -1*Q37_02 -1*Q37_03 -1*Q37_04
> -1*Q37_05 -3*Q38_00 -3*Q38_01 -3*Q38_02 -5*Q39_00 -2*Q40_00
> -2*Q40_01 -2*Q40_02 -2*Q40_03 -2*Q40_04 -2*Q40_05 -2*Q40_06
> -2*Q40_07 -2*Q40_08 -2*Q40_09 -2*Q40_10 -2*Q40_11 -2*Q40_12
> -2*Q40_13 -2*Q40_14 -4*Q41_00 -4*Q41_01 -4*Q41_02 -4*Q41_03
> -4*Q41_04 -4*Q41_05 -2*Q42_00 -2*Q42_01 -2*Q42_02 -2*Q42_03
> -2*Q42_04 -2*Q42_05 -4*Q43_00 -4*Q43_01 -4*Q43_02 -6*Q44_00
> -2*Q45_00 -2*Q45_01 -2*Q45_02 -2*Q45_03 -2*Q45_04 -2*Q45_05
> -2*Q45_06 -2*Q45_07 -2*Q45_08 -2*Q45_09 -3*Q30_00  >= 0,
>
```

```
> +1*Q22_00 +1*Q22_01 +1*Q22_02 +1*Q22_03 +1*Q22_04 +1*Q22_05
> +1*Q22_06 +1*Q22_07 +1*Q22_08 +1*Q22_09 +1*Q22_10 +1*Q22_11
> +1*Q22_12 +1*Q22_13 +1*Q22_14 -1*Q23_10 -1*Q23_11 -1*Q23_12
> -1*Q23_13 -1*Q23_14 -1*Q23_15 -1*Q23_16 -1*Q23_17 -1*Q23_18
> -1*Q23_19 -1*Q24_00 -1*Q24_01 -1*Q24_02 -1*Q24_03 -1*Q24_04
> -1*Q24_05 -2*Q24_06 -2*Q24_07 -2*Q24_08 -2*Q24_09 -2*Q24_10
> -2*Q24_11 -3*Q24_12 -3*Q24_13 -3*Q24_14 -3*Q24_15 -3*Q24_16
> -3*Q24_17 -1*Q25_06 -1*Q25_07 -1*Q25_08 -1*Q25_09 -1*Q25_10
> -1*Q25_11 -2*Q25_12 -2*Q25_13 -2*Q25_14 -2*Q25_15 -2*Q25_16
> -2*Q25_17 -1*Q26_00 -1*Q26_01 -1*Q26_02 -2*Q26_03 -2*Q26_04
> -2*Q26_05 -3*Q26_06 -3*Q26_07 -3*Q26_08 -4*Q26_09 -4*Q26_10
> -4*Q26_11 -1*Q27_00 -2*Q27_01 -3*Q27_02 -4*Q27_03 -5*Q27_04
> +2*Q34_00 +2*Q34_01 +2*Q34_02 +2*Q34_03 +2*Q34_04 +2*Q34_05
> +2*Q34_06 +2*Q34_07 +2*Q34_08 +2*Q34_09 +2*Q34_10 +2*Q34_11
> +2*Q34_12 +2*Q34_13 +2*Q34_14 +1*Q35_00 +1*Q35_01 +1*Q35_02
> +1*Q35_03 +1*Q35_04 +1*Q35_05 +1*Q35_06 +1*Q35_07 +1*Q35_08
> +1*Q35_09 +1*Q36_00 +1*Q36_01 +1*Q36_02 +1*Q36_03 +1*Q36_04
> +1*Q36_05 +2*Q40_00 +2*Q40_01 +2*Q40_02 +2*Q40_03 +2*Q40_04
> +2*Q40_05 +2*Q40_06 +2*Q40_07 +2*Q40_08 +2*Q40_09 +2*Q40_10
> +2*Q40_11 +2*Q40_12 +2*Q40_13 +2*Q40_14 +1*Q41_00 +1*Q41_01
> +1*Q41_02 +1*Q41_03 +1*Q41_04 +1*Q41_05 +1*Q45_00 +1*Q45_01
> +1*Q45_02 +1*Q45_03 +1*Q45_04 +1*Q45_05 +1*Q45_06 +1*Q45_07
> +1*Q45_08 +1*Q45_09  >= 0,
>
> +4*Q22_00 +3*Q22_01 +3*Q22_02 +2*Q22_03 +2*Q22_04 +2*Q22_05
> +1*Q22_06 +1*Q22_07 +1*Q22_08 +1*Q22_09 +3*Q23_00 +2*Q23_01
> +2*Q23_02 +1*Q23_03 +1*Q23_04 +1*Q23_05 +3*Q23_10 +2*Q23_11
> +2*Q23_12 +1*Q23_13 +1*Q23_14 +1*Q23_15 +2*Q24_00 +1*Q24_01
> +1*Q24_02 +2*Q24_06 +1*Q24_07 +1*Q24_08 +2*Q24_12 +1*Q24_13
> +1*Q24_14 +1*Q25_00 -1*Q25_03 -1*Q25_04 -1*Q25_05 +1*Q25_06
> -1*Q25_09 -1*Q25_10 -1*Q25_11 +1*Q25_12 -1*Q25_15 -1*Q25_16
> -1*Q25_17 -1*Q26_01 -1*Q26_02 -1*Q26_04 -1*Q26_05 -1*Q26_07
> -1*Q26_08 -1*Q26_10 -1*Q26_11 -2*Q27_00 -2*Q27_01 -2*Q27_02
> -2*Q27_03 -2*Q27_04 +4*Q34_00 +3*Q34_01 +3*Q34_02 +2*Q34_03
> +2*Q34_04 +2*Q34_05 +1*Q34_06 +1*Q34_07 +1*Q34_08 +1*Q34_09
> +3*Q35_00 +2*Q35_01 +2*Q35_02 +1*Q35_03 +1*Q35_04 +1*Q35_05
> +2*Q36_00 +1*Q36_01 +1*Q36_02 +2*Q37_00 +1*Q37_01 +1*Q37_02
> +1*Q38_00 +4*Q40_00 +3*Q40_01 +3*Q40_02 +2*Q40_03 +2*Q40_04
> +2*Q40_05 +1*Q40_06 +1*Q40_07 +1*Q40_08 +1*Q40_09 +2*Q41_00
> +1*Q41_01 +1*Q41_02 +2*Q42_00 +1*Q42_01 +1*Q42_02 +1*Q43_00
> +3*Q45_00 +2*Q45_01 +2*Q45_02 +1*Q45_03 +1*Q45_04 +1*Q45_05
> -1*Q30_00  >= 0,
>
> -4*Q22_00 -2*Q22_01 -3*Q22_02 -1*Q22_04 -2*Q22_05 +2*Q22_06
> +1*Q22_07 -1*Q22_09 +4*Q22_10 +3*Q22_11 +2*Q22_12 +1*Q22_13
> -4*Q23_00 -2*Q23_01 -3*Q23_02 -1*Q23_04 -2*Q23_05 +2*Q23_06
> +1*Q23_07 -1*Q23_09 -4*Q23_10 -2*Q23_11 -3*Q23_12 -1*Q23_14
> -2*Q23_15 +2*Q23_16 +1*Q23_17 -1*Q23_19 -4*Q24_00 -2*Q24_01
> -3*Q24_02 -1*Q24_04 -2*Q24_05 -4*Q24_06 -2*Q24_07 -3*Q24_08
> -1*Q24_10 -2*Q24_11 -4*Q24_12 -2*Q24_13 -3*Q24_14 -1*Q24_16
> -2*Q24_17 -2*Q25_00 -1*Q25_02 +2*Q25_03 +1*Q25_04 -2*Q25_06
> -1*Q25_08 +2*Q25_09 +1*Q25_10 -2*Q25_12 -1*Q25_14 +2*Q25_15
> +1*Q25_16 -2*Q26_00 -1*Q26_02 -2*Q26_03 -1*Q26_05 -2*Q26_06
> -1*Q26_08 -2*Q26_09 -1*Q26_11 -2*Q31_00 -4*Q34_00 -2*Q34_01
> -3*Q34_02 -1*Q34_04 -2*Q34_05 +2*Q34_06 +1*Q34_07 -1*Q34_09
> +4*Q34_10 +3*Q34_11 +2*Q34_12 +1*Q34_13 -3*Q35_00 -1*Q35_01
> -2*Q35_02 +1*Q35_03 -1*Q35_05 +3*Q35_06 +2*Q35_07 +1*Q35_08
> -3*Q36_00 -1*Q36_01 -2*Q36_02 +1*Q36_03 -1*Q36_05 -2*Q37_00
> -1*Q37_02 +2*Q37_03 +1*Q37_04 -2*Q38_00 -1*Q38_02 -2*Q39_00
```

```
> -5*Q40_00 -3*Q40_01 -4*Q40_02 -1*Q40_03 -2*Q40_04 -3*Q40_05
> +1*Q40_06 -1*Q40_08 -2*Q40_09 +3*Q40_10 +2*Q40_11 +1*Q40_12
> -1*Q40_14 -4*Q41_00 -2*Q41_01 -3*Q41_02 -1*Q41_04 -2*Q41_05
> -3*Q42_00 -1*Q42_01 -2*Q42_02 +1*Q42_03 -1*Q42_05 -3*Q43_00
> -1*Q43_01 -2*Q43_02 -3*Q44_00 -4*Q45_00 -2*Q45_01 -3*Q45_02
> -1*Q45_04 -2*Q45_05 +2*Q45_06 +1*Q45_07 -1*Q45_09  >= 0,
>
> -2*Q22_00 -3*Q22_01 -1*Q22_02 -4*Q22_03 -2*Q22_04 -5*Q22_06
> -3*Q22_07 -1*Q22_08 +1*Q22_09 -6*Q22_10 -4*Q22_11 -2*Q22_12
> +2*Q22_14 -1*Q23_00 -2*Q23_01 -3*Q23_03 -1*Q23_04 +1*Q23_05
> -4*Q23_06 -2*Q23_07 +2*Q23_09 -1*Q23_10 -2*Q23_11 -3*Q23_13
> -1*Q23_14 +1*Q23_15 -4*Q23_16 -2*Q23_17 +2*Q23_19 -1*Q24_01
> +1*Q24_02 -2*Q24_03 +2*Q24_05 -1*Q24_07 +1*Q24_08 -2*Q24_09
> +2*Q24_11 -1*Q24_13 +1*Q24_14 -2*Q24_15 +2*Q24_17 -1*Q25_00
> -2*Q25_01 -3*Q25_03 -1*Q25_04 +1*Q25_05 -1*Q25_06 -2*Q25_07
> -3*Q25_09 -1*Q25_10 +1*Q25_11 -1*Q25_12 -2*Q25_13 -3*Q25_15
> -1*Q25_16 +1*Q25_17 -1*Q26_01 +1*Q26_02 -1*Q26_04 +1*Q26_05
> -1*Q26_07 +1*Q26_08 -1*Q26_10 +1*Q26_11 -1*Q34_00 -2*Q34_01
> -3*Q34_03 -1*Q34_04 +1*Q34_05 -4*Q34_06 -2*Q34_07 +2*Q34_09
> -5*Q34_10 -3*Q34_11 -1*Q34_12 +1*Q34_13 +3*Q34_14 -2*Q35_00
> -3*Q35_01 -1*Q35_02 -4*Q35_03 -2*Q35_04 -5*Q35_06 -3*Q35_07
> -1*Q35_08 +1*Q35_09 -1*Q36_00 -2*Q36_01 -3*Q36_03 -1*Q36_04
> +1*Q36_05 -3*Q37_00 -4*Q37_01 -2*Q37_02 -5*Q37_03 -3*Q37_04
> -1*Q37_05 -2*Q38_00 -3*Q38_01 -1*Q38_02 -1*Q39_00 -1*Q40_01
> +1*Q40_02 -2*Q40_03 +2*Q40_05 -3*Q40_06 -1*Q40_07 +1*Q40_08
> +3*Q40_09 -4*Q40_10 -2*Q40_11 +2*Q40_13 +4*Q40_14 -1*Q41_01
> +1*Q41_02 -2*Q41_03 +2*Q41_05 -2*Q42_00 -3*Q42_01 -1*Q42_02
> -4*Q42_03 -2*Q42_04 -1*Q43_00 -2*Q43_01 -1*Q45_00 -2*Q45_01
> -3*Q45_03 -1*Q45_04 +1*Q45_05 -4*Q45_06 -2*Q45_07 +2*Q45_09  >= 0,
>
> -1*Q22_02 -1*Q22_04 -2*Q22_05 -1*Q22_07 -2*Q22_08 -3*Q22_09
> -1*Q22_11 -2*Q22_12 -3*Q22_13 -4*Q22_14 -1*Q23_02 -1*Q23_04
> -2*Q23_05 -1*Q23_07 -2*Q23_08 -3*Q23_09 -1*Q23_12 -1*Q23_14
> -2*Q23_15 -1*Q23_17 -2*Q23_18 -3*Q23_19 -1*Q24_02 -1*Q24_04
> -2*Q24_05 -1*Q24_08 -1*Q24_10 -2*Q24_11 -1*Q24_14 -1*Q24_16
> -2*Q24_17 -1*Q25_02 -1*Q25_04 -2*Q25_05 -1*Q25_08 -1*Q25_10
> -2*Q25_11 -1*Q25_14 -1*Q25_16 -2*Q25_17 -1*Q26_02 -1*Q26_05
> -1*Q26_08 -1*Q26_11 -2*Q34_00 -2*Q34_01 -3*Q34_02 -2*Q34_03
> -3*Q34_04 -4*Q34_05 -2*Q34_06 -3*Q34_07 -4*Q34_08 -5*Q34_09
> -2*Q34_10 -3*Q34_11 -4*Q34_12 -5*Q34_13 -6*Q34_14 -1*Q35_00
> -1*Q35_01 -2*Q35_02 -1*Q35_03 -2*Q35_04 -3*Q35_05 -1*Q35_06
> -2*Q35_07 -3*Q35_08 -4*Q35_09 -1*Q36_00 -1*Q36_01 -2*Q36_02
> -1*Q36_03 -2*Q36_04 -3*Q36_05 -1*Q37_02 -1*Q37_04 -2*Q37_05
> -1*Q38_02 -3*Q40_00 -3*Q40_01 -4*Q40_02 -3*Q40_03 -4*Q40_04
> -5*Q40_05 -3*Q40_06 -4*Q40_07 -5*Q40_08 -6*Q40_09 -3*Q40_10
> -4*Q40_11 -5*Q40_12 -6*Q40_13 -7*Q40_14 -2*Q41_00 -2*Q41_01
> -3*Q41_02 -2*Q41_03 -3*Q41_04 -4*Q41_05 -1*Q42_00 -1*Q42_01
> -2*Q42_02 -1*Q42_03 -2*Q42_04 -3*Q42_05 -1*Q43_00 -1*Q43_01
> -2*Q43_02 -1*Q44_00 -2*Q45_00 -2*Q45_01 -3*Q45_02 -2*Q45_03
> -3*Q45_04 -4*Q45_05 -2*Q45_06 -3*Q45_07 -4*Q45_08 -5*Q45_09  >= -1};

cnsts := {0 <= 2 Q23_10 + 2 Q23_11 + 2 Q23_12 + 2 Q23_13 + 2 Q23_14 + 2 Q23_15

     + 2 Q23_16 + 2 Q23_17 + 2 Q23_18 + 2 Q23_19 + 2 Q24_06 + 2 Q24_07

     + 2 Q24_08 + 2 Q24_09 + 2 Q24_10 + 2 Q24_11 + 4 Q24_12 + 4 Q24_13

     + 4 Q24_14 + 4 Q24_15 + 4 Q24_16 + 4 Q24_17 - 2 Q25_00 - 2 Q25_01
```

```
 - 2 Q25_02 - 2 Q25_03 - 2 Q25_04 - 2 Q25_05 + 2 Q25_12 + 2 Q25_13
 + 2 Q25_14 + 2 Q25_15 + 2 Q25_16 + 2 Q25_17 - 2 Q26_00 - 2 Q26_01
 - 2 Q26_02 + 2 Q26_06 + 2 Q26_07 + 2 Q26_08 + 4 Q26_09 + 4 Q26_10
 + 4 Q26_11 - 4 Q27_00 - 2 Q27_01 + 2 Q27_03 + 4 Q27_04 - Q34_00 - Q34_01
 - Q34_02 - Q34_03 - Q34_04 - Q34_05 - Q34_06 - Q34_07 - Q34_08 - Q34_09
 - Q34_10 - Q34_11 - Q34_12 - Q34_13 - Q34_14 - Q35_00 - Q35_01 - Q35_02
 - Q35_03 - Q35_04 - Q35_05 - Q35_06 - Q35_07 - Q35_08 - Q35_09 - 3 Q36_00
 - 3 Q36_01 - 3 Q36_02 - 3 Q36_03 - 3 Q36_04 - 3 Q36_05 - Q37_00 - Q37_01
 - Q37_02 - Q37_03 - Q37_04 - Q37_05 - 3 Q38_00 - 3 Q38_01 - 3 Q38_02
 - 2 Q40_00 - 2 Q40_01 - 2 Q40_02 - 2 Q40_03 - 2 Q40_04 - 2 Q40_05
 - 2 Q40_06 - 2 Q40_07 - 2 Q40_08 - 2 Q40_09 - 2 Q40_10 - 2 Q40_11
 - 2 Q40_12 - 2 Q40_13 - 2 Q40_14 - 4 Q41_00 - 4 Q41_01 - 4 Q41_02
 - 4 Q41_03 - 4 Q41_04 - 4 Q41_05 - 2 Q42_00 - 2 Q42_01 - 2 Q42_02
 - 2 Q42_03 - 2 Q42_04 - 2 Q42_05 - 4 Q43_00 - 4 Q43_01 - 4 Q43_02
 - 2 Q45_00 - 2 Q45_01 - 2 Q45_02 - 2 Q45_03 - 2 Q45_04 - 2 Q45_05
 - 2 Q45_06 - 2 Q45_07 - 2 Q45_08 - 2 Q45_09 - 4 Q31_00 - 5 Q39_00
 - 6 Q44_00 - 3 Q30_00, 0 <= Q22_01 + Q22_00 + Q22_03 + Q22_02 + Q22_04
 + Q22_05 + Q22_06 + Q22_07 + Q22_08 + Q22_09 + Q22_10 + Q22_11 + Q22_12
 + Q22_13 + Q22_14 - Q24_12 - Q24_13 - Q24_14 - Q24_15 - Q24_16 - Q24_17
 - Q25_12 - Q25_13 - Q25_14 - Q25_15 - Q25_16 - Q25_17 - Q26_06 - Q26_07
 - Q26_08 - 2 Q26_09 - 2 Q26_10 - 2 Q26_11 - Q27_02 - 2 Q27_03 - 3 Q27_04,
0 <= Q22_01 + Q22_00 + Q22_03 + Q22_02 + Q22_04 + Q22_05 + Q22_06 + Q22_07
 + Q22_08 + Q22_09 + Q22_10 + Q22_11 + Q22_12 + Q22_13 + Q22_14 - Q23_10
 - Q23_11 - Q23_12 - Q23_13 - Q23_14 - Q23_15 - Q23_16 - Q23_17 - Q23_18
 - Q23_19 - Q24_00 - Q24_01 - Q24_02 - Q24_03 - Q24_04 - Q24_05 - 2 Q24_06
 - 2 Q24_07 - 2 Q24_08 - 2 Q24_09 - 2 Q24_10 - 2 Q24_11 - 3 Q24_12
 - 3 Q24_13 - 3 Q24_14 - 3 Q24_15 - 3 Q24_16 - 3 Q24_17 - Q25_06 - Q25_07
 - Q25_08 - Q25_09 - Q25_10 - Q25_11 - 2 Q25_12 - 2 Q25_13 - 2 Q25_14
 - 2 Q25_15 - 2 Q25_16 - 2 Q25_17 - Q26_00 - Q26_01 - Q26_02 - 2 Q26_03
 - 2 Q26_04 - 2 Q26_05 - 3 Q26_06 - 3 Q26_07 - 3 Q26_08 - 4 Q26_09
```

- 4 Q26_10 - 4 Q26_11 - Q27_00 - 2 Q27_01 - 3 Q27_02 - 4 Q27_03 - 5 Q27_04
+ 2 Q34_00 + 2 Q34_01 + 2 Q34_02 + 2 Q34_03 + 2 Q34_04 + 2 Q34_05
+ 2 Q34_06 + 2 Q34_07 + 2 Q34_08 + 2 Q34_09 + 2 Q34_10 + 2 Q34_11
+ 2 Q34_12 + 2 Q34_13 + 2 Q34_14 + Q35_00 + Q35_01 + Q35_02 + Q35_03
+ Q35_04 + Q35_05 + Q35_06 + Q35_07 + Q35_08 + Q35_09 + Q36_00 + Q36_01
+ Q36_02 + Q36_03 + Q36_04 + Q36_05 + 2 Q40_00 + 2 Q40_01 + 2 Q40_02
+ 2 Q40_03 + 2 Q40_04 + 2 Q40_05 + 2 Q40_06 + 2 Q40_07 + 2 Q40_08
+ 2 Q40_09 + 2 Q40_10 + 2 Q40_11 + 2 Q40_12 + 2 Q40_13 + 2 Q40_14 + Q41_00
+ Q41_01 + Q41_02 + Q41_03 + Q41_04 + Q41_05 + Q45_00 + Q45_01 + Q45_02
+ Q45_03 + Q45_04 + Q45_05 + Q45_06 + Q45_07 + Q45_08 + Q45_09, 0 <=
3 Q22_01 + 4 Q22_00 + 2 Q22_03 + 3 Q22_02 + 2 Q22_04 + 2 Q22_05 + Q22_06
+ Q22_07 + Q22_08 + Q22_09 + 3 Q23_00 + 2 Q23_01 + 2 Q23_02 + Q23_03
+ Q23_04 + Q23_05 + 3 Q23_10 + 2 Q23_11 + 2 Q23_12 + Q23_13 + Q23_14
+ Q23_15 + 2 Q24_00 + Q24_01 + Q24_02 + 2 Q24_06 + Q24_07 + Q24_08
+ 2 Q24_12 + Q24_13 + Q24_14 + Q25_00 - Q25_03 - Q25_04 - Q25_05 + Q25_06
- Q25_09 - Q25_10 - Q25_11 + Q25_12 - Q25_15 - Q25_16 - Q25_17 - Q26_01
- Q26_02 - Q26_04 - Q26_05 - Q26_07 - Q26_08 - Q26_10 - Q26_11 - 2 Q27_00
- 2 Q27_01 - 2 Q27_02 - 2 Q27_03 - 2 Q27_04 + 4 Q34_00 + 3 Q34_01
+ 3 Q34_02 + 2 Q34_03 + 2 Q34_04 + 2 Q34_05 + Q34_06 + Q34_07 + Q34_08
+ Q34_09 + 3 Q35_00 + 2 Q35_01 + 2 Q35_02 + Q35_03 + Q35_04 + Q35_05
+ 2 Q36_00 + Q36_01 + Q36_02 + 2 Q37_00 + Q37_01 + Q37_02 + Q38_00
+ 4 Q40_00 + 3 Q40_01 + 3 Q40_02 + 2 Q40_03 + 2 Q40_04 + 2 Q40_05 + Q40_06
+ Q40_07 + Q40_08 + Q40_09 + 2 Q41_00 + Q41_01 + Q41_02 + 2 Q42_00
+ Q42_01 + Q42_02 + Q43_00 + 3 Q45_00 + 2 Q45_01 + 2 Q45_02 + Q45_03
+ Q45_04 + Q45_05 - Q30_00, 0 <= -2 Q22_01 - 4 Q22_00 - 3 Q22_02 - Q22_04
- 2 Q22_05 + 2 Q22_06 + Q22_07 - Q22_09 + 4 Q22_10 + 3 Q22_11 + 2 Q22_12
+ Q22_13 - 4 Q23_00 - 2 Q23_01 - 3 Q23_02 - Q23_04 - 2 Q23_05 + 2 Q23_06
+ Q23_07 - Q23_09 - 4 Q23_10 - 2 Q23_11 - 3 Q23_12 - Q23_14 - 2 Q23_15
+ 2 Q23_16 + Q23_17 - Q23_19 - 4 Q24_00 - 2 Q24_01 - 3 Q24_02 - Q24_04

- 2 Q24_05 - 4 Q24_06 - 2 Q24_07 - 3 Q24_08 - Q24_10 - 2 Q24_11 - 4 Q24_12
- 2 Q24_13 - 3 Q24_14 - Q24_16 - 2 Q24_17 - 2 Q25_00 - Q25_02 + 2 Q25_03
+ Q25_04 - 2 Q25_06 - Q25_08 + 2 Q25_09 + Q25_10 - 2 Q25_12 - Q25_14
+ 2 Q25_15 + Q25_16 - 2 Q26_00 - Q26_02 - 2 Q26_03 - Q26_05 - 2 Q26_06
- Q26_08 - 2 Q26_09 - Q26_11 - 4 Q34_00 - 2 Q34_01 - 3 Q34_02 - Q34_04
- 2 Q34_05 + 2 Q34_06 + Q34_07 - Q34_09 + 4 Q34_10 + 3 Q34_11 + 2 Q34_12
+ Q34_13 - 3 Q35_00 - Q35_01 - 2 Q35_02 + Q35_03 - Q35_05 + 3 Q35_06
+ 2 Q35_07 + Q35_08 - 3 Q36_00 - Q36_01 - 2 Q36_02 + Q36_03 - Q36_05
- 2 Q37_00 - Q37_02 + 2 Q37_03 + Q37_04 - 2 Q38_00 - Q38_02 - 5 Q40_00
- 3 Q40_01 - 4 Q40_02 - Q40_03 - 2 Q40_04 - 3 Q40_05 + Q40_06 - Q40_08
- 2 Q40_09 + 3 Q40_10 + 2 Q40_11 + Q40_12 - Q40_14 - 4 Q41_00 - 2 Q41_01
- 3 Q41_02 - Q41_04 - 2 Q41_05 - 3 Q42_00 - Q42_01 - 2 Q42_02 + Q42_03
- Q42_05 - 3 Q43_00 - Q43_01 - 2 Q43_02 - 4 Q45_00 - 2 Q45_01 - 3 Q45_02
- Q45_04 - 2 Q45_05 + 2 Q45_06 + Q45_07 - Q45_09 - 2 Q31_00 - 2 Q39_00
- 3 Q44_00, 0 <= -3 Q22_01 - 2 Q22_00 - 4 Q22_03 - Q22_02 - 2 Q22_04
- 5 Q22_06 - 3 Q22_07 - Q22_08 + Q22_09 - 6 Q22_10 - 4 Q22_11 - 2 Q22_12
+ 2 Q22_14 - Q23_00 - 2 Q23_01 - 3 Q23_03 - Q23_04 + Q23_05 - 4 Q23_06
- 2 Q23_07 + 2 Q23_09 - Q23_10 - 2 Q23_11 - 3 Q23_13 - Q23_14 + Q23_15
- 4 Q23_16 - 2 Q23_17 + 2 Q23_19 - Q24_01 + Q24_02 - 2 Q24_03 + 2 Q24_05
- Q24_07 + Q24_08 - 2 Q24_09 + 2 Q24_11 - Q24_13 + Q24_14 - 2 Q24_15
+ 2 Q24_17 - Q25_00 - 2 Q25_01 - 3 Q25_03 - Q25_04 + Q25_05 - Q25_06
- 2 Q25_07 - 3 Q25_09 - Q25_10 + Q25_11 - Q25_12 - 2 Q25_13 - 3 Q25_15
- Q25_16 + Q25_17 - Q26_01 + Q26_02 - Q26_04 + Q26_05 - Q26_07 + Q26_08
- Q26_10 + Q26_11 - Q34_00 - 2 Q34_01 - 3 Q34_03 - Q34_04 + Q34_05
- 4 Q34_06 - 2 Q34_07 + 2 Q34_09 - 5 Q34_10 - 3 Q34_11 - Q34_12 + Q34_13
+ 3 Q34_14 - 2 Q35_00 - 3 Q35_01 - Q35_02 - 4 Q35_03 - 2 Q35_04 - 5 Q35_06
- 3 Q35_07 - Q35_08 + Q35_09 - Q36_00 - 2 Q36_01 - 3 Q36_03 - Q36_04
+ Q36_05 - 3 Q37_00 - 4 Q37_01 - 2 Q37_02 - 5 Q37_03 - 3 Q37_04 - Q37_05
- 2 Q38_00 - 3 Q38_01 - Q38_02 - Q40_01 + Q40_02 - 2 Q40_03 + 2 Q40_05
- 3 Q40_06 - Q40_07 + Q40_08 + 3 Q40_09 - 4 Q40_10 - 2 Q40_11 + 2 Q40_13

```
     + 4 Q40_14 - Q41_01 + Q41_02 - 2 Q41_03 + 2 Q41_05 - 2 Q42_00 - 3 Q42_01
     - Q42_02 - 4 Q42_03 - 2 Q42_04 - Q43_00 - 2 Q43_01 - Q45_00 - 2 Q45_01
     - 3 Q45_03 - Q45_04 + Q45_05 - 4 Q45_06 - 2 Q45_07 + 2 Q45_09 - Q39_00, -1
     <= -Q22_02 - Q22_04 - 2 Q22_05 - Q22_07 - 2 Q22_08 - 3 Q22_09 - Q22_11
     - 2 Q22_12 - 3 Q22_13 - 4 Q22_14 - Q23_02 - Q23_04 - 2 Q23_05 - Q23_07
     - 2 Q23_08 - 3 Q23_09 - Q23_12 - Q23_14 - 2 Q23_15 - Q23_17 - 2 Q23_18
     - 3 Q23_19 - Q24_02 - Q24_04 - 2 Q24_05 - Q24_08 - Q24_10 - 2 Q24_11
     - Q24_14 - Q24_16 - 2 Q24_17 - Q25_02 - Q25_04 - 2 Q25_05 - Q25_08
     - Q25_10 - 2 Q25_11 - Q25_14 - Q25_16 - 2 Q25_17 - Q26_02 - Q26_05
     - Q26_08 - Q26_11 - 2 Q34_00 - 2 Q34_01 - 3 Q34_02 - 2 Q34_03 - 3 Q34_04
     - 4 Q34_05 - 2 Q34_06 - 3 Q34_07 - 4 Q34_08 - 5 Q34_09 - 2 Q34_10
     - 3 Q34_11 - 4 Q34_12 - 5 Q34_13 - 6 Q34_14 - Q35_00 - Q35_01 - 2 Q35_02
     - Q35_03 - 2 Q35_04 - 3 Q35_05 - Q35_06 - 2 Q35_07 - 3 Q35_08 - 4 Q35_09
     - Q36_00 - Q36_01 - 2 Q36_02 - Q36_03 - 2 Q36_04 - 3 Q36_05 - Q37_02
     - Q37_04 - 2 Q37_05 - Q38_02 - 3 Q40_00 - 3 Q40_01 - 4 Q40_02 - 3 Q40_03
     - 4 Q40_04 - 5 Q40_05 - 3 Q40_06 - 4 Q40_07 - 5 Q40_08 - 6 Q40_09
     - 3 Q40_10 - 4 Q40_11 - 5 Q40_12 - 6 Q40_13 - 7 Q40_14 - 2 Q41_00
     - 2 Q41_01 - 3 Q41_02 - 2 Q41_03 - 3 Q41_04 - 4 Q41_05 - Q42_00 - Q42_01
     - 2 Q42_02 - Q42_03 - 2 Q42_04 - 3 Q42_05 - Q43_00 - Q43_01 - 2 Q43_02
     - 2 Q45_00 - 2 Q45_01 - 3 Q45_02 - 2 Q45_03 - 3 Q45_04 - 4 Q45_05
     - 2 Q45_06 - 3 Q45_07 - 4 Q45_08 - 5 Q45_09 - Q44_00}
> maximize(obj,cnsts,NONNEGATIVE);
{Q25_11 = 0, Q25_12 = 0, Q25_13 = 0, Q25_14 = 0, Q25_15 = 0, Q25_16 = 0,
    Q25_17 = 0, Q26_00 = 0, Q26_01 = 0, Q26_02 = 0, Q26_03 = 0, Q26_04 = 0,
    Q24_13 = 0, Q24_14 = 0, Q24_15 = 0, Q24_16 = 0, Q25_00 = 0, Q25_01 = 0,
    Q25_02 = 0, Q25_03 = 0, Q25_04 = 0, Q25_05 = 0, Q25_06 = 0, Q25_07 = 0,
    Q25_08 = 0, Q25_09 = 0, Q25_10 = 0, Q23_17 = 0, Q23_18 = 0, Q23_19 = 0,
    Q24_00 = 0, Q24_01 = 0, Q24_02 = 0, Q24_03 = 0, Q24_04 = 0, Q24_05 = 0,
    Q24_06 = 0, Q24_07 = 0, Q24_08 = 0, Q24_09 = 0, Q24_10 = 0, Q24_12 = 0,
```

```
    Q23_12 = 0, Q23_13 = 0, Q23_14 = 0, Q23_15 = 0, Q23_16 = 0, Q22_06 = 0,
    Q22_07 = 0, Q22_08 = 0, Q22_09 = 0, Q22_10 = 0, Q22_11 = 0, Q22_12 = 0,
    Q22_13 = 0, Q22_00 = 0, Q22_03 = 0, Q22_02 = 0, Q22_04 = 0, Q22_05 = 0,
    Q45_09 = 0, Q39_00 = 0, Q44_00 = 0, Q30_00 = 0, Q24_11 = 1/6, Q40_11 = 1/6,
    Q40_10 = 0, Q40_09 = 0, Q24_17 = 0, Q22_01 = 0, Q31_00 = 0, Q43_01 = 0,
    Q43_02 = 0, Q45_00 = 0, Q45_01 = 0, Q45_02 = 0, Q45_03 = 0, Q45_04 = 0,
    Q45_05 = 0, Q45_06 = 0, Q45_07 = 0, Q45_08 = 0, Q40_13 = 0, Q40_14 = 0,
    Q41_00 = 0, Q41_01 = 0, Q41_02 = 0, Q41_03 = 0, Q41_04 = 0, Q41_05 = 0,
    Q42_00 = 0, Q42_01 = 0, Q42_02 = 0, Q42_03 = 0, Q42_04 = 0, Q42_05 = 0,
    Q43_00 = 0, Q40_04 = 0, Q40_05 = 0, Q40_06 = 0, Q40_07 = 0, Q40_08 = 0,
    Q40_12 = 0, Q38_00 = 0, Q38_01 = 0, Q38_02 = 0, Q40_00 = 0, Q40_01 = 0,
    Q40_02 = 0, Q40_03 = 0, Q37_05 = 0, Q37_00 = 0, Q37_01 = 0, Q37_02 = 0,
    Q37_03 = 0, Q37_04 = 0, Q35_07 = 0, Q35_08 = 0, Q35_09 = 0, Q36_00 = 0,
    Q36_01 = 0, Q36_02 = 0, Q36_03 = 0, Q36_04 = 0, Q36_05 = 0, Q34_14 = 0,
    Q35_00 = 0, Q35_01 = 0, Q35_02 = 0, Q35_03 = 0, Q35_04 = 0, Q35_05 = 0,
    Q35_06 = 0, Q34_08 = 0, Q34_09 = 0, Q34_10 = 0, Q34_11 = 0, Q34_12 = 0,
    Q34_13 = 0, Q34_02 = 0, Q34_03 = 0, Q34_04 = 0, Q34_05 = 0, Q34_06 = 0,
    Q34_07 = 0, Q27_00 = 0, Q27_01 = 0, Q27_02 = 0, Q27_03 = 0, Q27_04 = 0,
    Q34_00 = 0, Q34_01 = 0, Q26_05 = 0, Q26_06 = 0, Q26_07 = 0, Q26_08 = 0,
    Q26_09 = 0, Q26_10 = 0, Q26_11 = 0, Q23_00 = 0, Q22_14 = 0, Q23_01 = 0,
    Q23_02 = 0, Q23_03 = 0, Q23_04 = 0, Q23_05 = 0, Q23_06 = 0, Q23_07 = 0,
    Q23_08 = 0, Q23_09 = 0, Q23_10 = 0, Q23_11 = 0}
> subs(%,obj);
```



```
> (dualobj,dualcnsts):=dual(obj,cnsts,y);
dualobj, dualcnsts := y7, {3 <= 2 y1 - 2 y2 - y4 - y5 + y6 + 5 y7,
    3 <= 2 y1 - 2 y2 - y4 - 3 y5 + 2 y6 + 6 y7,
    3 <= 2 y1 - 2 y2 + 2 y5 - 2 y6 + 4 y7, 3 <= 2 y1 - 2 y2 - y6 + 5 y7,
    3 <= 2 y1 - 2 y2 + 4 y5 - 3 y6 + 3 y7, 3 <= 2 y1 - 2 y2 - 2 y5 + 6 y7,
    3 <= 2 y1 - 2 y2 - 4 y5 + y6 + 7 y7, 3 <= 2 y1 - y2 + 2 y5 - y6 + 3 y7,
```

3 <= 2 y1 - y2 + 4 y7, 3 <= 2 y1 - y2 - 2 y5 + y6 + 5 y7,

3 <= 2 y1 - y2 - y4 - y5 + 2 y6 + 4 y7, 3 <= 2 y1 - y2 + 4 y5 - 2 y6 + 2 y7,

3 <= 2 y1 - y2 - y4 + y5 + y6 + 3 y7, 3 <= 2 y1 - y2 - 2 y4 + 3 y6 + 3 y7,

3 <= 2 y1 - y2 - y4 + 3 y5 + 2 y7, 3 <= 2 y1 + 4 y5 - y6 + y7,

3 <= 2 y1 + 2 y5 + 2 y7, 3 <= 2 y1 + y6 + 3 y7, 3 <= 4 y1 - y2 + y6 + 3 y7,

2 <= y1 - y2 + y5 - y6 + 3 y7, 2 <= y1 - y2 - y5 + 4 y7,

1 <= 3 y1 - y2 - 2 y4 + y5 + 3 y6 + y7, 1 <= -y2 - y3 - y6 + 3 y7,

1 <= -y2 - y3 - 2 y5 + 4 y7, 1 <= -y2 - y3 + 2 y5 - 2 y6 + 2 y7,

1 <= -y2 - y3 + 6 y5 - 4 y6, 1 <= -y2 - y3 + 4 y5 - 3 y6 + y7,

1 <= -y2 - y3 - y4 - y5 + y6 + 3 y7, 1 <= -y2 - y3 - y4 + y5 + 2 y7,

1 <= -y2 - y3 - y4 + 3 y5 - y6 + y7, 1 <= -y2 - y3 - 2 y4 + 4 y5,

1 <= -y2 - y3 - 3 y4 + y5 + 3 y6 + y7, 1 <= -y2 - y3 - y4 + 5 y5 - 2 y6,

1 <= -y2 - y3 - 2 y4 + 2 y6 + 2 y7, 1 <= -y2 - y3 - 3 y4 + 3 y5 + 2 y6,

1 <= -2 y1 + y2 + 2 y7, 1 <= -2 y1 + y2 - 2 y5 + y6 + 3 y7,

1 <= -2 y1 + y2 + 2 y5 - y6 + y7, 1 <= -2 y1 + y2 + 4 y5 - 2 y6,

1 <= -2 y1 + y2 - y4 - y5 + 2 y6 + 2 y7,

1 <= -2 y1 + y2 - y4 + y5 + y6 + y7, 1 <= -2 y1 + y2 - y4 + 3 y5,

1 <= -2 y1 + y2 - 2 y4 + 3 y6 + y7, 1 <= -2 y1 + y2 - 2 y4 + 2 y5 + 2 y6,

1 <= -2 y1 + y2 - 3 y4 + y5 + 4 y6, 1 <= 2 y7, 1 <= -2 y5 + y6 + 3 y7,

1 <= 4 y5 - 2 y6, 1 <= 2 y5 - y6 + y7, 1 <= -y4 - y5 + 2 y6 + 2 y7,

1 <= -y4 + 3 y5, 1 <= -y4 + y5 + y6 + y7, 1 <= -2 y4 + 2 y5 + 2 y6,

1 <= -2 y4 + 3 y6 + y7, 1 <= -3 y4 + y5 + 4 y6,

1 <= -4 y1 + 3 y2 + y3 + 2 y5, 1 <= -4 y1 + 3 y2 + y3 - y4 - y5 + 3 y6 + y7,

1 <= -4 y1 + 3 y2 + y3 - y4 + y5 + 2 y6,

1 <= -4 y1 + 3 y2 + y3 - 2 y4 + 4 y6,

1 <= -2 y1 + 2 y2 - 2 y5 + 2 y6 + 2 y7, 1 <= -2 y1 + 2 y2 + y6 + y7,

1 <= -2 y1 + 2 y2 + 2 y5, 1 <= -2 y1 + 2 y2 - y4 - y5 + 3 y6 + y7,

1 <= -2 y1 + 2 y2 - y4 + y5 + 2 y6, 1 <= -2 y1 + 2 y2 - 2 y4 + 4 y6,

1 <= y2 - 2 y5 + 2 y6 + 2 y7, 1 <= y2 - y4 - y5 + 3 y6 + y7, 1 <= y2 + 2 y5,

1 <= y2 + y6 + y7, 1 <= y2 - 2 y4 + 4 y6, 1 <= y2 - y4 + y5 + 2 y6,

1 <= y2 + y4 - y5 + 2 y7, 1 <= y2 + y4 + y5 - y6 + y7,

1 <= y2 + y4 + 3 y5 - 2 y6, 1 <= 2 y1 + y4 - y5 + 2 y7,

1 <= 2 y1 + y4 + 3 y5 - 2 y6, 1 <= 2 y1 + y4 + y5 - y6 + y7,

1 <= 2 y1 + y6 + y7, 1 <= 2 y1 + 2 y5,

1 <= -4 y1 + 3 y2 + y3 - 2 y5 + 2 y6 + 2 y7, 1 <= 2 y1 - y4 + y5 + 2 y6,

1 <= -4 y1 + 3 y2 + y3 + y6 + y7, 1 <= -2 y1 + 3 y2 + y3 + y4 + y5,

1 <= 2 y2 + y4 - y5 + y6 + y7, 1 <= -2 y1 + 3 y2 + y3 + 2 y6,

1 <= 2 y2 + 2 y6, 1 <= 2 y2 + y4 + y5, 1 <= 2 y1 + y2 + y4 - y5 + y6 + y7,

1 <= 2 y1 + y2 + y4 + y5, 1 <= 2 y1 + y2 + 2 y6,

1 <= -2 y1 + 2 y2 + y3 + y4 - y5 + 2 y7,

1 <= -2 y1 + 2 y2 + y3 + y4 + y5 - y6 + y7,

1 <= -2 y1 + 2 y2 + y3 + y4 + 3 y5 - 2 y6, 1 <= -2 y1 + 2 y2 + y3 + 2 y5,

1 <= -2 y1 + 2 y2 + y3 + y6 + y7, 1 <= -2 y1 + 2 y2 + y3 - y4 + y5 + 2 y6,

1 <= 4 y1 + y2 + 2 y4, 1 <= -4 y1 + 4 y2 + 2 y3 + y4 - y5 + y6 + y7,

1 <= y1 - y4 + 2 y5 + y6 + y7, 1 <= y1 + 5 y5 - 2 y6,

1 <= -4 y1 + 4 y2 + 2 y3 + y4 + y5,

1 <= -2 y1 + 3 y2 + y3 + y4 - y5 + y6 + y7, 1 <= -4 y1 + 4 y2 + 2 y3 + 2 y6,

2 <= y1 - 2 y2 - 3 y4 + 2 y5 + 2 y6 + 2 y7,

2 <= y1 - 2 y2 - 3 y4 + 3 y6 + 3 y7,

2 <= y1 - 2 y2 - 4 y4 + y5 + 4 y6 + 2 y7, 1 <= -4 y1 + 5 y2 + 3 y3 + 2 y4,

1 <= -2 y1 + 4 y2 + 2 y3 + 2 y4, 1 <= 2 y1 + 2 y2 + 2 y4,

1 <= 3 y2 + y3 + 2 y4, 2 <= y1 - y2 - y4 + 4 y5 - y6 + y7,

2 <= y1 - y2 - y4 + 2 y5 + 2 y7, 2 <= y1 - y2 - y4 + y6 + 3 y7,

2 <= y1 - y2 + 5 y5 - 3 y6 + y7, 2 <= y1 - 2 y2 - 2 y4 + y5 + y6 + 3 y7,

2 <= y1 - 2 y2 - 2 y4 - y5 + 2 y6 + 4 y7,

2 <= y1 - 2 y2 - 2 y4 + 3 y5 + 2 y7,

2 <= y1 - 2 y2 - y4 + 4 y5 - 2 y6 + 2 y7,

2 <= y1 - 2 y2 - y4 + 2 y5 - y6 + 3 y7, 1 <= y1 - y4 + 4 y5,

2 <= y1 - y2 + 3 y5 - 2 y6 + 2 y7, 1 <= 3 y1 - y2 - y4 + 2 y6 + 2 y7,

```
    1 <= -y2 - y3 - 4 y4 + 2 y5 + 4 y6, 1 <= 3 y1 - y2 + 3 y5 - y6 + y7,

    1 <= 3 y1 - y2 + y5 + 2 y7, 1 <= 3 y1 - y2 - y5 + y6 + 3 y7,

    1 <= y1 - 2 y4 + 3 y5 + 2 y6, 1 <= -y2 - y3 - 2 y4 + 2 y5 + y6 + y7,

    2 <= y1 - 2 y2 - y4 + 4 y7, 2 <= y1 - 2 y2 - y4 - 2 y5 + y6 + 5 y7,

    2 <= y1 - 2 y2 + 5 y5 - 4 y6 + 2 y7, 2 <= y1 - 2 y2 + 3 y5 - 3 y6 + 3 y7,

    2 <= y1 - 2 y2 + y5 - 2 y6 + 4 y7, 2 <= y1 - 2 y2 - y5 - y6 + 5 y7,

    2 <= y1 - 2 y2 - 3 y5 + 6 y7, 2 <= y1 - y2 - 3 y4 + 2 y5 + 3 y6 + y7,

    2 <= y1 - y2 - 2 y4 + 3 y5 + y6 + y7,

    2 <= y1 - y2 - 2 y4 + y5 + 2 y6 + 2 y7,

    1 <= 3 y1 - y2 - y4 + 2 y5 + y6 + y7, 1 <= y1 + 3 y5 - y6 + y7,

    1 <= y1 + y5 + 2 y7, 2 <= 3 y1 - y4 + 2 y5 + 2 y6, 2 <= 3 y1 + 3 y5,

    2 <= 3 y1 + y5 + y6 + y7, 3 <= 2 y1 - 2 y2 - 4 y4 + 5 y6 + 3 y7,

    3 <= 2 y1 - 2 y2 - 3 y4 + y5 + 3 y6 + 3 y7,

    3 <= 2 y1 - 2 y2 - 3 y4 - y5 + 4 y6 + 4 y7,

    3 <= 2 y1 - 2 y2 - 2 y4 + 2 y5 + y6 + 3 y7,

    3 <= 2 y1 - 2 y2 - 2 y4 + 2 y6 + 4 y7,

    3 <= 2 y1 - 2 y2 - 2 y4 - 2 y5 + 3 y6 + 5 y7,

    3 <= 2 y1 - 2 y2 - y4 + 3 y5 - y6 + 3 y7, 3 <= 2 y1 - 2 y2 - y4 + y5 + 4 y7,

    3 <= 4 y1 - y2 - 2 y4 + 4 y6 + 2 y7, 3 <= 4 y1 - y2 - y4 + y5 + 2 y6 + 2 y7,

    3 <= 4 y1 - y2 - y4 - y5 + 3 y6 + 3 y7, 3 <= 4 y1 - y2 + 2 y5 + 2 y7,

    3 <= 4 y1 - y2 - 2 y5 + 2 y6 + 4 y7, 3 <= 2 y1 - 2 y4 + 2 y5 + 3 y6 + y7,

    3 <= 2 y1 - y4 + 3 y5 + y6 + y7, 3 <= 2 y1 - y4 + y5 + 2 y6 + 2 y7,

    3 <= 4 y1 - y4 + y5 + 3 y6 + y7, 3 <= 4 y1 + 2 y5 + y6 + y7,

    3 <= 4 y1 + 2 y6 + 2 y7, 3 <= 2 y1 - y2 - 3 y4 + y5 + 4 y6 + 2 y7,

    3 <= 2 y1 - y2 - 2 y4 + 2 y5 + 2 y6 + 2 y7, 2 <= 4 y1 + 2 y6,

    2 <= 5 y1 + y5 + 2 y6, 3 <= 6 y1 + 3 y6 + y7, 1 <= 3 y1 + y4}

>
> minimize(dualobj,dualcnsts,NONNEGATIVE);

     {y1 = 7/18, y2 = 4/9, y5 = 4/9, y6 = 2/9, y7 = 2/3, y4 = 0, y3 = 1/3}

> quit
```

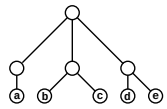
# MIDS

In all operations considered, the vertex at the "top" of the figure is the one that could possibly be selected by "some" algorithm for inclusion in the independent dominating set. The priorities are

- vertices with a neighbour of degree 1 over
- vertices of degree 2 (and their neighbours) that have 1 vertex at distance 2 over
- vertices of degree 2 (and their neighbours) that have 2 vertices at distance 2 over
- vertices of degree 2 (and their neighbours) that have 3 vertices at distance 2 over
- vertices of degree 2 (and their neighbours) that have 4 vertices at distance 2

In each case, choose the vertex that, if added to the set, would give the smallest ratio of vertices added to the set (which includes the vertex itself and all isolates created) to vertices removed from the graph (i.e., vertices that attain degree zero). Ties are broken arbitrarily. Operations written in red (with "***" in the last column) are excluded from $OPS_2$.

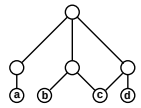
Z01:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z01-00: | $V_2$ | $V_2$ | $V_2$ | $V_2$ | $V_2$: | 3 | 6 | 0 | 0 | 0 | 5 | 1 |
| Z01-01: | $V_2$ | $V_2$ | $V_2$ | $V_2$ | $V_3$: | 4 | 5 | 0 | 0 | 1 | 4 | 1 |
| Z01-02: | $V_2$ | $V_2$ | $V_2$ | $V_3$ | $V_3$: | 5 | 4 | 0 | 0 | 2 | 3 | 1 |
| Z01-03: | $V_2$ | $V_2$ | $V_3$ | $V_2$ | $V_3$: | 5 | 4 | 0 | 0 | 2 | 3 | 1 |
| Z01-04: | $V_2$ | $V_2$ | $V_3$ | $V_3$ | $V_3$: | 6 | 3 | 0 | 0 | 3 | 2 | 1 |
| Z01-05: | $V_2$ | $V_3$ | $V_3$ | $V_3$ | $V_3$: | 7 | 2 | 0 | 0 | 4 | 1 | 1 |
| Z01-06: | $V_3$ | $V_2$ | $V_2$ | $V_2$ | $V_2$: | 4 | 5 | 0 | 0 | 1 | 4 | 1 |
| Z01-07: | $V_3$ | $V_2$ | $V_2$ | $V_2$ | $V_3$: | 5 | 4 | 0 | 0 | 2 | 3 | 1 |
| Z01-08: | $V_3$ | $V_2$ | $V_2$ | $V_3$ | $V_3$: | 6 | 3 | 0 | 0 | 3 | 2 | 1 |
| Z01-09: | $V_3$ | $V_2$ | $V_3$ | $V_2$ | $V_3$: | 6 | 3 | 0 | 0 | 3 | 2 | 1 |
| Z01-10: | $V_3$ | $V_2$ | $V_3$ | $V_3$ | $V_3$: | 7 | 2 | 0 | 0 | 4 | 1 | 1 |
| Z01-11: | $V_3$ | $V_3$ | $V_3$ | $V_3$ | $V_3$: | 8 | 1 | 0 | 0 | 5 | 0 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

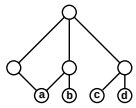
Z02:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z02-00: | $V_2$ | $V_2$ | $V_2$ | $V_2$ | : | 3 | 5 | 0 | 0 | 0 | 3 | 2*** |
| Z02-01: | $V_2$ | $V_2$ | $V_2$ | $V_3$ | : | 4 | 4 | 0 | 0 | 1 | 2 | 2*** |
| Z02-02: | $V_2$ | $V_2$ | $V_3$ | $V_2$ | : | 4 | 4 | 0 | 0 | 0 | 4 | 1 |
| Z02-03: | $V_2$ | $V_2$ | $V_3$ | $V_3$ | : | 5 | 3 | 0 | 0 | 1 | 3 | 1 |
| Z02-04: | $V_2$ | $V_3$ | $V_2$ | $V_3$ | : | 5 | 3 | 0 | 0 | 2 | 1 | 2*** |
| Z02-05: | $V_2$ | $V_3$ | $V_3$ | $V_3$ | : | 6 | 2 | 0 | 0 | 2 | 2 | 1 |
| Z02-06: | $V_3$ | $V_2$ | $V_2$ | $V_2$ | : | 4 | 4 | 0 | 0 | 1 | 2 | 2*** |
| Z02-07: | $V_3$ | $V_2$ | $V_2$ | $V_3$ | : | 5 | 3 | 0 | 0 | 2 | 1 | 2*** |
| Z02-08: | $V_3$ | $V_2$ | $V_3$ | $V_2$ | : | 5 | 3 | 0 | 0 | 1 | 3 | 1 |
| Z02-09: | $V_3$ | $V_2$ | $V_3$ | $V_3$ | : | 6 | 2 | 0 | 0 | 2 | 2 | 1 |
| Z02-10: | $V_3$ | $V_3$ | $V_2$ | $V_3$ | : | 6 | 2 | 0 | 0 | 3 | 0 | 2*** |
| Z02-11: | $V_3$ | $V_3$ | $V_3$ | $V_3$ | : | 7 | 1 | 0 | 0 | 3 | 1 | 1 |

The vertex $c$ cannot have degree 2. The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours). If the vertex $c$ had degree 2, adding the selected vertex would give a ratio of 2/5 whereas adding the neighbour of $b$ would give a ratio of 1/4. The only way an isolate may be generated is if $b$ has a neighbour of degree 1, which is not the case.

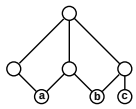
Z03:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z03-00: | $V_2$ | $V_2$ | $V_2$ | $V_2$ | | : | 3 | 5 | 0 | 0 | 0 | 3 | 2*** |
| Z03-01: | $V_2$ | $V_2$ | $V_2$ | $V_3$ | | : | 4 | 4 | 0 | 0 | 1 | 2 | 2*** |
| Z03-02: | $V_2$ | $V_2$ | $V_3$ | $V_3$ | | : | 5 | 3 | 0 | 0 | 2 | 1 | 2*** |
| Z03-03: | $V_2$ | $V_3$ | $V_2$ | $V_2$ | | : | 4 | 4 | 0 | 0 | 1 | 2 | 2*** |
| Z03-04: | $V_2$ | $V_3$ | $V_2$ | $V_3$ | | : | 5 | 3 | 0 | 0 | 2 | 1 | 2*** |
| Z03-05: | $V_2$ | $V_3$ | $V_3$ | $V_3$ | | : | 6 | 2 | 0 | 0 | 3 | 0 | 2*** |
| Z03-06: | $V_3$ | $V_2$ | $V_2$ | $V_2$ | | : | 4 | 4 | 0 | 0 | 0 | 4 | 1 |
| Z03-07: | $V_3$ | $V_2$ | $V_2$ | $V_3$ | | : | 5 | 3 | 0 | 0 | 1 | 3 | 1 |
| Z03-08: | $V_3$ | $V_2$ | $V_3$ | $V_3$ | | : | 6 | 2 | 0 | 0 | 2 | 2 | 1 |
| Z03-09: | $V_3$ | $V_3$ | $V_2$ | $V_2$ | | : | 5 | 3 | 0 | 0 | 1 | 3 | 1 |
| Z03-10: | $V_3$ | $V_3$ | $V_2$ | $V_3$ | | : | 6 | 2 | 0 | 0 | 2 | 2 | 1 |
| Z03-11: | $V_3$ | $V_3$ | $V_3$ | $V_3$ | | : | 7 | 1 | 0 | 0 | 3 | 1 | 1 |

The vertex $a$ cannot have degree 2. The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours). If the vertex $a$ had degree 2, adding the selected vertex would give a ratio of 2/5 whereas adding the neighbour of $a$ of degree 2 would give a ratio of 1/3.

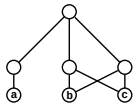
Z04:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z04-00: | $V_2$ | $V_2$ | $V_2$ | | | : | 3 | 4 | 0 | 0 | 0 | 1 | 3*** |
| Z04-01: | $V_2$ | $V_2$ | $V_3$ | | | : | 4 | 3 | 0 | 0 | 1 | 0 | 3*** |
| Z04-02: | $V_2$ | $V_3$ | $V_2$ | | | : | 4 | 3 | 0 | 0 | 0 | 2 | 2*** |
| Z04-03: | $V_2$ | $V_3$ | $V_3$ | | | : | 5 | 2 | 0 | 0 | 1 | 1 | 2*** |
| Z04-04: | $V_3$ | $V_3$ | $V_2$ | | | : | 5 | 2 | 0 | 0 | 0 | 3 | 1 |
| Z04-05: | $V_3$ | $V_3$ | $V_3$ | | | : | 6 | 1 | 0 | 0 | 1 | 2 | 1 |

The vertices $a$ and $b$ cannot have degree 2. The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours). If one of the vertices $a$ or $b$ had degree 2 (respectively both of the vertices $a$ and $b$ had degree 2), adding the selected vertex would give a ratio of 2/5 (respectively 3/5) whereas adding the neighbour of $a$ of degree 2 would give a ratio of 1/3.

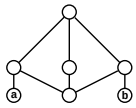
Z05:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z05-00: | $V_2$ | $V_2$ | $V_2$ | | | : | 3 | 4 | 0 | 0 | 0 | 1 | 3*** |
| Z05-01: | $V_2$ | $V_2$ | $V_3$ | | | : | 4 | 3 | 0 | 0 | 0 | 2 | 2*** |
| Z05-02: | $V_2$ | $V_3$ | $V_3$ | | | : | 5 | 2 | 0 | 0 | 0 | 3 | 1 |
| Z05-03: | $V_3$ | $V_2$ | $V_2$ | | | : | 4 | 3 | 0 | 0 | 1 | 0 | 3*** |
| Z05-04: | $V_3$ | $V_2$ | $V_3$ | | | : | 5 | 2 | 0 | 0 | 1 | 1 | 2*** |
| Z05-05: | $V_3$ | $V_3$ | $V_3$ | | | : | 6 | 1 | 0 | 0 | 1 | 2 | 1 |

The vertices $a$ and $b$ cannot have degree 2. The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours). The selected vertex has one neighbour of degree 2 which has at least three vertices at distance 2 from it. If either of the vertices $b$ or $c$ had degree 2, that vertex would have precisely two vertices at distance 2 from it. That vertex or one of its neighbours has higher priority.

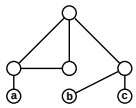
Z06:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z06-00: | $V_2$ | $V_2$ | | | | : | 4 | 3 | 0 | 0 | 0 | 2 | 2*** |
| Z06-01: | $V_2$ | $V_3$ | | | | : | 5 | 2 | 0 | 0 | 1 | 1 | 2*** |
| Z06-02: | $V_3$ | $V_3$ | | | | : | 6 | 1 | 0 | 0 | 2 | 0 | 2*** |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours). Adding the selected vertex would give a ratio of 2/5 whereas adding the neighbour of the selected vertex of degree 2 would give a ratio of 1/3.

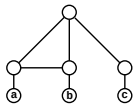
Z07:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z07-00: | $V_2$ | $V_2$ | $V_2$ | | | : | 3 | 4 | 0 | 0 | 0 | 3 | 1 |
| Z07-01: | $V_2$ | $V_2$ | $V_3$ | | | : | 4 | 3 | 0 | 0 | 1 | 2 | 1 |
| Z07-02: | $V_2$ | $V_3$ | $V_3$ | | | : | 5 | 2 | 0 | 0 | 2 | 1 | 1 |
| Z07-03: | $V_3$ | $V_2$ | $V_2$ | | | : | 4 | 3 | 0 | 0 | 1 | 2 | 1 |
| Z07-04: | $V_3$ | $V_2$ | $V_3$ | | | : | 5 | 2 | 0 | 0 | 2 | 1 | 1 |
| Z07-05: | $V_3$ | $V_3$ | $V_3$ | | | : | 6 | 1 | 0 | 0 | 3 | 0 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

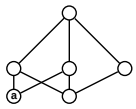
Z08:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z08-00: | $V_2$ | $V_2$ | $V_2$ | | | : | 3 | 4 | 0 | 0 | 0 | 3 | 1 |
| Z08-01: | $V_2$ | $V_2$ | $V_3$ | | | : | 4 | 3 | 0 | 0 | 1 | 2 | 1 |
| Z08-02: | $V_2$ | $V_3$ | $V_2$ | | | : | 4 | 3 | 0 | 0 | 1 | 2 | 1 |
| Z08-03: | $V_2$ | $V_3$ | $V_3$ | | | : | 5 | 2 | 0 | 0 | 2 | 1 | 1 |
| Z08-04: | $V_3$ | $V_3$ | $V_2$ | | | : | 5 | 2 | 0 | 0 | 2 | 1 | 1 |
| Z08-05: | $V_3$ | $V_3$ | $V_3$ | | | : | 6 | 1 | 0 | 0 | 3 | 0 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

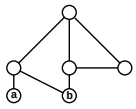
Z09:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z09-00: | $V_2$ | | | | | : | 4 | 2 | 0 | 0 | 0 | 0 | 3*** |
| Z09-01: | $V_3$ | | | | | : | 5 | 1 | 0 | 0 | 0 | 1 | 2*** |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours). Adding the selected vertex would give a ratio of 2/5 or 3/6 (depending on the degree of $a$) whereas adding the neighbour of the selected vertex of degree 2 would give a ratio of 1/3.

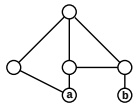
Z10:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z10-00: | $V_2$ | $V_2$ | | | | : | 3 | 3 | 0 | 0 | 0 | 1 | 2 |
| Z10-01: | $V_2$ | $V_3$ | | | | : | 4 | 2 | 0 | 0 | 0 | 2 | 1 |
| Z10-02: | $V_3$ | $V_2$ | | | | : | 4 | 2 | 0 | 0 | 1 | 0 | 2 |
| Z10-03: | $V_3$ | $V_3$ | | | | : | 5 | 1 | 0 | 0 | 1 | 1 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

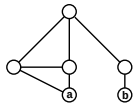
Z11:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z11-00: | $V_2$ | $V_2$ | | | : | 3 | 3 | 0 | 0 | 0 | 1 | 2 |
| Z11-01: | $V_2$ | $V_3$ | | | : | 4 | 2 | 0 | 0 | 1 | 0 | 2 |
| Z11-02: | $V_3$ | $V_2$ | | | : | 4 | 2 | 0 | 0 | 0 | 2 | 1 |
| Z11-03: | $V_3$ | $V_3$ | | | : | 5 | 1 | 0 | 0 | 1 | 1 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

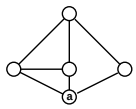
Z12:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z12-00: | $V_2$ | $V_2$ | | | : | 3 | 3 | 0 | 0 | 0 | 1 | 2 |
| Z12-01: | $V_2$ | $V_3$ | | | : | 4 | 2 | 0 | 0 | 1 | 0 | 2 |
| Z12-02: | $V_3$ | $V_2$ | | | : | 4 | 2 | 0 | 0 | 0 | 2 | 1 |
| Z12-03: | $V_3$ | $V_3$ | | | : | 5 | 1 | 0 | 0 | 1 | 1 | 1 |

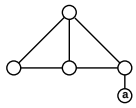
The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

Z13:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z13-00: | | | | | : | 4 | 1 | 0 | 0 | 0 | 0 | 2 |

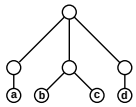
Z14:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z14-00: | $V_2$ | | | | : | 3 | 2 | 0 | 0 | 0 | 1 | 1 |
| Z14-01: | $V_3$ | | | | : | 4 | 1 | 0 | 0 | 1 | 0 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

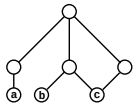
Z15:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z15-00: | $V_2$ | $V_2$ | $V_2$ | $V_2$ | : | 2 | 6 | 0 | 0 | 0 | 4 | 1 |
| Z15-01: | $V_2$ | $V_2$ | $V_2$ | $V_3$ | : | 3 | 5 | 0 | 0 | 1 | 3 | 1 |
| Z15-02: | $V_2$ | $V_2$ | $V_3$ | $V_2$ | : | 3 | 5 | 0 | 0 | 1 | 3 | 1 |
| Z15-03: | $V_2$ | $V_2$ | $V_3$ | $V_3$ | : | 4 | 4 | 0 | 0 | 2 | 2 | 1 |
| Z15-04: | $V_2$ | $V_3$ | $V_3$ | $V_2$ | : | 4 | 4 | 0 | 0 | 2 | 2 | 1 |
| Z15-05: | $V_2$ | $V_3$ | $V_3$ | $V_3$ | : | 5 | 3 | 0 | 0 | 3 | 1 | 1 |
| Z15-06: | $V_3$ | $V_2$ | $V_2$ | $V_3$ | : | 4 | 4 | 0 | 0 | 2 | 2 | 1 |
| Z15-07: | $V_3$ | $V_2$ | $V_3$ | $V_3$ | : | 5 | 3 | 0 | 0 | 3 | 1 | 1 |
| Z15-08: | $V_3$ | $V_3$ | $V_3$ | $V_3$ | : | 6 | 2 | 0 | 0 | 4 | 0 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

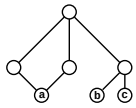
Z16:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z16-00: | $V_2$ | $V_2$ | $V_2$ | | : | 2 | 5 | 0 | 0 | 0 | 2 | 2 |
| Z16-01: | $V_2$ | $V_2$ | $V_3$ | | : | 3 | 4 | 0 | 0 | 0 | 3 | 1 |
| Z16-02: | $V_2$ | $V_3$ | $V_2$ | | : | 3 | 4 | 0 | 0 | 1 | 1 | 2 |
| Z16-03: | $V_2$ | $V_3$ | $V_3$ | | : | 4 | 3 | 0 | 0 | 1 | 2 | 1 |
| Z16-04: | $V_3$ | $V_2$ | $V_2$ | | : | 3 | 4 | 0 | 0 | 1 | 1 | 2 |
| Z16-05: | $V_3$ | $V_2$ | $V_3$ | | : | 4 | 3 | 0 | 0 | 1 | 2 | 1 |
| Z16-06: | $V_3$ | $V_3$ | $V_2$ | | : | 4 | 3 | 0 | 0 | 2 | 0 | 2 |
| Z16-07: | $V_3$ | $V_3$ | $V_3$ | | : | 5 | 2 | 0 | 0 | 2 | 1 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

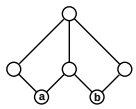
Z17:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z17-00: | $V_2$ | $V_2$ | $V_2$ | | : | 2 | 5 | 0 | 0 | 0 | 2 | 2 |
| Z17-01: | $V_2$ | $V_2$ | $V_3$ | | : | 3 | 4 | 0 | 0 | 1 | 1 | 2 |
| Z17-02: | $V_2$ | $V_3$ | $V_3$ | | : | 4 | 3 | 0 | 0 | 2 | 0 | 2 |
| Z17-03: | $V_3$ | $V_2$ | $V_2$ | | : | 3 | 4 | 0 | 0 | 0 | 3 | 1 |
| Z17-04: | $V_3$ | $V_2$ | $V_3$ | | : | 4 | 3 | 0 | 0 | 1 | 2 | 1 |
| Z17-05: | $V_3$ | $V_3$ | $V_3$ | | : | 5 | 2 | 0 | 0 | 2 | 1 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

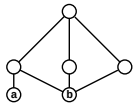
Z18:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z18-00: | $V_2$ | $V_2$ | | | : | 2 | 4 | 0 | 0 | 0 | 0 | 3 |
| Z18-01: | $V_2$ | $V_3$ | | | : | 3 | 3 | 0 | 0 | 0 | 1 | 2 |
| Z18-02: | $V_3$ | $V_3$ | | | : | 4 | 2 | 0 | 0 | 0 | 2 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

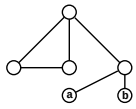
Z19:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z19-00: | $V_2$ | | | | : | 3 | 3 | 0 | 0 | 0 | 1 | 2 |
| Z19-01: | $V_3$ | | | | : | 4 | 2 | 0 | 0 | 1 | 0 | 2 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

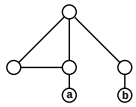
Z20:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z20-00: | $V_2$ | $V_2$ | | | : | 2 | 4 | 0 | 0 | 0 | 2 | 1 |
| Z20-01: | $V_2$ | $V_3$ | | | : | 3 | 3 | 0 | 0 | 1 | 1 | 1 |
| Z20-02: | $V_3$ | $V_3$ | | | : | 4 | 2 | 0 | 0 | 2 | 0 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

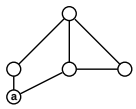
Z21:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z21-00: | $V_2$ | $V_2$ | | | | : | 2 | 4 | 0 | 0 | 0 | 2 | 1 |
| Z21-01: | $V_2$ | $V_3$ | | | | : | 3 | 3 | 0 | 0 | 1 | 1 | 1 |
| Z21-02: | $V_3$ | $V_2$ | | | | : | 3 | 3 | 0 | 0 | 1 | 1 | 1 |
| Z21-03: | $V_3$ | $V_3$ | | | | : | 4 | 2 | 0 | 0 | 2 | 0 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

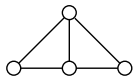
Z22:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z22-00: | $V_2$ | | | | | : | 2 | 3 | 0 | 0 | 0 | 0 | 2 |
| Z22-01: | $V_3$ | | | | | : | 3 | 2 | 0 | 0 | 0 | 1 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

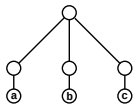
Z23:



| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z23-00: | | | | | | : | 2 | 2 | 0 | 0 | 0 | 0 | 1 |

| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z24-00: | $V_2$ | $V_2$ | $V_2$ | | | : | 1 | 6 | 0 | 0 | 0 | 3 | 1 |
| Z24-01: | $V_2$ | $V_2$ | $V_3$ | | | : | 2 | 5 | 0 | 0 | 1 | 2 | 1 |
| Z24-02: | $V_2$ | $V_3$ | $V_3$ | | | : | 3 | 4 | 0 | 0 | 2 | 1 | 1 |
| Z24-03: | $V_3$ | $V_3$ | $V_3$ | | | : | 4 | 3 | 0 | 0 | 3 | 0 | 1 |

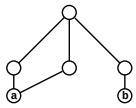
Z24:




The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

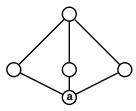
| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z25-00: | $V_2$ | $V_2$ | | | | : | 1 | 5 | 0 | 0 | 0 | 1 | 2 |
| Z25-01: | $V_2$ | $V_3$ | | | | : | 2 | 4 | 0 | 0 | 1 | 0 | 2 |
| Z25-02: | $V_3$ | $V_2$ | | | | : | 2 | 4 | 0 | 0 | 0 | 2 | 1 |
| Z25-03: | $V_3$ | $V_3$ | | | | : | 3 | 3 | 0 | 0 | 1 | 1 | 1 |

Z25:



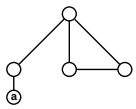

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

Z26:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z26-00: | | | | | | : | 1 | 4 | 0 | 0 | 0 | 0 | 2 |

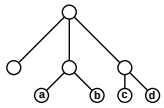
Z27:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z27-00: | $V_2$ | | | | | : | 1 | 4 | 0 | 0 | 0 | 1 | 1 |
| Z27-01: | $V_3$ | | | | | : | 2 | 3 | 0 | 0 | 1 | 0 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

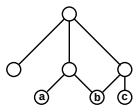
Z28:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z28-00: | $V_1$ | $V_1$ | $V_1$ | $V_1$ | | : | 3 | 0 | 5 | 0 | 0 | 0 | 5*** |
| Z28-01: | $V_1$ | $V_1$ | $V_1$ | $V_2$ | | : | 3 | 1 | 4 | 0 | 0 | 1 | 4*** |
| Z28-02: | $V_1$ | $V_1$ | $V_1$ | $V_3$ | | : | 4 | 0 | 4 | 0 | 1 | 0 | 4*** |
| Z28-03: | $V_1$ | $V_1$ | $V_2$ | $V_2$ | | : | 3 | 2 | 3 | 0 | 0 | 2 | 3*** |
| Z28-04: | $V_1$ | $V_1$ | $V_2$ | $V_3$ | | : | 4 | 1 | 3 | 0 | 1 | 1 | 3*** |
| Z28-05: | $V_1$ | $V_1$ | $V_3$ | $V_3$ | | : | 5 | 0 | 3 | 0 | 2 | 0 | 3*** |
| Z28-06: | $V_1$ | $V_2$ | $V_1$ | $V_2$ | | : | 3 | 2 | 3 | 0 | 0 | 2 | 3 |
| Z28-07: | $V_1$ | $V_2$ | $V_1$ | $V_3$ | | : | 4 | 1 | 3 | 0 | 1 | 1 | 3 |
| Z28-08: | $V_1$ | $V_2$ | $V_2$ | $V_2$ | | : | 3 | 3 | 2 | 0 | 0 | 3 | 2 |
| Z28-09: | $V_1$ | $V_2$ | $V_2$ | $V_3$ | | : | 4 | 2 | 2 | 0 | 1 | 2 | 2 |
| Z28-10: | $V_1$ | $V_2$ | $V_3$ | $V_3$ | | : | 5 | 1 | 2 | 0 | 2 | 1 | 2 |
| Z28-11: | $V_1$ | $V_3$ | $V_1$ | $V_3$ | | : | 5 | 0 | 3 | 0 | 2 | 0 | 3 |
| Z28-12: | $V_1$ | $V_3$ | $V_2$ | $V_2$ | | : | 4 | 2 | 2 | 0 | 1 | 2 | 2 |
| Z28-13: | $V_1$ | $V_3$ | $V_2$ | $V_3$ | | : | 5 | 1 | 2 | 0 | 2 | 1 | 2 |
| Z28-14: | $V_1$ | $V_3$ | $V_3$ | $V_3$ | | : | 6 | 0 | 2 | 0 | 3 | 0 | 2 |
| Z28-15: | $V_2$ | $V_2$ | $V_2$ | $V_2$ | | : | 3 | 4 | 1 | 0 | 0 | 4 | 1 |
| Z28-16: | $V_2$ | $V_2$ | $V_2$ | $V_3$ | | : | 4 | 3 | 1 | 0 | 1 | 3 | 1 |
| Z28-17: | $V_2$ | $V_2$ | $V_3$ | $V_3$ | | : | 5 | 2 | 1 | 0 | 2 | 2 | 1 |
| Z28-18: | $V_2$ | $V_3$ | $V_2$ | $V_3$ | | : | 5 | 2 | 1 | 0 | 2 | 2 | 1 |
| Z28-19: | $V_2$ | $V_3$ | $V_3$ | $V_3$ | | : | 6 | 1 | 1 | 0 | 3 | 1 | 1 |
| Z28-20: | $V_3$ | $V_3$ | $V_3$ | $V_3$ | | : | 7 | 0 | 1 | 0 | 4 | 0 | 1 |

Both $a$ and $b$ cannot have degree 1 (similarly both $c$ and $d$ cannot have degree 1). If both, say, $a$ and $b$ had degree 1, adding the selected vertex would give a ratio of at least 3/6 (depending on the degrees of $c$ and $d$) whereas adding the neighbour of $a$ would give a ratio of 2/5.

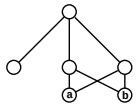
Z29:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z29-00: | $V_1$ | $V_2$ | $V_1$ | | | : | 3 | 1 | 3 | 0 | 0 | 0 | 4*** |
| Z29-01: | $V_1$ | $V_2$ | $V_2$ | | | : | 3 | 2 | 2 | 0 | 0 | 1 | 3*** |
| Z29-02: | $V_1$ | $V_2$ | $V_3$ | | | : | 4 | 1 | 2 | 0 | 1 | 0 | 3*** |
| Z29-03: | $V_1$ | $V_3$ | $V_1$ | | | : | 4 | 0 | 3 | 0 | 0 | 1 | 3 |
| Z29-04: | $V_1$ | $V_3$ | $V_2$ | | | : | 4 | 1 | 2 | 0 | 0 | 2 | 2 |
| Z29-05: | $V_1$ | $V_3$ | $V_3$ | | | : | 5 | 0 | 2 | 0 | 1 | 1 | 2 |
| Z29-06: | $V_2$ | $V_2$ | $V_2$ | | | : | 3 | 3 | 1 | 0 | 0 | 2 | 2 |
| Z29-07: | $V_2$ | $V_2$ | $V_3$ | | | : | 4 | 2 | 1 | 0 | 1 | 1 | 2 |
| Z29-08: | $V_2$ | $V_3$ | $V_2$ | | | : | 4 | 2 | 1 | 0 | 0 | 3 | 1 |
| Z29-09: | $V_2$ | $V_3$ | $V_3$ | | | : | 5 | 1 | 1 | 0 | 1 | 2 | 1 |
| Z29-10: | $V_3$ | $V_2$ | $V_3$ | | | : | 5 | 1 | 1 | 0 | 2 | 0 | 2 |
| Z29-11: | $V_3$ | $V_3$ | $V_3$ | | | : | 6 | 0 | 1 | 0 | 2 | 1 | 1 |

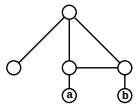
If $b$ has degree 2 then $a$ and $c$ cannot have degree 1. If, say, $a$ has degree 1 and $b$ has degree 2, adding the selected vertex would give a ratio of at least 3/6 (depending on the degree of $c$) whereas adding the neighbour of $a$ would give a ratio of 2/5.

Z30:



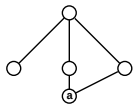

| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z30-00: | $V_2$ | $V_2$ | | | : | 3 | 2 | 1 | 0 | 0 | 0 | 3 |
| Z30-01: | $V_2$ | $V_3$ | | | : | 4 | 1 | 1 | 0 | 0 | 1 | 2 |
| Z30-02: | $V_3$ | $V_3$ | | | : | 5 | 0 | 1 | 0 | 0 | 2 | 1 |

Z31:



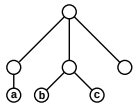

| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z31-00: | $V_1$ | $V_1$ | | | : | 3 | 0 | 3 | 0 | 0 | 0 | 3 |
| Z31-01: | $V_1$ | $V_2$ | | | : | 3 | 1 | 2 | 0 | 0 | 1 | 2 |
| Z31-02: | $V_1$ | $V_3$ | | | : | 4 | 0 | 2 | 0 | 1 | 0 | 2 |
| Z31-03: | $V_2$ | $V_2$ | | | : | 3 | 2 | 1 | 0 | 0 | 2 | 1 |
| Z31-04: | $V_2$ | $V_3$ | | | : | 4 | 1 | 1 | 0 | 1 | 1 | 1 |
| Z31-05: | $V_3$ | $V_3$ | | | : | 5 | 0 | 1 | 0 | 2 | 0 | 1 |

Z32:



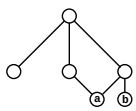

| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z32-00: | $V_2$ | | | | : | 3 | 1 | 1 | 0 | 0 | 0 | 2 |
| Z32-01: | $V_3$ | | | | : | 4 | 0 | 1 | 0 | 0 | 1 | 1 |

Z33:



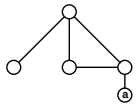

| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z33-00: | $V_1$ | $V_1$ | $V_1$ | | : | 2 | 1 | 4 | 0 | 0 | 0 | 4 |
| Z33-01: | $V_1$ | $V_1$ | $V_2$ | | : | 2 | 2 | 3 | 0 | 0 | 1 | 3 |
| Z33-02: | $V_1$ | $V_1$ | $V_3$ | | : | 3 | 1 | 3 | 0 | 1 | 0 | 3 |
| Z33-03: | $V_1$ | $V_2$ | $V_2$ | | : | 2 | 3 | 2 | 0 | 0 | 2 | 2 |
| Z33-04: | $V_1$ | $V_2$ | $V_3$ | | : | 3 | 2 | 2 | 0 | 1 | 1 | 2 |
| Z33-05: | $V_1$ | $V_3$ | $V_3$ | | : | 4 | 1 | 2 | 0 | 2 | 0 | 2 |
| Z33-06: | $V_2$ | $V_1$ | $V_1$ | | : | 2 | 2 | 3 | 0 | 0 | 1 | 3 |
| Z33-07: | $V_2$ | $V_1$ | $V_2$ | | : | 2 | 3 | 2 | 0 | 0 | 2 | 2 |
| Z33-08: | $V_2$ | $V_1$ | $V_3$ | | : | 3 | 2 | 2 | 0 | 1 | 1 | 2 |
| Z33-09: | $V_2$ | $V_2$ | $V_2$ | | : | 2 | 4 | 1 | 0 | 0 | 3 | 1 |
| Z33-10: | $V_2$ | $V_2$ | $V_3$ | | : | 3 | 3 | 1 | 0 | 1 | 2 | 1 |
| Z33-11: | $V_2$ | $V_3$ | $V_3$ | | : | 4 | 2 | 1 | 0 | 2 | 1 | 1 |
| Z33-12: | $V_3$ | $V_1$ | $V_1$ | | : | 3 | 1 | 3 | 0 | 1 | 0 | 3 |
| Z33-13: | $V_3$ | $V_1$ | $V_2$ | | : | 3 | 2 | 2 | 0 | 1 | 1 | 2 |
| Z33-14: | $V_3$ | $V_1$ | $V_3$ | | : | 4 | 1 | 2 | 0 | 2 | 0 | 2 |
| Z33-15: | $V_3$ | $V_2$ | $V_2$ | | : | 3 | 3 | 1 | 0 | 1 | 2 | 1 |
| Z33-16: | $V_3$ | $V_2$ | $V_3$ | | : | 4 | 2 | 1 | 0 | 2 | 1 | 1 |
| Z33-17: | $V_3$ | $V_3$ | $V_3$ | | : | 5 | 1 | 1 | 0 | 3 | 0 | 1 |

Z34:



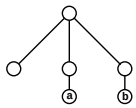

| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z34-00: | $V_2$ | $V_1$ | | | : | 2 | 2 | 2 | 0 | 0 | 0 | 3 |
| Z34-01: | $V_2$ | $V_2$ | | | : | 2 | 3 | 1 | 0 | 0 | 1 | 2 |
| Z34-02: | $V_2$ | $V_3$ | | | : | 3 | 2 | 1 | 0 | 1 | 0 | 2 |
| Z34-03: | $V_3$ | $V_1$ | | | : | 3 | 1 | 2 | 0 | 0 | 1 | 2 |
| Z34-04: | $V_3$ | $V_2$ | | | : | 3 | 2 | 1 | 0 | 0 | 2 | 1 |
| Z34-05: | $V_3$ | $V_3$ | | | : | 4 | 1 | 1 | 0 | 1 | 1 | 1 |

Z35:



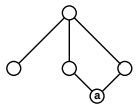

| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z35-00: | $V_1$ | | | | | : | 2 | 1 | 2 | 0 | 0 | 0 | 2 |
| Z35-01: | $V_2$ | | | | | : | 2 | 2 | 1 | 0 | 0 | 1 | 1 |
| Z35-02: | $V_3$ | | | | | : | 3 | 1 | 1 | 0 | 1 | 0 | 1 |

Z36:



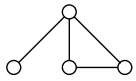

| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z36-00: | $V_1$ | $V_1$ | | | | : | 1 | 2 | 3 | 0 | 0 | 0 | 3 |
| Z36-01: | $V_1$ | $V_2$ | | | | : | 1 | 3 | 2 | 0 | 0 | 1 | 2 |
| Z36-02: | $V_1$ | $V_3$ | | | | : | 2 | 2 | 2 | 0 | 1 | 0 | 2 |
| Z36-03: | $V_2$ | $V_2$ | | | | : | 1 | 4 | 1 | 0 | 0 | 2 | 1 |
| Z36-04: | $V_2$ | $V_3$ | | | | : | 2 | 3 | 1 | 0 | 1 | 1 | 1 |
| Z36-05: | $V_3$ | $V_3$ | | | | : | 3 | 2 | 1 | 0 | 2 | 0 | 1 |

Z37:



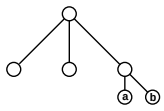

| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z37-00: | $V_2$ | | | | | : | 1 | 3 | 1 | 0 | 0 | 0 | 2 |
| Z37-01: | $V_3$ | | | | | : | 2 | 2 | 1 | 0 | 0 | 1 | 1 |

Z38:



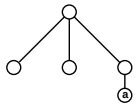
| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z38-00: | | | | | | : | 1 | 2 | 1 | 0 | 0 | 0 | 1 |

Z39:



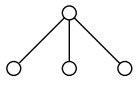

| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z39-00: | $V_1$ | $V_1$ | | | | : | 2 | 0 | 4 | 0 | 0 | 0 | 3 |
| Z39-01: | $V_1$ | $V_2$ | | | | : | 2 | 1 | 3 | 0 | 0 | 1 | 2 |
| Z39-02: | $V_1$ | $V_3$ | | | | : | 3 | 0 | 3 | 0 | 1 | 0 | 2 |
| Z39-03: | $V_2$ | $V_2$ | | | | : | 2 | 2 | 2 | 0 | 0 | 2 | 1 |
| Z39-04: | $V_2$ | $V_3$ | | | | : | 3 | 1 | 2 | 0 | 1 | 1 | 1 |
| Z39-05: | $V_3$ | $V_3$ | | | | : | 4 | 0 | 2 | 0 | 2 | 0 | 1 |

Z40:



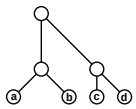

| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z40-00: | $V_1$ | | | | | : | 1 | 1 | 3 | 0 | 0 | 0 | 2 |
| Z40-01: | $V_2$ | | | | | : | 1 | 2 | 2 | 0 | 0 | 1 | 1 |
| Z40-02: | $V_3$ | | | | | : | 2 | 1 | 2 | 0 | 1 | 0 | 1 |

Z41:



| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z41-00: | | | | | | : | 1 | 0 | 3 | 0 | 0 | 0 | 1 |

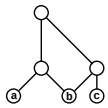
Y01:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y01-00: | $V_2$ | $V_2$ | $V_2$ | $V_2$ | | : | 2 | 5 | 0 | 0 | 0 | 4 | 1*** |
| Y01-01: | $V_2$ | $V_2$ | $V_2$ | $V_3$ | | : | 3 | 4 | 0 | 0 | 1 | 3 | 1*** |
| Y01-02: | $V_2$ | $V_2$ | $V_3$ | $V_3$ | | : | 4 | 3 | 0 | 0 | 2 | 2 | 1*** |
| Y01-03: | $V_2$ | $V_3$ | $V_2$ | $V_3$ | | : | 4 | 3 | 0 | 0 | 2 | 2 | 1*** |
| Y01-04: | $V_2$ | $V_3$ | $V_3$ | $V_3$ | | : | 5 | 2 | 0 | 0 | 3 | 1 | 1*** |
| Y01-05: | $V_3$ | $V_3$ | $V_3$ | $V_3$ | | : | 6 | 1 | 0 | 0 | 4 | 0 | 1*** |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours). Adding the selected vertex would give a ratio of 1/3. The only way that adding a neighbour of the selected vertex would give a ratio of more than 1/4 is if there existed a higher priority instance somewhere else in the graph, i.e., a vertex of degree 2 with less than four vertices at distance 2 from it.

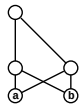
Y02:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y02-00: | $V_2$ | $V_2$ | $V_2$ | | | : | 2 | 4 | 0 | 0 | 0 | 2 | 2*** |
| Y02-01: | $V_2$ | $V_2$ | $V_3$ | | | : | 3 | 3 | 0 | 0 | 1 | 1 | 2*** |
| Y02-02: | $V_2$ | $V_3$ | $V_2$ | | | : | 3 | 3 | 0 | 0 | 0 | 3 | 1*** |
| Y02-03: | $V_2$ | $V_3$ | $V_3$ | | | : | 4 | 2 | 0 | 0 | 1 | 2 | 1*** |
| Y02-04: | $V_3$ | $V_2$ | $V_3$ | | | : | 4 | 2 | 0 | 0 | 2 | 0 | 2*** |
| Y02-05: | $V_3$ | $V_3$ | $V_3$ | | | : | 5 | 1 | 0 | 0 | 2 | 1 | 1*** |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours). Adding the selected vertex would give a ratio of 1/3 or 2/4 (depending on the degree of $b$). The only way that adding a neighbour of the selected vertex, the neighbour of $a$, say, would give a ratio of more than 1/4 is if there existed another vertex common to both $a$ and $b$ of degree 2. In which instance, adding the neighbour of $c$ would give a ratio of 1/4.

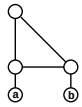
Y03:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y03-00: | $V_2$ | $V_2$ | | | | : | 2 | 3 | 0 | 0 | 0 | 0 | 3*** |
| Y03-01: | $V_2$ | $V_3$ | | | | : | 3 | 2 | 0 | 0 | 0 | 1 | 2*** |
| Y03-02: | $V_3$ | $V_3$ | | | | : | 4 | 1 | 0 | 0 | 0 | 2 | 1 |

The vertices $a$ and $b$ cannot have degree 2. The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours). If either $a$ or $b$ had degree 2, adding the selected vertex would give a ratio of 2/3 or 2/4 (depending on whether one of $a$ or $b$ had degree 2 or both had degree 2). In which instance, adding either neighbour of the selected vertex would give a ratio of 2/5 (isolating the neighbour that was not selected).

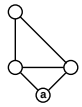
Y04:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y04-00: | $V_2$ | $V_2$ | | | | : | 2 | 3 | 0 | 0 | 0 | 2 | 1 |
| Y04-01: | $V_2$ | $V_3$ | | | | : | 3 | 2 | 0 | 0 | 1 | 1 | 1 |
| Y04-02: | $V_3$ | $V_3$ | | | | : | 4 | 1 | 0 | 0 | 2 | 0 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

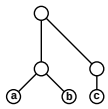
Y05:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y05-00: | $V_2$ | | | | : | 2 | 2 | 0 | 0 | 0 | 0 | 2*** |
| Y05-01: | $V_3$ | | | | : | 3 | 1 | 0 | 0 | 0 | 1 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours). Adding the selected vertex would give a ratio of 1/3 or 2/4 (depending on the degree of $a$). Adding a neighbour of the selected vertex gives a ratio of 1/4.

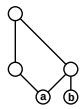
Y06:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y06-00: | $V_2$ | $V_2$ | $V_2$ | | : | 1 | 5 | 0 | 0 | 0 | 3 | 1 |
| Y06-01: | $V_2$ | $V_2$ | $V_3$ | | : | 2 | 4 | 0 | 0 | 1 | 2 | 1 |
| Y06-02: | $V_2$ | $V_3$ | $V_2$ | | : | 2 | 4 | 0 | 0 | 1 | 2 | 1 |
| Y06-03: | $V_2$ | $V_3$ | $V_3$ | | : | 3 | 3 | 0 | 0 | 2 | 1 | 1 |
| Y06-04: | $V_3$ | $V_3$ | $V_2$ | | : | 3 | 3 | 0 | 0 | 2 | 1 | 1 |
| Y06-05: | $V_3$ | $V_3$ | $V_3$ | | : | 4 | 2 | 0 | 0 | 3 | 0 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

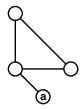
Y07:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y07-00: | $V_2$ | $V_2$ | | | : | 1 | 4 | 0 | 0 | 0 | 1 | 2*** |
| Y07-01: | $V_2$ | $V_3$ | | | : | 2 | 3 | 0 | 0 | 1 | 0 | 2*** |
| Y07-02: | $V_3$ | $V_2$ | | | : | 2 | 3 | 0 | 0 | 0 | 2 | 1 |
| Y07-03: | $V_3$ | $V_3$ | | | : | 3 | 2 | 0 | 0 | 1 | 1 | 1 |

The vertex $a$ cannot have degree 2. The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours). If the vertex $a$ had degree 2, adding the selected vertex would give a ratio of 2/4 whereas adding the neighbour of $a$ of degree 2 would give a ratio of 1/3.

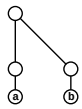
Y08:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y08-00: | $V_2$ | | | | : | 1 | 3 | 0 | 0 | 0 | 1 | 1 |
| Y08-01: | $V_3$ | | | | : | 2 | 2 | 0 | 0 | 1 | 0 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

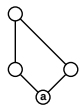
Y09:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y09-00: | $V_2$ | $V_2$ | | | : | 0 | 5 | 0 | 0 | 0 | 2 | 1 |
| Y09-01: | $V_2$ | $V_3$ | | | : | 1 | 4 | 0 | 0 | 1 | 1 | 1 |
| Y09-02: | $V_3$ | $V_3$ | | | : | 2 | 3 | 0 | 0 | 2 | 0 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

Y10:



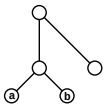

| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y10-00: | $V_2$ | | | | | : | 0 | 4 | 0 | 0 | 0 | 0 | 2 |
| Y10-01: | $V_3$ | | | | | : | 1 | 3 | 0 | 0 | 0 | 1 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

Y11:

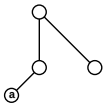
| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y11-00: | | | | | | : | 0 | 3 | 0 | 0 | 0 | 0 | 1 |

Y12:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y12-00: | $V_1$ | $V_1$ | | | | : | 1 | 1 | 3 | 0 | 0 | 0 | 3 |
| Y12-01: | $V_1$ | $V_2$ | | | | : | 1 | 2 | 2 | 0 | 0 | 1 | 2 |
| Y12-02: | $V_1$ | $V_3$ | | | | : | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| Y12-03: | $V_2$ | $V_2$ | | | | : | 1 | 3 | 1 | 0 | 0 | 2 | 1 |
| Y12-04: | $V_2$ | $V_3$ | | | | : | 2 | 2 | 1 | 0 | 1 | 1 | 1 |
| Y12-05: | $V_3$ | $V_3$ | | | | : | 3 | 1 | 1 | 0 | 2 | 0 | 1 |

Y13:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y13-00: | $V_1$ | | | | | : | 0 | 2 | 2 | 0 | 0 | 0 | 2 |
| Y13-01: | $V_2$ | | | | | : | 0 | 3 | 1 | 0 | 0 | 1 | 1 |
| Y13-02: | $V_3$ | | | | | : | 1 | 2 | 1 | 0 | 1 | 0 | 1 |

Y14:

| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y14-00: | | | | | | : | 0 | 1 | 2 | 0 | 0 | 0 | 1 |

X01:

| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| X01-00: | | | | | | : | 0 | 0 | 2 | 0 | 0 | 0 | 1 |

```
    |\^/|     Maple V Release 5.1 (Macquarie University)
._|\|   |/|_. Copyright (c) 1981-1998 by Waterloo Maple Inc. All rights
 \  MAPLE  /  reserved. Maple and Maple V are registered trademarks of
 <____ ____>  Waterloo Maple Inc.
      |       Type ? for help.
> with(simplex):
Warning, new definition for maximize
Warning, new definition for minimize
>
> obj:= 1*Z01_00 +1*Z01_01 +1*Z01_02 +1*Z01_03 +1*Z01_04 +1*Z01_05
+1*Z01_06 +1*Z01_07 +1*Z01_08 +1*Z01_09 +1*Z01_10 +1*Z01_11 +1*Z02_02
+1*Z02_03 +1*Z02_05 +1*Z02_08 +1*Z02_09 +1*Z02_11 +1*Z03_06 +1*Z03_07
+1*Z03_08 +1*Z03_09 +1*Z03_10 +1*Z03_11 +1*Z04_04 +1*Z04_05 +1*Z05_02
+1*Z05_05 +1*Z07_00 +1*Z07_01 +1*Z07_02 +1*Z07_03 +1*Z07_04 +1*Z07_05
+1*Z08_00 +1*Z08_01 +1*Z08_02 +1*Z08_03 +1*Z08_04 +1*Z08_05 +2*Z10_00
+1*Z10_01 +2*Z10_02 +1*Z10_03 +2*Z11_00 +2*Z11_01 +1*Z11_02 +1*Z11_03
+2*Z12_00 +2*Z12_01 +1*Z12_02 +1*Z12_03 +2*Z13_00 +1*Z14_00 +1*Z14_01
+1*Z15_00 +1*Z15_01 +1*Z15_02 +1*Z15_03 +1*Z15_04 +1*Z15_05 +1*Z15_06
+1*Z15_07 +1*Z15_08 +2*Z16_00 +1*Z16_01 +2*Z16_02 +1*Z16_03 +2*Z16_04
+1*Z16_05 +2*Z16_06 +1*Z16_07 +2*Z17_00 +2*Z17_01 +2*Z17_02 +1*Z17_03
+1*Z17_04 +1*Z17_05 +3*Z18_00 +2*Z18_01 +1*Z18_02 +2*Z19_00 +2*Z19_01
+1*Z20_00 +1*Z20_01 +1*Z20_02 +1*Z21_00 +1*Z21_01 +1*Z21_02 +1*Z21_03
+2*Z22_00 +1*Z22_01 +1*Z23_00 +1*Z24_00 +1*Z24_01 +1*Z24_02 +1*Z24_03
+2*Z25_00 +2*Z25_01 +1*Z25_02 +1*Z25_03 +2*Z26_00 +1*Z27_00 +1*Z27_01
+3*Z28_06 +3*Z28_07 +2*Z28_08 +2*Z28_09 +2*Z28_10 +3*Z28_11 +2*Z28_12
+2*Z28_13 +2*Z28_14 +1*Z28_15 +1*Z28_16 +1*Z28_17 +1*Z28_18 +1*Z28_19
+1*Z28_20 +3*Z29_03 +2*Z29_04 +2*Z29_05 +2*Z29_06 +2*Z29_07 +1*Z29_08
+1*Z29_09 +2*Z29_10 +1*Z29_11 +3*Z30_00 +2*Z30_01 +1*Z30_02 +3*Z31_00
+2*Z31_01 +2*Z31_02 +1*Z31_03 +1*Z31_04 +1*Z31_05 +2*Z32_00 +1*Z32_01
+4*Z33_00 +3*Z33_01 +3*Z33_02 +2*Z33_03 +2*Z33_04 +2*Z33_05 +3*Z33_06
+2*Z33_07 +2*Z33_08 +1*Z33_09 +1*Z33_10 +1*Z33_11 +3*Z33_12 +2*Z33_13
+2*Z33_14 +1*Z33_15 +1*Z33_16 +1*Z33_17 +3*Z34_00 +2*Z34_01 +2*Z34_02
+2*Z34_03 +1*Z34_04 +1*Z34_05 +2*Z35_00 +1*Z35_01 +1*Z35_02 +3*Z36_00
+2*Z36_01 +2*Z36_02 +1*Z36_03 +1*Z36_04 +1*Z36_05 +2*Z37_00 +1*Z37_01
+1*Z38_00 +3*Z39_00 +2*Z39_01 +2*Z39_02 +1*Z39_03 +1*Z39_04 +1*Z39_05
+2*Z40_00 +1*Z40_01 +1*Z40_02 +1*Z41_00 +1*Y03_02 +1*Y04_00 +1*Y04_01
+1*Y04_02 +1*Y05_01 +1*Y06_00 +1*Y06_01 +1*Y06_02 +1*Y06_03 +1*Y06_04
+1*Y06_05 +1*Y07_02 +1*Y07_03 +1*Y08_00 +1*Y08_01 +1*Y09_00 +1*Y09_01
+1*Y09_02 +2*Y10_00 +1*Y10_01 +1*Y11_00 +3*Y12_00 +2*Y12_01 +2*Y12_02
+1*Y12_03 +1*Y12_04 +1*Y12_05 +2*Y13_00 +1*Y13_01 +1*Y13_02 +1*Y14_00
+1*X01_00;
```

```
obj := Z01_01 + Z01_00 + Z01_03 + Z01_02 + Z01_04 + Z01_05 + Z01_06 + Z01_07
     + Z01_08 + Z01_09 + Z01_10 + Z01_11 + Z02_02 + Z02_03 + Z02_05 + Z02_08
     + Z02_09 + Z02_11 + Z03_06 + Z03_07 + Z03_08 + Z03_09 + Z03_10 + Z03_11
     + Z04_04 + Z04_05 + Z05_02 + Z05_05 + Z07_00 + Z07_01 + Z07_02 + Z07_03
     + Z07_04 + Z07_05 + Z08_00 + Z08_01 + Z08_02 + Z08_03 + Z08_04 + Z08_05
     + 2 Z10_00 + Z10_01 + 2 Z10_02 + Z10_03 + 2 Z11_00 + 2 Z11_01 + Z11_02
     + Z11_03 + 2 Z12_00 + 2 Z12_01 + Z12_02 + Z12_03 + 2 Z13_00 + Z14_00
     + Z14_01 + Z15_00 + Z15_01 + Z15_02 + Z15_03 + Z15_04 + Z15_05 + Z15_06
     + Z15_07 + Z15_08 + 2 Z16_00 + Z16_01 + 2 Z16_02 + Z16_03 + 2 Z16_04
     + Z16_05 + 2 Z16_06 + Z16_07 + 2 Z17_00 + 2 Z17_01 + 2 Z17_02 + Z17_03
     + Z17_04 + Z17_05 + 3 Z18_00 + 2 Z18_01 + Z18_02 + 2 Z19_00 + 2 Z19_01
     + Z20_00 + Z20_01 + Z20_02 + Z21_00 + Z21_01 + Z21_02 + 2 Z22_00 + Z22_01
     + Z23_00 + Z24_00 + Z24_01 + Z24_02 + Z24_03 + 2 Z25_00 + 2 Z25_01
     + Z21_03 + Z25_02 + Z25_03 + 2 Z26_00 + Z27_00 + Z27_01 + 3 Z28_06
     + 3 Z28_07 + 2 Z28_08 + 2 Z28_09 + 2 Z28_10 + 3 Z28_11 + 2 Z28_12
     + 2 Z28_13 + 2 Z28_14 + Z28_15 + Z28_16 + Z28_17 + Z28_18 + Z28_19
     + Z28_20 + 3 Z29_03 + 2 Z29_04 + 2 Z29_05 + 2 Z29_06 + 2 Z29_07 + Z29_08
     + Z29_09 + 2 Z29_10 + Z29_11 + 3 Z30_00 + 2 Z30_01 + Z30_02 + 3 Z31_00
     + 2 Z31_01 + 2 Z31_02 + Z31_03 + Z31_04 + Z31_05 + 2 Z32_00 + Z32_01
     + 4 Z33_00 + 3 Z33_01 + 3 Z33_02 + 2 Z33_03 + 2 Z33_04 + 2 Z33_05
     + 3 Z33_06 + 2 Z33_07 + 2 Z33_08 + Z33_09 + Z33_10 + Z33_11 + 3 Z33_12
     + 2 Z33_13 + 2 Z33_14 + Z33_15 + Z33_16 + Z33_17 + 3 Z34_00 + 2 Z34_01
     + 2 Z34_02 + 2 Z34_03 + Z34_04 + Z34_05 + 2 Z35_00 + Z35_01 + Z35_02
     + 3 Z36_00 + 2 Z36_01 + 2 Z36_02 + Z36_03 + Z36_04 + Z36_05 + 2 Z37_00
     + Z37_01 + Z38_00 + 3 Z39_00 + 2 Z39_01 + 2 Z39_02 + Z39_03 + Z39_04
     + Z39_05 + 2 Z40_00 + Z40_01 + Z40_02 + Z41_00 + Y03_02 + Y04_00 + Y04_01
     + Y04_02 + Y05_01 + Y06_00 + Y06_01 + Y06_02 + Y06_03 + Y06_04 + Y06_05
     + Y07_02 + Y07_03 + Y08_00 + Y08_01 + Y09_00 + Y09_01 + Y09_02 + 2 Y10_00
     + Y10_01 + Y11_00 + 3 Y12_00 + 2 Y12_01 + 2 Y12_02 + Y12_03 + Y12_04
     + Y12_05 + 2 Y13_00 + Y13_01 + Y13_02 + Y14_00 + X01_00
```

```
> cnsts:={
+4*Z01_00 +3*Z01_01 +2*Z01_02 +2*Z01_03 +1*Z01_04 +3*Z01_06 +2*Z01_07
+1*Z01_08 +1*Z01_09 +3*Z02_02 +2*Z02_03 +1*Z02_05 +2*Z02_08 +1*Z02_09
+3*Z03_06 +2*Z03_07 +1*Z03_08 +2*Z03_09 +1*Z03_10 +2*Z04_04 +1*Z04_05
+2*Z05_02 +1*Z05_05 +2*Z07_00 +1*Z07_01 +1*Z07_03 +2*Z08_00 +1*Z08_01
+1*Z08_02 +1*Z10_01 +1*Z11_02 +1*Z12_02 +3*Z15_00 +2*Z15_01 +2*Z15_02
+1*Z15_03 +1*Z15_04 +1*Z15_06 +1*Z16_00 +2*Z16_01 +1*Z16_03 +1*Z16_05
+1*Z17_00 +2*Z17_03 +1*Z17_04 +1*Z18_02 +1*Z20_00 +1*Z21_00 +2*Z24_00
+1*Z24_01 +1*Z25_02 -1*Z28_06 -2*Z28_07 +1*Z28_08 -1*Z28_10 -2*Z28_11
-1*Z28_13 -1*Z28_14 +3*Z28_15 +2*Z28_16 +1*Z28_17 +1*Z28_18 -2*Z29_03
-1*Z29_05 +1*Z29_06 +2*Z29_08 +1*Z29_09 +1*Z30_02 -2*Z31_00 -1*Z31_01
-1*Z31_02 +1*Z31_03 -3*Z33_00 -2*Z33_01 -2*Z33_02 -1*Z33_04 -1*Z33_05
-2*Z33_06 -1*Z33_08 +2*Z33_09 +1*Z33_10 -2*Z33_12 -1*Z33_13 -1*Z33_14
+1*Z33_15 -1*Z34_00 -1*Z34_03 +1*Z34_04 -1*Z35_00 -2*Z36_00 -1*Z36_01
-1*Z36_02 +1*Z36_03 -3*Z39_00 -2*Z39_01 -2*Z39_02 -1*Z39_04 -1*Z39_05
-2*Z40_00 -1*Z40_01 -1*Z40_02 -2*Z41_00 +1*Y03_02 +1*Y04_00 +2*Y06_00
+1*Y06_01 +1*Y06_02 +1*Y07_02 +1*Y09_00 -2*Y12_00 -1*Y12_01 -1*Y12_02
+1*Y12_03 -1*Y13_00 -1*Y14_00 -1*X01_00 >= 0,
>
>  +3*Z01_00 +2*Z01_01 +1*Z01_02 +1*Z01_03 +2*Z01_06 +1*Z01_07
+2*Z02_02 +1*Z02_03 +1*Z02_08 +2*Z03_06 +1*Z03_07 +1*Z03_09 +1*Z04_04
+1*Z05_02 +1*Z07_00 +1*Z08_00 +2*Z15_00 +1*Z15_01 +1*Z15_02 +1*Z16_01
+1*Z17_03 +1*Z24_00 -1*Z28_06 -1*Z28_07 +1*Z28_08 -1*Z28_11 +3*Z28_15
+2*Z28_16 +1*Z28_17 +1*Z28_18 -1*Z29_03 +1*Z29_06 +2*Z29_08 +1*Z29_09
+1*Z30_02 -1*Z31_00 +1*Z31_03 -2*Z33_00 -1*Z33_01 -1*Z33_02 -1*Z33_06
+2*Z33_09 +1*Z33_10 -1*Z33_12 +1*Z33_15 +1*Z34_04 -1*Z36_00 +1*Z36_03
-2*Z39_00 -1*Z39_01 -1*Z39_02 -1*Z40_00 -1*Z41_00 +1*Y06_00 -1*Y12_00
+1*Y12_03 >= 0,
>
>  +2*Z01_00 +1*Z01_01 +1*Z01_06 +1*Z02_02 +1*Z03_06 +1*Z15_00
+1*Z28_08 +3*Z28_15 +2*Z28_16 +1*Z28_17 +1*Z28_18 +1*Z29_06 +2*Z29_08
+1*Z29_09 +1*Z30_02 +1*Z31_03 -1*Z33_00 +2*Z33_09 +1*Z33_10 +1*Z33_15
+1*Z34_04 +1*Z36_03 -1*Z39_00 +1*Y12_03 >= 0,
>
>  +5*Z01_00 +4*Z01_01 +3*Z01_02 +3*Z01_03 +2*Z01_04 +1*Z01_05
+4*Z01_06 +3*Z01_07 +2*Z01_08 +2*Z01_09 +1*Z01_10 +4*Z02_02 +3*Z02_03
+2*Z02_05 +3*Z02_08 +2*Z02_09 +1*Z02_11 +4*Z03_06 +3*Z03_07 +2*Z03_08
+3*Z03_09 +2*Z03_10 +1*Z03_11 +3*Z04_04 +2*Z04_05 +3*Z05_02 +2*Z05_05
+3*Z07_00 +2*Z07_01 +1*Z07_02 +2*Z07_03 +1*Z07_04 +3*Z08_00 +2*Z08_01
+2*Z08_02 +1*Z08_03 +1*Z08_04 +1*Z10_00 +2*Z10_01 +1*Z10_03 +1*Z11_00
+2*Z11_02 +1*Z11_03 +1*Z12_00 +2*Z12_02 +1*Z12_03 +1*Z14_00 +4*Z15_00
+3*Z15_01 +3*Z15_02 +2*Z15_03 +2*Z15_04 +1*Z15_05 +2*Z15_06 +1*Z15_07
+2*Z16_00 +3*Z16_01 +1*Z16_02 +2*Z16_03 +1*Z16_04 +2*Z16_05 +1*Z16_07
+2*Z17_00 +1*Z17_01 +3*Z17_03 +2*Z17_04 +1*Z17_05 +1*Z18_01 +2*Z18_02
+1*Z19_00 +2*Z20_00 +1*Z20_01 +2*Z21_00 +1*Z21_01 +1*Z21_02 +1*Z22_01
+3*Z24_00 +2*Z24_01 +1*Z24_02 +1*Z25_00 +2*Z25_02 +1*Z25_03 +1*Z27_00
-1*Z28_06 -2*Z28_07 +1*Z28_08 -1*Z28_10 -3*Z28_11 -1*Z28_13 -2*Z28_14
+3*Z28_15 +2*Z28_16 +1*Z28_17 +1*Z28_18 -1*Z28_20 -2*Z29_03 -1*Z29_05
+1*Z29_06 +2*Z29_08 +1*Z29_09 -1*Z29_10 -1*Z30_00 +1*Z30_02 -3*Z31_00
-1*Z31_01 -2*Z31_02 +1*Z31_03 -1*Z31_05 -1*Z32_00 -4*Z33_00 -2*Z33_01
-3*Z33_02 -1*Z33_04 -2*Z33_05 -2*Z33_06 -1*Z33_08 +2*Z33_09 +1*Z33_10
-3*Z33_12 -1*Z33_13 -2*Z33_14 +1*Z33_15 -1*Z33_17 -2*Z34_00 -1*Z34_02
-1*Z34_03 +1*Z34_04 -2*Z35_00 -1*Z35_02 -3*Z36_00 -1*Z36_01 -2*Z36_02
+1*Z36_03 -1*Z36_05 -1*Z37_00 -1*Z38_00 -4*Z39_00 -2*Z39_01 -3*Z39_02
-1*Z39_04 -2*Z39_05 -3*Z40_00 -1*Z40_01 -2*Z40_02 -3*Z41_00 +2*Y03_02
+2*Y04_00 +1*Y04_01 +1*Y05_01 +3*Y06_00 +2*Y06_01 +2*Y06_02 +1*Y06_03
+1*Y06_04 +2*Y07_02 +1*Y07_03 +1*Y08_00 +2*Y09_00 +1*Y09_01 +1*Y10_01
-3*Y12_00 -1*Y12_01 -2*Y12_02 +1*Y12_03 -1*Y12_05 -2*Y13_00 -1*Y13_02
-2*Y14_00 -2*X01_00 >= 0,
```

```
> -2*Z01_00 -2*Z01_01 -1*Z01_02 -1*Z01_03 -1*Z01_04 -2*Z01_06
-1*Z01_07 -1*Z01_08 -1*Z01_09 -2*Z02_02 -1*Z02_03 -1*Z02_05 -1*Z02_08
-1*Z02_09 -2*Z03_06 -1*Z03_07 -1*Z03_08 -1*Z03_09 -1*Z03_10 -1*Z04_04
-1*Z04_05 -1*Z05_02 -1*Z05_05 -1*Z07_00 -1*Z07_01 -1*Z07_03 -1*Z08_00
-1*Z08_01 -1*Z08_02 -1*Z10_01 -1*Z11_02 -1*Z12_02 -2*Z15_00 -1*Z15_01
-1*Z15_02 -1*Z15_03 -1*Z15_04 -1*Z15_06 -1*Z16_00 -1*Z16_01 -1*Z16_03
-1*Z16_05 -1*Z17_00 -1*Z17_03 -1*Z17_04 -1*Z18_02 -1*Z20_00 -1*Z21_00
-1*Z24_00 -1*Z24_01 -1*Z25_02 +1*Z28_07 -1*Z28_08 +1*Z28_11 +1*Z28_14
-2*Z28_15 -1*Z28_16 -1*Z28_17 -1*Z28_18 +1*Z29_03 -1*Z29_06 -1*Z29_08
-1*Z29_09 -1*Z30_02 +1*Z31_00 +1*Z31_02 -1*Z31_03 +2*Z33_00 +1*Z33_01
+1*Z33_02 +1*Z33_05 +1*Z33_06 -1*Z33_09 -1*Z33_10 +1*Z33_12 +1*Z33_14
-1*Z33_15 +1*Z34_00 -1*Z34_04 +1*Z35_00 +1*Z36_00 +1*Z36_02 -1*Z36_03
+2*Z39_00 +1*Z39_01 +1*Z39_02 +1*Z39_05 +1*Z40_00 +1*Z40_02 +1*Z41_00
-1*Y03_02 -1*Y04_00 -1*Y06_00 -1*Y06_01 -1*Y06_02 -1*Y07_02 -1*Y09_00
+1*Y12_00 +1*Y12_02 -1*Y12_03 +1*Y13_00 +1*Y14_00 +1*X01_00  <= 0,
>
> -1*Z01_00 -1*Z01_01 -1*Z01_02 -1*Z01_03 -1*Z01_06 -1*Z01_07
-1*Z02_02 -1*Z02_03 -1*Z02_08 -1*Z03_06 -1*Z03_07 -1*Z03_09 -1*Z04_04
-1*Z05_02 -1*Z07_00 -1*Z08_00 -1*Z15_00 -1*Z15_01 -1*Z15_02 -1*Z16_01
-1*Z17_03 -1*Z24_00 -1*Z28_08 +1*Z28_11 -1*Z28_15 -1*Z28_16 -1*Z28_17
-1*Z28_18 -1*Z29_06 -1*Z29_08 -1*Z29_09 -1*Z30_02 +1*Z31_00 -1*Z31_03
+1*Z33_00 +1*Z33_02 -1*Z33_09 -1*Z33_10 +1*Z33_12 -1*Z33_15 -1*Z34_04
+1*Z36_00 -1*Z36_03 +1*Z39_00 +1*Z39_02 +1*Z40_00 +1*Z41_00 -1*Y06_00
+1*Y12_00 -1*Y12_03  <= 0,
>
> -1*Z01_00 -1*Z01_01 -1*Z01_06 -1*Z02_02 -1*Z03_06 -1*Z15_00
-1*Z28_08 -1*Z28_15 -1*Z28_16 -1*Z28_17 -1*Z28_18 -1*Z29_06 -1*Z29_08
-1*Z29_09 -1*Z30_02 -1*Z31_03 +1*Z33_00 -1*Z33_09 -1*Z33_10 -1*Z33_15
-1*Z34_04 -1*Z36_03 +1*Z39_00 -1*Y12_03  <= 0,
>
> -3*Z01_00 -4*Z01_01 -5*Z01_02 -5*Z01_03 -6*Z01_04 -7*Z01_05
-4*Z01_06 -5*Z01_07 -6*Z01_08 -6*Z01_09 -7*Z01_10 -8*Z01_11 -4*Z02_02
-5*Z02_03 -6*Z02_05 -5*Z02_08 -6*Z02_09 -7*Z02_11 -4*Z03_06 -5*Z03_07
-6*Z03_08 -5*Z03_09 -6*Z03_10 -7*Z03_11 -5*Z04_04 -6*Z04_05 -5*Z05_02
-6*Z05_05 -3*Z07_00 -4*Z07_01 -5*Z07_02 -4*Z07_03 -5*Z07_04 -6*Z07_05
-3*Z08_00 -4*Z08_01 -4*Z08_02 -5*Z08_03 -5*Z08_04 -6*Z08_05 -3*Z10_00
-4*Z10_01 -4*Z10_02 -5*Z10_03 -3*Z11_00 -4*Z11_01 -4*Z11_02 -5*Z11_03
-3*Z12_00 -4*Z12_01 -4*Z12_02 -5*Z12_03 -4*Z13_00 -3*Z14_00 -4*Z14_01
-2*Z15_00 -3*Z15_01 -3*Z15_02 -4*Z15_03 -4*Z15_04 -5*Z15_05 -4*Z15_06
-5*Z15_07 -6*Z15_08 -2*Z16_00 -3*Z16_01 -3*Z16_02 -4*Z16_03 -3*Z16_04
-4*Z16_05 -4*Z16_06 -5*Z16_07 -2*Z17_00 -3*Z17_01 -4*Z17_02 -3*Z17_03
-4*Z17_04 -5*Z17_05 -2*Z18_00 -3*Z18_01 -4*Z18_02 -3*Z19_00 -4*Z19_01
-2*Z20_00 -3*Z20_01 -4*Z20_02 -2*Z21_00 -3*Z21_01 -3*Z21_02 -4*Z21_03
-2*Z22_00 -3*Z22_01 -2*Z23_00 -1*Z24_00 -2*Z24_01 -3*Z24_02 -4*Z24_03
-1*Z25_00 -2*Z25_01 -2*Z25_02 -3*Z25_03 -1*Z26_00 -1*Z27_00 -2*Z27_01
-3*Z28_06 -4*Z28_07 -3*Z28_08 -4*Z28_09 -5*Z28_10 -5*Z28_11 -4*Z28_12
-5*Z28_13 -6*Z28_14 -3*Z28_15 -4*Z28_16 -5*Z28_17 -5*Z28_18 -6*Z28_19
-7*Z28_20 -4*Z29_03 -4*Z29_04 -5*Z29_05 -3*Z29_06 -4*Z29_07 -4*Z29_08
-5*Z29_09 -5*Z29_10 -6*Z29_11 -3*Z30_00 -4*Z30_01 -5*Z30_02 -3*Z31_00
-3*Z31_01 -4*Z31_02 -3*Z31_03 -4*Z31_04 -5*Z31_05 -3*Z32_00 -4*Z32_01
-2*Z33_00 -2*Z33_01 -3*Z33_02 -2*Z33_03 -3*Z33_04 -4*Z33_05 -2*Z33_06
-2*Z33_07 -3*Z33_08 -2*Z33_09 -3*Z33_10 -4*Z33_11 -3*Z33_12 -3*Z33_13
-4*Z33_14 -3*Z33_15 -4*Z33_16 -5*Z33_17 -2*Z34_00 -2*Z34_01 -3*Z34_02
-3*Z34_03 -3*Z34_04 -4*Z34_05 -2*Z35_00 -2*Z35_01 -3*Z35_02 -1*Z36_00
-1*Z36_01 -2*Z36_02 -1*Z36_03 -2*Z36_04 -3*Z36_05 -1*Z37_00 -2*Z37_01
-1*Z38_00 -2*Z39_00 -2*Z39_01 -3*Z39_02 -2*Z39_03 -3*Z39_04 -4*Z39_05
-1*Z40_00 -1*Z40_01 -2*Z40_02 -1*Z41_00 -4*Y03_02 -2*Y04_00 -3*Y04_01
-4*Y04_02 -3*Y05_01 -1*Y06_00 -2*Y06_01 -2*Y06_02 -3*Y06_03 -3*Y06_04
-4*Y06_05 -2*Y07_02 -3*Y07_03 -1*Y08_00 -2*Y08_01 -1*Y09_01 -2*Y09_02
```

```
-1*Y10_01 -1*Y12_00 -1*Y12_01 -2*Y12_02 -1*Y12_03 -2*Y12_04 -3*Y12_05
-1*Y13_02  >= -1,
>
> -6*Z01_00 -4*Z01_01 -2*Z01_02 -2*Z01_03 +2*Z01_05 -4*Z01_06
-2*Z01_07 +2*Z01_10 +4*Z01_11 -4*Z02_02 -2*Z02_03 -2*Z02_08 +2*Z02_11
-4*Z03_06 -2*Z03_07 -2*Z03_09 +2*Z03_11 -2*Z04_04 -2*Z05_02 -4*Z07_00
-2*Z07_01 -2*Z07_03 +2*Z07_05 -4*Z08_00 -2*Z08_01 -2*Z08_02 +2*Z08_05
-3*Z10_00 -2*Z10_01 -1*Z10_02 -3*Z11_00 -1*Z11_01 -2*Z11_02 -3*Z12_00
-1*Z12_01 -2*Z12_02 -1*Z13_00 -2*Z14_00 -6*Z15_00 -4*Z15_01 -4*Z15_02
-2*Z15_03 -2*Z15_04 -2*Z15_06 +2*Z15_08 -5*Z16_00 -4*Z16_01 -3*Z16_02
-2*Z16_03 -3*Z16_04 -2*Z16_05 -1*Z16_06 -5*Z17_00 -3*Z17_01 -1*Z17_02
-4*Z17_03 -2*Z17_04 -4*Z18_00 -3*Z18_01 -2*Z18_02 -3*Z19_00 -1*Z19_01
-4*Z20_00 -2*Z20_01 -4*Z21_00 -2*Z21_01 -2*Z21_02 -3*Z22_00 -2*Z22_01
-2*Z23_00 -6*Z24_00 -4*Z24_01 -2*Z24_02 -5*Z25_00 -3*Z25_01 -4*Z25_02
-2*Z25_03 -4*Z26_00 -4*Z27_00 -2*Z27_01 -2*Z28_06 -3*Z28_08 -1*Z28_09
+1*Z28_10 +2*Z28_11 -1*Z28_12 +1*Z28_13 +3*Z28_14 -4*Z28_15 -2*Z28_16
+2*Z28_19 +4*Z28_20 -1*Z29_04 +1*Z29_05 -3*Z29_06 -1*Z29_07 -2*Z29_08
+1*Z29_10 +2*Z29_11 -2*Z30_00 -1*Z30_01 -1*Z31_01 +1*Z31_02 -2*Z31_03
+2*Z31_05 -1*Z32_00 -1*Z33_00 -2*Z33_01 -3*Z33_03 -1*Z33_04 +1*Z33_05
-2*Z33_06 -3*Z33_07 -1*Z33_08 -4*Z33_09 -2*Z33_10 -1*Z33_13 +1*Z33_14
-2*Z33_15 +2*Z33_17 -2*Z34_00 -3*Z34_01 -1*Z34_02 -1*Z34_03 -2*Z34_04
-1*Z35_00 -2*Z35_01 -2*Z36_00 -3*Z36_01 -1*Z36_02 -4*Z36_03 -2*Z36_04
-3*Z37_00 -2*Z37_01 -2*Z38_00 -1*Z39_01 +1*Z39_02 -2*Z39_03 +2*Z39_05
-1*Z40_00 -2*Z40_01 -1*Y03_02 -3*Y04_00 -1*Y04_01 +1*Y04_02 -1*Y05_01
-5*Y06_00 -3*Y06_01 -3*Y06_02 -1*Y06_03 -1*Y06_04 +1*Y06_05 -3*Y07_02
-1*Y07_03 -3*Y08_00 -1*Y08_01 -5*Y09_00 -3*Y09_01 -1*Y09_02 -4*Y10_00
-3*Y10_01 -3*Y11_00 -1*Y12_00 -2*Y12_01 -3*Y12_03 -1*Y12_04 +1*Y12_05
-2*Y13_00 -3*Y13_01 -1*Y13_02 -1*Y14_00  >= 0,
>
> +5*Z01_00 +4*Z01_01 +3*Z01_02 +3*Z01_03 +2*Z01_04 +1*Z01_05
> +4*Z01_06 +3*Z01_07 +2*Z01_08 +2*Z01_09 +1*Z01_10 +4*Z02_02
> +3*Z02_03 +2*Z02_05 +3*Z02_08 +2*Z02_09 +1*Z02_11 +4*Z03_06
> +3*Z03_07 +2*Z03_08 +3*Z03_09 +2*Z03_10 +1*Z03_11 +3*Z04_04
> +2*Z04_05 +3*Z05_02 +2*Z05_05 +3*Z07_00 +2*Z07_01 +1*Z07_02
> +2*Z07_03 +1*Z07_04 +3*Z08_00 +2*Z08_01 +2*Z08_02 +1*Z08_03
> +1*Z08_04 +1*Z10_00 +2*Z10_01 +1*Z10_03 +1*Z11_00 +2*Z11_02
> +1*Z11_03 +1*Z12_00 +2*Z12_02 +1*Z12_03 +1*Z14_00 +4*Z15_00
> +3*Z15_01 +3*Z15_02 +2*Z15_03 +2*Z15_04 +1*Z15_05 +2*Z15_06
> +1*Z15_07 +2*Z16_00 +3*Z16_01 +1*Z16_02 +2*Z16_03 +1*Z16_04
> +2*Z16_05 +1*Z16_07 +2*Z17_00 +1*Z17_01 +3*Z17_03 +2*Z17_04
> +1*Z17_05 +1*Z18_01 +2*Z18_02 +1*Z19_00 +2*Z20_00 +1*Z20_01
> +2*Z21_00 +1*Z21_01 +1*Z21_02 +1*Z22_01 +3*Z24_00 +2*Z24_01
> +1*Z24_02 +1*Z25_00 +2*Z25_02 +1*Z25_03 +1*Z27_00 -1*Z28_06
> -2*Z28_07 +1*Z28_08 -1*Z28_10 -3*Z28_11 -1*Z28_13 -2*Z28_14
> +3*Z28_15 +2*Z28_16 +1*Z28_17 +1*Z28_18 -1*Z28_20 -2*Z29_03
> -1*Z29_05 +1*Z29_06 +2*Z29_08 +1*Z29_09 -1*Z29_10 -1*Z30_00
> +1*Z30_02 -3*Z31_00 -1*Z31_01 -2*Z31_02 +1*Z31_03 -1*Z31_05
> -1*Z32_00 -4*Z33_00 -2*Z33_01 -3*Z33_02 -1*Z33_04 -2*Z33_05
> -2*Z33_06 -1*Z33_08 +2*Z33_09 +1*Z33_10 -3*Z33_12 -1*Z33_13
> -2*Z33_14 +1*Z33_15 -1*Z33_17 -2*Z34_00 -1*Z34_02 -1*Z34_03
> +1*Z34_04 -2*Z35_00 -1*Z35_02 -3*Z36_00 -1*Z36_01 -2*Z36_02
> +1*Z36_03 -1*Z36_05 -1*Z37_00 -1*Z38_00 -4*Z39_00 -2*Z39_01
> -3*Z39_02 -1*Z39_04 -2*Z39_05 -3*Z40_00 -1*Z40_01 -2*Z40_02
> -3*Z41_00 +2*Y03_02 +2*Y04_00 +1*Y04_01 +1*Y05_01 +3*Y06_00
> +2*Y06_01 +2*Y06_02 +1*Y06_03 +1*Y06_04 +2*Y07_02 +1*Y07_03
> +1*Y08_00 +2*Y09_00 +1*Y09_01 +1*Y10_01 -3*Y12_00 -1*Y12_01
> -2*Y12_02 +1*Y12_03 -1*Y12_05 -2*Y13_00 -1*Y13_02 -2*Y14_00
> -2*X01_00  >= 0};
```

```
cnsts := {0 <= 2 Z01_01 + 3 Z01_00 + Z01_03 + Z01_02 + 2 Z01_06 + Z01_07
     + 2 Z02_02 + Z02_03 + Z02_08 + 2 Z03_06 + Z03_07 + Z03_09 + Z04_04
     + Z05_02 + Z07_00 + Z08_00 + 2 Z15_00 + Z15_01 + Z15_02 + Z16_01 + Z17_03
     + Z24_00 - Z28_06 - Z28_07 + Z28_08 - Z28_11 + 3 Z28_15 + 2 Z28_16
     + Z28_17 + Z28_18 - Z29_03 + Z29_06 + 2 Z29_08 + Z29_09 + Z30_02 - Z31_00
     + Z31_03 - 2 Z33_00 - Z33_01 - Z33_02 - Z33_06 + 2 Z33_09 + Z33_10
     - Z33_12 + Z33_15 + Z34_04 - Z36_00 + Z36_03 - 2 Z39_00 - Z39_01 - Z39_02
     - Z40_00 - Z41_00 + Y06_00 - Y12_00 + Y12_03, 0 <= 2 Z01_00 + Z01_01
     + Z01_06 + Z02_02 + Z03_06 + Z15_00 + Z28_08 + 3 Z28_15 + 2 Z28_16
     + Z28_17 + Z28_18 + Z29_06 + 2 Z29_08 + Z29_09 + Z30_02 + Z31_03 - Z33_00
     + 2 Z33_09 + Z33_10 + Z33_15 + Z34_04 + Z36_03 - Z39_00 + Y12_03, 0 <=
    3 Z01_01 + 4 Z01_00 + 2 Z01_03 + 2 Z01_02 + Z01_04 + 3 Z01_06 + 2 Z01_07
     + Z01_08 + Z01_09 + 3 Z02_02 + 2 Z02_03 + Z02_05 + 2 Z02_08 + Z02_09
     + 3 Z03_06 + 2 Z03_07 + Z03_08 + 2 Z03_09 + Z03_10 + 2 Z04_04 + Z04_05
     + 2 Z05_02 + Z05_05 + 2 Z07_00 + Z07_01 + Z07_03 + 2 Z08_00 + Z08_01
     + Z08_02 + Z10_01 + Z11_02 + Z12_02 + 3 Z15_00 + 2 Z15_01 + 2 Z15_02
     + Z15_03 + Z15_04 + Z15_06 + Z16_00 + 2 Z16_01 + Z16_03 + Z16_05 + Z17_00
     + 2 Z17_03 + Z17_04 + Z18_02 + Z20_00 + Z21_00 + 2 Z24_00 + Z24_01
     + Z25_02 - Z28_06 - 2 Z28_07 + Z28_08 - Z28_10 - 2 Z28_11 - Z28_13
     - Z28_14 + 3 Z28_15 + 2 Z28_16 + Z28_17 + Z28_18 - 2 Z29_03 - Z29_05
     + Z29_06 + 2 Z29_08 + Z29_09 + Z30_02 - 2 Z31_00 - Z31_01 - Z31_02
     + Z31_03 - 3 Z33_00 - 2 Z33_01 - 2 Z33_02 - Z33_04 - Z33_05 - 2 Z33_06
     - Z33_08 + 2 Z33_09 + Z33_10 - 2 Z33_12 - Z33_13 - Z33_14 + Z33_15
     - Z34_00 - Z34_03 + Z34_04 - Z35_00 - 2 Z36_00 - Z36_01 - Z36_02 + Z36_03
     - 3 Z39_00 - 2 Z39_01 - 2 Z39_02 - Z39_04 - Z39_05 - 2 Z40_00 - Z40_01
     - Z40_02 - 2 Z41_00 + Y03_02 + Y04_00 + 2 Y06_00 + Y06_01 + Y06_02
     + Y07_02 + Y09_00 - 2 Y12_00 - Y12_01 - Y12_02 + Y12_03 - Y13_00 - Y14_00
     - X01_00, -2 Z01_01 - 2 Z01_00 - Z01_03 - Z01_02 - Z01_04 - 2 Z01_06
     - Z01_07 - Z01_08 - Z01_09 - 2 Z02_02 - Z02_03 - Z02_05 - Z02_08 - Z02_09
     - 2 Z03_06 - Z03_07 - Z03_08 - Z03_09 - Z03_10 - Z04_04 - Z04_05 - Z05_02
     - Z05_05 - Z07_00 - Z07_01 - Z07_03 - Z08_00 - Z08_01 - Z08_02 - Z10_01
     - Z11_02 - Z12_02 - 2 Z15_00 - Z15_01 - Z15_02 - Z15_03 - Z15_04 - Z15_06
     - Z16_00 - Z16_01 - Z16_03 - Z16_05 - Z17_00 - Z17_03 - Z17_04 - Z18_02
     - Z20_00 - Z21_00 - Z24_00 - Z24_01 - Z25_02 + Z28_07 - Z28_08 + Z28_11
     + Z28_14 - 2 Z28_15 - Z28_16 - Z28_17 - Z28_18 + Z29_03 - Z29_06 - Z29_08
     - Z29_09 - Z30_02 + Z31_00 + Z31_02 - Z31_03 + 2 Z33_00 + Z33_01 + Z33_02
     + Z33_05 + Z33_06 - Z33_09 - Z33_10 + Z33_12 + Z33_14 - Z33_15 + Z34_00
     - Z34_04 + Z35_00 + Z36_00 + Z36_02 - Z36_03 + 2 Z39_00 + Z39_01 + Z39_02
     + Z39_05 + Z40_00 + Z40_02 + Z41_00 - Y03_02 - Y04_00 - Y06_00 - Y06_01
     - Y06_02 - Y07_02 - Y09_00 + Y12_00 + Y12_02 - Y12_03 + Y13_00 + Y14_00
     + X01_00 <= 0, -Z01_01 - Z01_00 - Z01_03 - Z01_02 - Z01_06 - Z01_07
     - Z02_02 - Z02_03 - Z02_08 - Z03_06 - Z03_07 - Z03_09 - Z04_04 - Z05_02
     - Z07_00 - Z08_00 - Z15_00 - Z15_01 - Z15_02 - Z16_01 - Z17_03 - Z24_00
     - Z28_08 + Z28_11 - Z28_15 - Z28_16 - Z28_17 - Z28_18 - Z29_06 - Z29_08
     - Z29_09 - Z30_02 + Z31_00 - Z31_03 + Z33_00 + Z33_02 - Z33_09 - Z33_10
     + Z33_12 - Z33_15 - Z34_04 + Z36_00 - Z36_03 + Z39_00 + Z39_02 + Z40_00
     + Z41_00 - Y06_00 + Y12_00 - Y12_03 <= 0, 0 <= 4 Z01_01 + 5 Z01_00
     + 3 Z01_03 + 3 Z01_02 + 2 Z01_04 + Z01_05 + 4 Z01_06 + 3 Z01_07 + 2 Z01_08
     + 2 Z01_09 + Z01_10 + 4 Z02_02 + 3 Z02_03 + 2 Z02_05 + 3 Z02_08 + 2 Z02_09
     + Z02_11 + 4 Z03_06 + 3 Z03_07 + 2 Z03_08 + 3 Z03_09 + 2 Z03_10 + Z03_11
     + 3 Z04_04 + 2 Z04_05 + 3 Z05_02 + 2 Z05_05 + 3 Z07_00 + 2 Z07_01 + Z07_02
     + 2 Z07_03 + Z07_04 + 3 Z08_00 + 2 Z08_01 + 2 Z08_02 + Z08_03 + Z08_04
     + Z10_00 + 2 Z10_01 + Z10_03 + Z11_00 + 2 Z11_02 + Z11_03 + Z12_00
     + 2 Z12_02 + Z12_03 + Z14_00 + 4 Z15_00 + 3 Z15_01 + 3 Z15_02 + 2 Z15_03
     + 2 Z15_04 + Z15_05 + 2 Z15_06 + Z15_07 + 2 Z16_00 + 3 Z16_01 + Z16_02
     + 2 Z16_03 + Z16_04 + 2 Z16_05 + Z16_07 + 2 Z17_00 + Z17_01 + 3 Z17_03
     + 2 Z17_04 + Z17_05 + Z18_01 + 2 Z18_02 + Z19_00 + 2 Z20_00 + Z20_01
     + 2 Z21_00 + Z21_01 + Z21_02 + Z22_01 + 3 Z24_00 + 2 Z24_01 + Z24_02
     + Z25_00 + 2 Z25_02 + Z25_03 + Z27_00 - Z28_06 - 2 Z28_07 + Z28_08
```

```
 - Z28_10 - 3 Z28_11 - Z28_13 - 2 Z28_14 + 3 Z28_15 + 2 Z28_16 + Z28_17
 + Z28_18 - Z28_20 - 2 Z29_03 - Z29_05 + Z29_06 + 2 Z29_08 + Z29_09
 - Z29_10 - Z30_00 + Z30_02 - 3 Z31_00 - Z31_01 - 2 Z31_02 + Z31_03
 - Z31_05 - Z32_00 - 4 Z33_00 - 2 Z33_01 - 3 Z33_02 - Z33_04 - 2 Z33_05
 - 2 Z33_06 - Z33_08 + 2 Z33_09 + Z33_10 - 3 Z33_12 - Z33_13 - 2 Z33_14
 + Z33_15 - Z33_17 - 2 Z34_00 - Z34_02 - Z34_03 + Z34_04 - 2 Z35_00
 - Z35_02 - 3 Z36_00 - Z36_01 - 2 Z36_02 + Z36_03 - Z36_05 - Z37_00
 - Z38_00 - 4 Z39_00 - 2 Z39_01 - 3 Z39_02 - Z39_04 - 2 Z39_05 - 3 Z40_00
 - Z40_01 - 2 Z40_02 - 3 Z41_00 + 2 Y03_02 + 2 Y04_00 + Y04_01 + Y05_01
 + 3 Y06_00 + 2 Y06_01 + 2 Y06_02 + Y06_03 + Y06_04 + 2 Y07_02 + Y07_03
 + Y08_00 + 2 Y09_00 + Y09_01 + Y10_01 - 3 Y12_00 - Y12_01 - 2 Y12_02
 + Y12_03 - Y12_05 - 2 Y13_00 - Y13_02 - 2 Y14_00 - 2 X01_00, -1 <=
-4 Z01_01 - 3 Z01_00 - 5 Z01_03 - 5 Z01_02 - 6 Z01_04 - 7 Z01_05 - 4 Z01_06
 - 5 Z01_07 - 6 Z01_08 - 6 Z01_09 - 7 Z01_10 - 8 Z01_11 - 4 Z02_02
 - 5 Z02_03 - 6 Z02_05 - 5 Z02_08 - 6 Z02_09 - 7 Z02_11 - 4 Z03_06
 - 5 Z03_07 - 6 Z03_08 - 5 Z03_09 - 6 Z03_10 - 7 Z03_11 - 5 Z04_04
 - 6 Z04_05 - 5 Z05_02 - 6 Z05_05 - 3 Z07_00 - 4 Z07_01 - 5 Z07_02
 - 4 Z07_03 - 5 Z07_04 - 6 Z07_05 - 3 Z08_00 - 4 Z08_01 - 4 Z08_02
 - 5 Z08_03 - 5 Z08_04 - 6 Z08_05 - 3 Z10_00 - 4 Z10_01 - 4 Z10_02
 - 5 Z10_03 - 3 Z11_00 - 4 Z11_01 - 4 Z11_02 - 5 Z11_03 - 3 Z12_00
 - 4 Z12_01 - 4 Z12_02 - 5 Z12_03 - 4 Z13_00 - 3 Z14_00 - 4 Z14_01
 - 2 Z15_00 - 3 Z15_01 - 3 Z15_02 - 4 Z15_03 - 4 Z15_04 - 5 Z15_05
 - 4 Z15_06 - 5 Z15_07 - 6 Z15_08 - 2 Z16_00 - 3 Z16_01 - 3 Z16_02
 - 4 Z16_03 - 3 Z16_04 - 4 Z16_05 - 4 Z16_06 - 5 Z16_07 - 2 Z17_00
 - 3 Z17_01 - 4 Z17_02 - 3 Z17_03 - 4 Z17_04 - 5 Z17_05 - 2 Z18_00
 - 3 Z18_01 - 4 Z18_02 - 3 Z19_00 - 4 Z19_01 - 2 Z20_00 - 3 Z20_01
 - 4 Z20_02 - 2 Z21_00 - 3 Z21_01 - 3 Z21_02 - 2 Z22_00 - 3 Z22_01
 - 2 Z23_00 - Z24_00 - 2 Z24_01 - 3 Z24_02 - 4 Z24_03 - Z25_00 - 2 Z25_01
 - 4 Z21_03 - 2 Z25_02 - 3 Z25_03 - Z26_00 - Z27_00 - 2 Z27_01 - 3 Z28_06
 - 4 Z28_07 - 3 Z28_08 - 4 Z28_09 - 5 Z28_10 - 5 Z28_11 - 4 Z28_12
 - 5 Z28_13 - 6 Z28_14 - 3 Z28_15 - 4 Z28_16 - 5 Z28_17 - 5 Z28_18
 - 6 Z28_19 - 7 Z28_20 - 4 Z29_03 - 4 Z29_04 - 5 Z29_05 - 3 Z29_06
 - 4 Z29_07 - 4 Z29_08 - 5 Z29_09 - 5 Z29_10 - 6 Z29_11 - 3 Z30_00
 - 4 Z30_01 - 5 Z30_02 - 3 Z31_00 - 3 Z31_01 - 4 Z31_02 - 3 Z31_03
 - 4 Z31_04 - 5 Z31_05 - 3 Z32_00 - 4 Z32_01 - 2 Z33_00 - 2 Z33_01
 - 3 Z33_02 - 2 Z33_03 - 3 Z33_04 - 4 Z33_05 - 2 Z33_06 - 2 Z33_07
 - 3 Z33_08 - 2 Z33_09 - 3 Z33_10 - 4 Z33_11 - 3 Z33_12 - 3 Z33_13
 - 4 Z33_14 - 3 Z33_15 - 4 Z33_16 - 5 Z33_17 - 2 Z34_00 - 2 Z34_01
 - 3 Z34_02 - 3 Z34_03 - 3 Z34_04 - 4 Z34_05 - 2 Z35_00 - 2 Z35_01
 - 3 Z35_02 - Z36_00 - Z36_01 - 2 Z36_02 - Z36_03 - 2 Z36_04 - 3 Z36_05
 - Z37_00 - 2 Z37_01 - Z38_00 - 2 Z39_00 - 2 Z39_01 - 3 Z39_02 - 2 Z39_03
 - 3 Z39_04 - 4 Z39_05 - Z40_00 - Z40_01 - 2 Z40_02 - Z41_00 - 4 Y03_02
 - 2 Y04_00 - 3 Y04_01 - 4 Y04_02 - 3 Y05_01 - Y06_00 - 2 Y06_01 - 2 Y06_02
 - 3 Y06_03 - 3 Y06_04 - 4 Y06_05 - 2 Y07_02 - 3 Y07_03 - Y08_00 - 2 Y08_01
 - Y09_01 - 2 Y09_02 - Y10_01 - Y12_00 - Y12_01 - 2 Y12_02 - Y12_03
 - 2 Y12_04 - 3 Y12_05 - Y13_02, -Z01_00 - Z01_01 - Z01_06 - Z02_02
 - Z03_06 - Z15_00 - Z28_08 - Z28_15 - Z28_16 - Z28_17 - Z28_18 - Z29_06
 - Z29_08 - Z29_09 - Z30_02 - Z31_03 + Z33_00 - Z33_09 - Z33_10 - Z33_15
 - Z34_04 - Z36_03 + Z39_00 - Y12_03 <= 0, 0 <= -4 Z01_01 - 6 Z01_00
 - 2 Z01_03 - 2 Z01_02 + 2 Z01_05 - 4 Z01_06 - 2 Z01_07 + 2 Z01_10
 + 4 Z01_11 - 4 Z02_02 - 2 Z02_03 - 2 Z02_08 + 2 Z02_11 - 4 Z03_06
 - 2 Z03_07 - 2 Z03_09 + 2 Z03_11 - 2 Z04_04 - 2 Z05_02 - 4 Z07_00
 - 2 Z07_01 - 2 Z07_03 + 2 Z07_05 - 4 Z08_00 - 2 Z08_01 - 2 Z08_02
 + 2 Z08_05 - 3 Z10_00 - 2 Z10_01 - Z10_02 - 3 Z11_00 - Z11_01 - 2 Z11_02
 - 3 Z12_00 - Z12_01 - 2 Z12_02 - Z13_00 - 2 Z14_00 - 6 Z15_00 - 4 Z15_01
 - 4 Z15_02 - 2 Z15_03 - 2 Z15_04 - 2 Z15_06 + 2 Z15_08 - 5 Z16_00
 - 4 Z16_01 - 3 Z16_02 - 2 Z16_03 - 3 Z16_04 - 2 Z16_05 - Z16_06 - 5 Z17_00
 - 3 Z17_01 - Z17_02 - 4 Z17_03 - 2 Z17_04 - 4 Z18_00 - 3 Z18_01 - 2 Z18_02
 - 3 Z19_00 - Z19_01 - 4 Z20_00 - 2 Z20_01 - 4 Z21_00 - 2 Z21_01 - 2 Z21_02
```

```
      - 3 Z22_00 - 2 Z22_01 - 2 Z23_00 - 6 Z24_00 - 4 Z24_01 - 2 Z24_02
      - 5 Z25_00 - 3 Z25_01 - 4 Z25_02 - 2 Z25_03 - 4 Z26_00 - 4 Z27_00
      - 2 Z27_01 - 2 Z28_06 - 3 Z28_08 - Z28_09 + Z28_10 + 2 Z28_11 - Z28_12
      + Z28_13 + 3 Z28_14 - 4 Z28_15 - 2 Z28_16 + 2 Z28_19 + 4 Z28_20 - Z29_04
      + Z29_05 - 3 Z29_06 - Z29_07 - 2 Z29_08 + Z29_10 + 2 Z29_11 - 2 Z30_00
      - Z30_01 - Z31_01 + Z31_02 - 2 Z31_03 + 2 Z31_05 - Z32_00 - Z33_00
      - 2 Z33_01 - 3 Z33_03 - Z33_04 + Z33_05 - 2 Z33_06 - 3 Z33_07 - Z33_08
      - 4 Z33_09 - 2 Z33_10 - Z33_13 + Z33_14 - 2 Z33_15 + 2 Z33_17 - 2 Z34_00
      - 3 Z34_01 - Z34_02 - Z34_03 - 2 Z34_04 - Z35_00 - 2 Z35_01 - 2 Z36_00
      - 3 Z36_01 - Z36_02 - 4 Z36_03 - 2 Z36_04 - 3 Z37_00 - 2 Z37_01 - 2 Z38_00
      - Z39_01 + Z39_02 - 2 Z39_03 + 2 Z39_05 - Z40_00 - 2 Z40_01 - Y03_02
      - 3 Y04_00 - Y04_01 + Y04_02 - Y05_01 - 5 Y06_00 - 3 Y06_01 - 3 Y06_02
      - Y06_03 - Y06_04 + Y06_05 - 3 Y07_02 - Y07_03 - 3 Y08_00 - Y08_01
      - 5 Y09_00 - 3 Y09_01 - Y09_02 - 4 Y10_00 - 3 Y10_01 - 3 Y11_00 - Y12_00
      - 2 Y12_01 - 3 Y12_03 - Y12_04 + Y12_05 - 2 Y13_00 - 3 Y13_01 - Y13_02
      - Y14_00}
>
> maximize(obj,cnsts,NONNEGATIVE);
>
{Z28_14 = 1/14, Z28_07 = 0, Z03_10 = 0, Z03_11 = 0, Z04_04 = 0, Z04_05 = 0,
    Z03_06 = 0, Z03_07 = 0, Z03_08 = 0, Z03_09 = 0, Z01_11 = 0, Z02_02 = 0,
    Z02_03 = 0, Z02_05 = 0, Z01_07 = 0, Z01_08 = 0, Z01_09 = 0, Z01_10 = 0,
    Z01_03 = 0, Z01_02 = 0, Z01_04 = 0, Z01_05 = 0, Z01_06 = 0, Z01_01 = 0,
    Z01_00 = 0, Z12_03 = 0, Z13_00 = 0, Z12_02 = 0, Z11_02 = 0, Z11_03 = 0,
    Z12_00 = 0, Z12_01 = 0, Z10_02 = 0, Z10_03 = 0, Z11_00 = 0, Z11_01 = 0,
    Z08_04 = 0, Z08_05 = 0, Z10_00 = 0, Z10_01 = 0, Z08_01 = 0, Z08_02 = 0,
    Z08_03 = 0, Z07_03 = 0, Z07_04 = 0, Z07_05 = 0, Z08_00 = 0, Z05_05 = 0,
    Z07_00 = 0, Z07_01 = 0, Z07_02 = 0, Z05_02 = 0, Z20_01 = 0, Z20_00 = 0,
    Z20_02 = 0, Z18_02 = 0, Z19_00 = 0, Z19_01 = 0, Z02_09 = 0, Z02_11 = 0,
    Z02_08 = 0, Z17_03 = 0, Z17_04 = 0, Z17_05 = 0, Z18_00 = 0, Z18_01 = 0,
    Z16_07 = 0, Z17_01 = 0, Z17_02 = 0, Z17_00 = 0, Z16_03 = 0, Z16_04 = 0,
    Z16_05 = 0, Z16_06 = 0, Z15_08 = 0, Z16_00 = 0, Z16_01 = 0, Z16_02 = 0,
    Z15_07 = 0, Z15_03 = 0, Z15_02 = 0, Z15_04 = 0, Z15_05 = 0, Z15_06 = 0,
    Z14_00 = 0, Z14_01 = 0, Z15_00 = 0, Z15_01 = 0, Z28_12 = 0, Z28_09 = 0,
    Z28_10 = 0, Z28_13 = 0, Z28_15 = 0, Z28_08 = 0, Z27_00 = 0, Z27_01 = 0,
    Z28_06 = 0, Z26_00 = 0, Z25_01 = 0, Z21_03 = 0, Z25_02 = 0, Z25_03 = 0,
    Z24_03 = 0, Z25_00 = 0, Z24_00 = 0, Z24_01 = 0, Z24_02 = 0, Z23_00 = 0,
    Z21_00 = 0, Z21_01 = 0, Z21_02 = 0, Z22_00 = 0, Z22_01 = 0, Z34_01 = 0,
    Z34_02 = 0, Z34_03 = 0, Z34_04 = 0, Z33_16 = 0, Z33_17 = 0, Z34_00 = 0,
    Z33_15 = 0, Z33_12 = 0, Z33_13 = 0, Z33_14 = 0, Z33_11 = 0, Z33_07 = 0,
    Z33_08 = 0, Z33_09 = 0, Z33_10 = 0, Z33_03 = 0, Z33_04 = 0, Z33_05 = 0,
    Z33_06 = 0, Z32_01 = 0, Z33_00 = 0, Z33_01 = 0, Z33_02 = 0, Z31_03 = 0,
    Z31_04 = 0, Z31_05 = 0, Z32_00 = 0, Z30_02 = 0, Z31_00 = 0, Z31_01 = 0,
    Z31_02 = 0, Z29_08 = 0, Z29_09 = 0, Z29_10 = 0, Z29_11 = 0, Z30_01 = 0,
    Z29_03 = 0, Z29_04 = 0, Z29_05 = 0, Z29_06 = 0, Z29_07 = 0, Z28_17 = 0,
    Z28_18 = 0, Z28_19 = 0, Z28_20 = 0, Z28_16 = 0, Z39_03 = 0, Z39_05 = 0,
    Z40_00 = 0, Z39_04 = 0, Z39_00 = 0, Z38_00 = 0, Z39_01 = 0, Z39_02 = 0,
    Z36_05 = 0, Z37_00 = 0, Z37_01 = 0, Z36_00 = 0, Z36_01 = 0, Z36_02 = 0,
    Z36_03 = 0, Z36_04 = 0, Z34_05 = 0, Z35_00 = 0, Z35_01 = 0, Z35_02 = 0,
    Y10_01 = 0, Y11_00 = 0, Y12_00 = 0, Y09_02 = 0, Y10_00 = 0, Y09_01 = 0,
    Y07_02 = 0, Y07_03 = 0, Y08_00 = 0, Y08_01 = 0, Y09_00 = 0, Y04_02 = 0,
    Y05_01 = 0, Y04_01 = 0, Y06_04 = 0, Y06_05 = 0, Y06_03 = 0, Y06_00 = 0,
    Y06_01 = 0, Y06_02 = 0, Z40_01 = 0, Z41_00 = 0, Y04_00 = 0, Z40_02 = 0,
    Y14_00 = 0, X01_00 = 0, Y13_02 = 0, Y12_03 = 0, Y12_04 = 0, Y12_05 = 0,
    Y13_00 = 0, Y13_01 = 0, Y12_01 = 0, Y12_02 = 0, Z28_11 = 0, Y03_02 = 1/10,
    Z30_00 = 2/35}
>
```

```
> subs(%,obj);
                                        29
                                        --
                                        70
>
> (dualobj,dualcnsts):=dual(obj,cnsts,y);

dualobj, dualcnsts := y8, {2 <= -y1 - y6 - 2 y7 + 2 y8 + 5 y9, 3 <= 2 y8 + 4 y9,
    1 <= -y7 + y8 + 4 y9, 3 <= y4 + y6 + y7 + 3 y8 + 2 y9, 2 <= y8 + 4 y9,
    2 <= -y7 + y8 + 5 y9, 1 <= 2 y8 + 2 y9,
    1 <= -y1 - y2 - y4 - 2 y6 - 3 y7 + y8 + 6 y9, 2 <= 2 y8 + 3 y9,
    1 <= -y1 - y6 - 2 y7 + 2 y8 + 4 y9,
    3 <= y1 + y2 + y4 + 2 y6 + 3 y7 + 5 y8 - 2 y9,
    3 <= y1 + y4 + 2 y6 + 2 y7 + 4 y8,
    2 <= -y1 - y2 - y3 - y4 - y5 - y6 - y7 + 3 y8 + 3 y9,
    2 <= y6 + y7 + 5 y8 - y9, 1 <= y7 + 7 y8 - 4 y9,
    1 <= -y1 - y2 - y3 - 2 y4 - 2 y5 - 2 y6 - 2 y7 + 4 y8 + 2 y9,
    1 <= -y1 - y2 - y3 - y4 - y5 - y6 - y7 + 5 y8,
    2 <= y1 + y6 + 2 y7 + 6 y8 - 3 y9,
    1 <= -2 y1 - y2 - y3 - 3 y4 - 3 y5 - 3 y6 - 3 y7 + 3 y8 + 4 y9,
    4 <= 2 y1 + y2 + y3 + 2 y4 + y5 + 3 y6 + 4 y7 + 2 y8 + y9,
    3 <= y1 + y4 + 2 y6 + 2 y7 + 2 y8 + 2 y9, 1 <= y7 + 5 y8 - 2 y9,
    2 <= y7 + 3 y8 + y9, 2 <= y1 + y6 + 2 y7 + 4 y8 - y9,
    1 <= -y1 - y2 - y3 - y4 - y5 - y6 - y7 + 3 y8 + 2 y9,
    3 <= y1 + y2 + y4 + 2 y6 + 3 y7 + 3 y8, 2 <= y6 + y7 + 3 y8 + y9,
    3 <= y7 + 3 y8 + 2 y9, 2 <= y7 + 5 y8 - y9,
    2 <= y1 + y2 + y4 + 2 y6 + 3 y7 + y8 + y9, 1 <= y6 + y7 + y8 + 2 y9,
    2 <= y1 + y2 + y4 + 2 y6 + 3 y7 + 3 y8 - y9, 1 <= y6 + y7 + 3 y8,
    1 <= y1 + y6 + 2 y7 + 4 y8 - 2 y9, 2 <= y7 + y8 + 3 y9, 1 <= y7 + y8 + 2 y9,
    3 <= 2 y1 + y2 + y3 + 2 y4 + y5 + 3 y6 + 4 y7 + 2 y8,
    2 <= y1 + y4 + 2 y6 + 2 y7 + 2 y8 + y9,
    1 <= -y1 - y2 - y3 - y4 - y5 - y6 - y7 + y8 + 4 y9,
    2 <= y6 + y7 + y8 + 3 y9, 1 <= y7 + 3 y8,
    3 <= y1 + y2 + y4 + 2 y6 + 3 y7 + y8 + 2 y9,
    2 <= y1 + y6 + 2 y7 + 2 y8 + y9, 3 <= y1 + y6 + 2 y7 + 2 y8 + 2 y9,
    1 <= -y1 - y2 - y3 - 2 y4 - 2 y5 - 2 y6 - 2 y7 + 2 y8 + 4 y9,
    1 <= y7 + y8 + y9, 1 <= y1 + y6 + 2 y7 + y9, 1 <= y1 + y6 + 2 y7,
    1 <= y7 + 3 y8 - y9, 2 <= y1 + y6 + 2 y7 + 2 y9, 2 <= y1 + y6 + 2 y7 + 2 y8,
    1 <= -y1 - y2 - y3 - y4 - y5 - y6 - y7 + y8 + 3 y9,
    2 <= y6 + y7 + y8 + 2 y9, 1 <= 3 y9,
    3 <= y1 + y2 + y4 + 2 y6 + 3 y7 + y8 + y9, 2 <= 4 y9, 1 <= 2 y8 + y9,
    1 <= -y1 - y6 - 2 y7 + 5 y9, 1 <= -y7 + y8 + 3 y9,
    1 <= -y1 - y2 - y4 - 2 y6 - 3 y7 + y8 + 5 y9, 1 <= -y7 + 3 y8 + y9,
    1 <= 4 y8 - y9, 1 <= -y1 - y6 - 2 y7 + 2 y8 + 3 y9,
    1 <= y1 + y2 + y4 + 2 y6 + 3 y7 + y8, 1 <= -y1 - y6 - 2 y7 + 4 y8 + y9,
    1 <= y1 + y6 + 2 y7 + 2 y8, 1 <= -y7 + 7 y8 - 2 y9,
    1 <= -y1 - y2 - y4 - 2 y6 - 3 y7 + 5 y8 + 2 y9, 1 <= -y1 - y6 - 2 y7 + 6 y8,
    1 <= -2 y1 - y2 - y3 - 2 y4 - y5 - 3 y6 - 4 y7 + 4 y8 + 4 y9,
    2 <= -y7 + 3 y8 + 3 y9, 2 <= 4 y8 + y9,
    1 <= -2 y1 - y2 - y3 - 3 y4 - 2 y5 - 4 y6 - 5 y7 + 3 y8 + 6 y9, 1 <= 4 y8,
    1 <= -2 y1 - y2 - y3 - 2 y4 - y5 - 3 y6 - 4 y7 + 2 y8 + 6 y9,
    1 <= 8 y8 - 4 y9, 1 <= -y7 + 3 y8 + 2 y9, 1 <= 6 y8 - 2 y9,
    1 <= -y1 - y2 - y4 - 2 y6 - 3 y7 + 3 y8 + 4 y9,
    1 <= -y1 - y6 - 2 y7 + 4 y8 + 2 y9, 1 <= -y7 + 5 y8}
>
```

```
> minimize(dualobj,dualcnsts,NONNEGATIVE);
>
                29
{y6 = 0, y8 = --, y9 = 4/7, y3 = 0, y5 = 0, y1 = 0, y2 = 4/35, y4 = 4/35,
                70

          43
    y7 = --}
          70
>
> quit
```

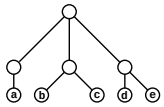
# MIDS girth 5

In all operations considered, the vertex at the "top" of the figure is the one that could possibly be selected by "some" algorithm for inclusion in the independent dominating set. The priorities are

- vertices with a neighbour of degree 1 over
- vertices of degree 2 and their neighbours

In each case, choose the vertex that, if added to the set, would give the smallest ratio of vertices added to the set (which includes the vertex itself and all isolates created) to vertices removed from the graph (i.e., vertices that attain degree zero). Ties with the same ratio are broken by the number of edges removed (with those removing the fewest edges having highest priority). Ties with the same ratio that remove the same number of edges are broken arbitrarily. Operations written in red (with "***" in the last column) are are excluded from $OPS_2$.

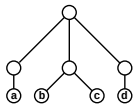
Z01:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z01-00: | $V_2$ | $V_2$ | $V_2$ | $V_2$ | $V_2$: | 3 | 6 | 0 | 0 | 0 | 5 | 1*** |
| Z01-01: | $V_2$ | $V_2$ | $V_2$ | $V_2$ | $V_3$: | 4 | 5 | 0 | 0 | 1 | 4 | 1*** |
| Z01-02: | $V_2$ | $V_2$ | $V_2$ | $V_3$ | $V_3$: | 5 | 4 | 0 | 0 | 2 | 3 | 1*** |
| Z01-03: | $V_2$ | $V_2$ | $V_3$ | $V_2$ | $V_3$: | 5 | 4 | 0 | 0 | 2 | 3 | 1 |
| Z01-04: | $V_2$ | $V_2$ | $V_3$ | $V_3$ | $V_3$: | 6 | 3 | 0 | 0 | 3 | 2 | 1 |
| Z01-05: | $V_2$ | $V_3$ | $V_3$ | $V_3$ | $V_3$: | 7 | 2 | 0 | 0 | 4 | 1 | 1 |
| Z01-06: | $V_3$ | $V_2$ | $V_2$ | $V_2$ | $V_2$: | 4 | 5 | 0 | 0 | 1 | 4 | 1*** |
| Z01-07: | $V_3$ | $V_2$ | $V_2$ | $V_2$ | $V_3$: | 5 | 4 | 0 | 0 | 2 | 3 | 1*** |
| Z01-08: | $V_3$ | $V_2$ | $V_2$ | $V_3$ | $V_3$: | 6 | 3 | 0 | 0 | 3 | 2 | 1*** |
| Z01-09: | $V_3$ | $V_2$ | $V_3$ | $V_2$ | $V_3$: | 6 | 3 | 0 | 0 | 3 | 2 | 1 |
| Z01-10: | $V_3$ | $V_2$ | $V_3$ | $V_3$ | $V_3$: | 7 | 2 | 0 | 0 | 4 | 1 | 1 |
| Z01-11: | $V_3$ | $V_3$ | $V_3$ | $V_3$ | $V_3$: | 8 | 1 | 0 | 0 | 5 | 0 | 1 |

Both $b$ and $c$ cannot have degree 2 (similarly both $d$ and $e$ cannot have degree 2). The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours). If both, say, $b$ and $c$ had degree 2, adding the selected vertex would give a ratio of 1/4 and remove eight edges whereas adding the neighbour of $b$ would give a ratio of 1/4 and remove seven edges.

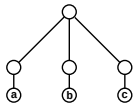
Z15:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z15-00: | $V_2$ | $V_2$ | $V_2$ | $V_2$ | : | 2 | 6 | 0 | 0 | 0 | 4 | 1 |
| Z15-01: | $V_2$ | $V_2$ | $V_2$ | $V_3$ | : | 3 | 5 | 0 | 0 | 1 | 3 | 1 |
| Z15-02: | $V_2$ | $V_2$ | $V_3$ | $V_2$ | : | 3 | 5 | 0 | 0 | 1 | 3 | 1 |
| Z15-03: | $V_2$ | $V_2$ | $V_3$ | $V_3$ | : | 4 | 4 | 0 | 0 | 2 | 2 | 1 |
| Z15-04: | $V_2$ | $V_3$ | $V_3$ | $V_2$ | : | 4 | 4 | 0 | 0 | 2 | 2 | 1 |
| Z15-05: | $V_2$ | $V_3$ | $V_3$ | $V_3$ | : | 5 | 3 | 0 | 0 | 3 | 1 | 1 |
| Z15-06: | $V_3$ | $V_2$ | $V_2$ | $V_3$ | : | 4 | 4 | 0 | 0 | 2 | 2 | 1 |
| Z15-07: | $V_3$ | $V_2$ | $V_3$ | $V_3$ | : | 5 | 3 | 0 | 0 | 3 | 1 | 1 |
| Z15-08: | $V_3$ | $V_3$ | $V_3$ | $V_3$ | : | 6 | 2 | 0 | 0 | 4 | 0 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

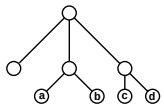
Z24:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z24-00: | $V_2$ | $V_2$ | $V_2$ | | : | 1 | 6 | 0 | 0 | 0 | 3 | 1 |
| Z24-01: | $V_2$ | $V_2$ | $V_3$ | | : | 2 | 5 | 0 | 0 | 1 | 2 | 1 |
| Z24-02: | $V_2$ | $V_3$ | $V_3$ | | : | 3 | 4 | 0 | 0 | 2 | 1 | 1 |
| Z24-03: | $V_3$ | $V_3$ | $V_3$ | | : | 4 | 3 | 0 | 0 | 3 | 0 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

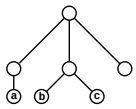
Z28:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z28-00: | $V_1$ | $V_1$ | $V_1$ | $V_1$ | | : | 3 | 0 | 5 | 0 | 0 | 0 | 5*** |
| Z28-01: | $V_1$ | $V_1$ | $V_1$ | $V_2$ | | : | 3 | 1 | 4 | 0 | 0 | 1 | 4*** |
| Z28-02: | $V_1$ | $V_1$ | $V_1$ | $V_3$ | | : | 4 | 0 | 4 | 0 | 1 | 0 | 4*** |
| Z28-03: | $V_1$ | $V_1$ | $V_2$ | $V_2$ | | : | 3 | 2 | 3 | 0 | 0 | 2 | 3*** |
| Z28-04: | $V_1$ | $V_1$ | $V_2$ | $V_3$ | | : | 4 | 1 | 3 | 0 | 1 | 1 | 3*** |
| Z28-05: | $V_1$ | $V_1$ | $V_3$ | $V_3$ | | : | 5 | 0 | 3 | 0 | 2 | 0 | 3*** |
| Z28-06: | $V_1$ | $V_2$ | $V_1$ | $V_2$ | | : | 3 | 2 | 3 | 0 | 0 | 2 | 3*** |
| Z28-07: | $V_1$ | $V_2$ | $V_1$ | $V_3$ | | : | 4 | 1 | 3 | 0 | 1 | 1 | 3*** |
| Z28-08: | $V_1$ | $V_2$ | $V_2$ | $V_2$ | | : | 3 | 3 | 2 | 0 | 0 | 3 | 2 |
| Z28-09: | $V_1$ | $V_2$ | $V_2$ | $V_3$ | | : | 4 | 2 | 2 | 0 | 1 | 2 | 2 |
| Z28-10: | $V_1$ | $V_2$ | $V_3$ | $V_3$ | | : | 5 | 1 | 2 | 0 | 2 | 1 | 2 |
| Z28-11: | $V_1$ | $V_3$ | $V_1$ | $V_3$ | | : | 5 | 0 | 3 | 0 | 2 | 0 | 3*** |
| Z28-12: | $V_1$ | $V_3$ | $V_2$ | $V_2$ | | : | 4 | 2 | 2 | 0 | 1 | 2 | 2 |
| Z28-13: | $V_1$ | $V_3$ | $V_2$ | $V_3$ | | : | 5 | 1 | 2 | 0 | 2 | 1 | 2 |
| Z28-14: | $V_1$ | $V_3$ | $V_3$ | $V_3$ | | : | 6 | 0 | 2 | 0 | 3 | 0 | 2 |
| Z28-15: | $V_2$ | $V_2$ | $V_2$ | $V_2$ | | : | 3 | 4 | 1 | 0 | 0 | 4 | 1 |
| Z28-16: | $V_2$ | $V_2$ | $V_2$ | $V_3$ | | : | 4 | 3 | 1 | 0 | 1 | 3 | 1 |
| Z28-17: | $V_2$ | $V_2$ | $V_3$ | $V_3$ | | : | 5 | 2 | 1 | 0 | 2 | 2 | 1 |
| Z28-18: | $V_2$ | $V_3$ | $V_2$ | $V_3$ | | : | 5 | 2 | 1 | 0 | 2 | 2 | 1 |
| Z28-19: | $V_2$ | $V_3$ | $V_3$ | $V_3$ | | : | 6 | 1 | 1 | 0 | 3 | 1 | 1 |
| Z28-20: | $V_3$ | $V_3$ | $V_3$ | $V_3$ | | : | 7 | 0 | 1 | 0 | 4 | 0 | 1 |

There cannot be more than one vertex of degree 1 amongst the vertices $a$, $b$, $c$ and $d$. If, say, $a$ has degree 1 and $b$ has degree 1, adding the selected vertex would give a ratio of at least 3/6 (depending on the degrees of $c$ and $d$) whereas adding the neighbour of $a$ would give a ratio of 2/5. A similar argument holds if $c$ and $d$ have degree 1. If, say, $a$ and $c$ had degree 1 (or $a$ and $d$, or $b$ and $c$, or $b$ and $d$), then there either exists a vertex somewhere else in the graph whose addition to the set gives a smaller ratio, or the same ratio by removing fewer edges, or we have one of the special cases $ZZ_i$ listed below.

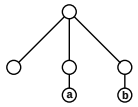
Z33:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z33-00: | $V_1$ | $V_1$ | $V_1$ | | | : | 2 | 1 | 4 | 0 | 0 | 0 | 4*** |
| Z33-01: | $V_1$ | $V_1$ | $V_2$ | | | : | 2 | 2 | 3 | 0 | 0 | 1 | 3 |
| Z33-02: | $V_1$ | $V_1$ | $V_3$ | | | : | 3 | 1 | 3 | 0 | 1 | 0 | 3 |
| Z33-03: | $V_1$ | $V_2$ | $V_2$ | | | : | 2 | 3 | 2 | 0 | 0 | 2 | 2 |
| Z33-04: | $V_1$ | $V_2$ | $V_3$ | | | : | 3 | 2 | 2 | 0 | 1 | 1 | 2 |
| Z33-05: | $V_1$ | $V_3$ | $V_3$ | | | : | 4 | 1 | 2 | 0 | 2 | 0 | 2 |
| Z33-06: | $V_2$ | $V_1$ | $V_1$ | | | : | 2 | 2 | 3 | 0 | 0 | 1 | 3*** |
| Z33-07: | $V_2$ | $V_1$ | $V_2$ | | | : | 2 | 3 | 2 | 0 | 0 | 2 | 2 |
| Z33-08: | $V_2$ | $V_1$ | $V_3$ | | | : | 3 | 2 | 2 | 0 | 1 | 1 | 2 |
| Z33-09: | $V_2$ | $V_2$ | $V_2$ | | | : | 2 | 4 | 1 | 0 | 0 | 3 | 1 |
| Z33-10: | $V_2$ | $V_2$ | $V_3$ | | | : | 3 | 3 | 1 | 0 | 1 | 2 | 1 |
| Z33-11: | $V_2$ | $V_3$ | $V_3$ | | | : | 4 | 2 | 1 | 0 | 2 | 1 | 1 |
| Z33-12: | $V_3$ | $V_1$ | $V_1$ | | | : | 3 | 1 | 3 | 0 | 1 | 0 | 3*** |
| Z33-13: | $V_3$ | $V_1$ | $V_2$ | | | : | 3 | 2 | 2 | 0 | 1 | 1 | 2 |
| Z33-14: | $V_3$ | $V_1$ | $V_3$ | | | : | 4 | 1 | 2 | 0 | 2 | 0 | 2 |
| Z33-15: | $V_3$ | $V_2$ | $V_2$ | | | : | 3 | 3 | 1 | 0 | 1 | 2 | 1 |
| Z33-16: | $V_3$ | $V_2$ | $V_3$ | | | : | 4 | 2 | 1 | 0 | 2 | 1 | 1 |
| Z33-17: | $V_3$ | $V_3$ | $V_3$ | | | : | 5 | 1 | 1 | 0 | 3 | 0 | 1 |

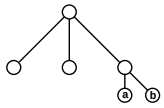
Both $b$ and $c$ cannot have degree 1. If both $b$ and $c$ had degree 1, adding the selected vertex would give a ratio of at least 3/6 whereas adding the neighbour of $b$ would give a ratio of 2/5.

Z36:



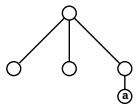

| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z36-00: | $V_1$ | $V_1$ | | | : | 1 | 2 | 3 | 0 | 0 | 0 | 3 |
| Z36-01: | $V_1$ | $V_2$ | | | : | 1 | 3 | 2 | 0 | 0 | 1 | 2 |
| Z36-02: | $V_1$ | $V_3$ | | | : | 2 | 2 | 2 | 0 | 1 | 0 | 2 |
| Z36-03: | $V_2$ | $V_2$ | | | : | 1 | 4 | 1 | 0 | 0 | 2 | 1 |
| Z36-04: | $V_2$ | $V_3$ | | | : | 2 | 3 | 1 | 0 | 1 | 1 | 1 |
| Z36-05: | $V_3$ | $V_3$ | | | : | 3 | 2 | 1 | 0 | 2 | 0 | 1 |

Z39:



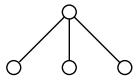

| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z39-00: | $V_1$ | $V_1$ | | | : | 2 | 0 | 4 | 0 | 0 | 0 | 3 |
| Z39-01: | $V_1$ | $V_2$ | | | : | 2 | 1 | 3 | 0 | 0 | 1 | 2 |
| Z39-02: | $V_1$ | $V_3$ | | | : | 3 | 0 | 3 | 0 | 1 | 0 | 2 |
| Z39-03: | $V_2$ | $V_2$ | | | : | 2 | 2 | 2 | 0 | 0 | 2 | 1 |
| Z39-04: | $V_2$ | $V_3$ | | | : | 3 | 1 | 2 | 0 | 1 | 1 | 1 |
| Z39-05: | $V_3$ | $V_3$ | | | : | 4 | 0 | 2 | 0 | 2 | 0 | 1 |

Z40:



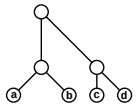

| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z40-00: | $V_1$ | | | | : | 1 | 1 | 3 | 0 | 0 | 0 | 2 |
| Z40-01: | $V_2$ | | | | : | 1 | 2 | 2 | 0 | 0 | 1 | 1 |
| Z40-02: | $V_3$ | | | | : | 2 | 1 | 2 | 0 | 1 | 0 | 1 |

Z41:



| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z41-00: | | | | | : | 1 | 0 | 3 | 0 | 0 | 0 | 1 |

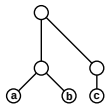
Y01:




| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y01-00: | $V_2$ | $V_2$ | $V_2$ | $V_2$ | : | 2 | 5 | 0 | 0 | 0 | 4 | 1*** |
| Y01-01: | $V_2$ | $V_2$ | $V_2$ | $V_3$ | : | 3 | 4 | 0 | 0 | 1 | 3 | 1*** |
| Y01-02: | $V_2$ | $V_2$ | $V_3$ | $V_3$ | : | 4 | 3 | 0 | 0 | 2 | 2 | 1*** |
| Y01-03: | $V_2$ | $V_3$ | $V_2$ | $V_3$ | : | 4 | 3 | 0 | 0 | 2 | 2 | 1*** |
| Y01-04: | $V_2$ | $V_3$ | $V_3$ | $V_3$ | : | 5 | 2 | 0 | 0 | 3 | 1 | 1*** |
| Y01-05: | $V_3$ | $V_3$ | $V_3$ | $V_3$ | : | 6 | 1 | 0 | 0 | 4 | 0 | 1*** |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours). Adding the selected vertex would give a ratio of 1/3. Adding the neighbour of the selected vertex that has degree 3 would give a ratio of 1/4.

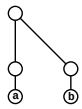
Y06:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y06-00: | $V_2$ | $V_2$ | $V_2$ | | | : | 1 | 5 | 0 | 0 | 0 | 3 | 1*** |
| Y06-01: | $V_2$ | $V_2$ | $V_3$ | | | : | 2 | 4 | 0 | 0 | 1 | 2 | 1*** |
| Y06-02: | $V_2$ | $V_3$ | $V_2$ | | | : | 2 | 4 | 0 | 0 | 1 | 2 | 1*** |
| Y06-03: | $V_2$ | $V_3$ | $V_3$ | | | : | 3 | 3 | 0 | 0 | 2 | 1 | 1*** |
| Y06-04: | $V_3$ | $V_3$ | $V_2$ | | | : | 3 | 3 | 0 | 0 | 2 | 1 | 1*** |
| Y06-05: | $V_3$ | $V_3$ | $V_3$ | | | : | 4 | 2 | 0 | 0 | 3 | 0 | 1*** |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours). Adding the selected vertex would give a ratio of 1/3 whereas adding a neighbour of the selected vertex gives a ratio of 1/4.

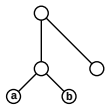
Y09:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y09-00: | $V_2$ | $V_2$ | | | | : | 0 | 5 | 0 | 0 | 0 | 2 | 1 |
| Y09-01: | $V_2$ | $V_3$ | | | | : | 1 | 4 | 0 | 0 | 1 | 1 | 1 |
| Y09-02: | $V_3$ | $V_3$ | | | | : | 2 | 3 | 0 | 0 | 2 | 0 | 1 |

The selected vertex has degree more than 1 and none of its neighbours has degree 1, indicating that the minimum degree in the graph is 2 (as we only ever select a vertex of minimum degree or one of its neighbours).

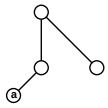
Y12:




| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y12-00: | $V_1$ | $V_1$ | | | | : | 1 | 1 | 3 | 0 | 0 | 0 | 3*** |
| Y12-01: | $V_1$ | $V_2$ | | | | : | 1 | 2 | 2 | 0 | 0 | 1 | 2 |
| Y12-02: | $V_1$ | $V_3$ | | | | : | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| Y12-03: | $V_2$ | $V_2$ | | | | : | 1 | 3 | 1 | 0 | 0 | 2 | 1 |
| Y12-04: | $V_2$ | $V_3$ | | | | : | 2 | 2 | 1 | 0 | 1 | 1 | 1 |
| Y12-05: | $V_3$ | $V_3$ | | | | : | 3 | 1 | 1 | 0 | 2 | 0 | 1 |

Both $a$ and $b$ cannot have degree 1. If both $a$ and $b$ had degree 1, adding the selected vertex would give a ratio of 3/5 whereas adding the neighbour of $b$ would give a ratio of 2/5.

Y13:

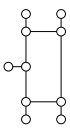



| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y13-00: | $V_1$ | | | | | : | 0 | 2 | 2 | 0 | 0 | 0 | 2 |
| Y13-01: | $V_2$ | | | | | : | 0 | 3 | 1 | 0 | 0 | 1 | 1 |
| Y13-02: | $V_3$ | | | | | : | 1 | 2 | 1 | 0 | 1 | 0 | 1 |

Y14:

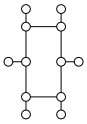
| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y14-00: | | | | | | : | 0 | 1 | 2 | 0 | 0 | 0 | 1 |

ZZ5:



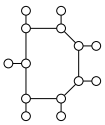
| variable | a | b | c | d | e | | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ZZ5-00: | | | | | | : | 5 | 0 | 5 | 0 | 0 | 0 | 5 |

ZZ6:



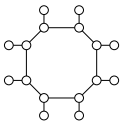
| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ZZ6-00: | | | | | : | 6 | 0 | 6 | 0 | 0 | 0 | 6 |

ZZ7:



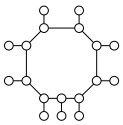
| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ZZ7-00: | | | | | : | 7 | 0 | 7 | 0 | 0 | 0 | 7 |

ZZ8:



| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ZZ8-00: | | | | | : | 8 | 0 | 8 | 0 | 0 | 0 | 8 |

ZZ9:



| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ZZ9-00: | | | | | : | 9 | 0 | 9 | 0 | 0 | 0 | 9 |

X01:

| variable | a | b | c | d | e | $Y_3^-$ | $Y_2^-$ | $Y_1^-$ | $Y_3^+$ | $Y_2^+$ | $Y_1^+$ | $\Delta\mathcal{I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| X01-00: | | | | | : | 0 | 0 | 2 | 0 | 0 | 0 | 1 |

```
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 \  MAPLE  /  reserved. Maple and Maple V are registered trademarks of
 <____ ____>  Waterloo Maple Inc.
      |       Type ? for help.
> with(simplex):
Warning, new definition for maximize
Warning, new definition for minimize
>
> obj:= 1*Z01_03 +1*Z01_04 +1*Z01_05 +1*Z01_09 +1*Z01_10 +1*Z01_11
+1*Z15_00 +1*Z15_01 +1*Z15_02 +1*Z15_03 +1*Z15_04 +1*Z15_05 +1*Z15_06
+1*Z15_07 +1*Z15_08 +1*Z24_00 +1*Z24_01 +1*Z24_02 +1*Z24_03 +2*Z28_08
+2*Z28_09 +2*Z28_10 +2*Z28_12 +2*Z28_13 +2*Z28_14 +1*Z28_15 +1*Z28_16
+1*Z28_17 +1*Z28_18 +1*Z28_19 +1*Z28_20 +3*Z33_01 +3*Z33_02 +2*Z33_03
+2*Z33_04 +2*Z33_05 +2*Z33_07 +2*Z33_08 +1*Z33_09 +1*Z33_10 +1*Z33_11
+2*Z33_13 +2*Z33_14 +1*Z33_15 +1*Z33_16 +1*Z33_17 +3*Z36_00 +2*Z36_01
+2*Z36_02 +1*Z36_03 +1*Z36_04 +1*Z36_05 +3*Z39_00 +2*Z39_01 +2*Z39_02
+1*Z39_03 +1*Z39_04 +1*Z39_05 +2*Z40_00 +1*Z40_01 +1*Z40_02 +1*Z41_00
+1*Y09_00 +1*Y09_01 +1*Y09_02 +2*Y12_01 +2*Y12_02 +1*Y12_03 +1*Y12_04
+1*Y12_05 +2*Y13_00 +1*Y13_01 +1*Y13_02 +1*Y14_00 +5*ZZ5_00 +6*ZZ6_00
+7*ZZ7_00 +8*ZZ8_00 +9*ZZ9_00 +1*X01_00;
>
obj := Z01_04 + Z01_03 + Z01_09 + Z01_05 + Z01_10 + Z01_11 + Z15_00 + Z15_01
     + Z15_02 + Z15_03 + Z15_04 + Z15_05 + Z15_06 + Z15_07 + Z15_08 + Z24_00
     + Z24_01 + Z24_02 + Z24_03 + 2 Z28_08 + 2 Z28_09 + 2 Z28_10 + 2 Z28_12
     + 2 Z28_13 + 2 Z28_14 + Z28_15 + Z28_16 + Z28_17 + Z28_18 + Z28_19
     + Z28_20 + 3 Z33_01 + 3 Z33_02 + 2 Z33_03 + 2 Z33_04 + 2 Z33_05 + 2 Z33_07
     + 2 Z33_08 + Z33_09 + Z33_10 + Z33_11 + 2 Z33_13 + 2 Z33_14 + Z33_15
     + Z33_16 + Z33_17 + 3 Z36_00 + 2 Z36_01 + 2 Z36_02 + Z36_03 + Z36_04
     + Z36_05 + 3 Z39_00 + 2 Z39_01 + 2 Z39_02 + Z39_03 + Z39_04 + Z39_05
     + 2 Z40_00 + Z40_01 + Z40_02 + Z41_00 + Y09_00 + Y09_01 + Y09_02
     + 2 Y12_01 + 2 Y12_02 + Y12_03 + Y12_04 + Y12_05 + 2 Y13_00 + Y13_01
     + Y13_02 + Y14_00 + 5 ZZ5_00 + 6 ZZ6_00 + 7 ZZ7_00 + 8 ZZ8_00 + 9 ZZ9_00
     + X01_00
>
> cnsts:={
+2*Z01_03 +1*Z01_04 +1*Z01_09 +3*Z15_00 +2*Z15_01 +2*Z15_02
+1*Z15_03 +1*Z15_04 +1*Z15_06 +2*Z24_00 +1*Z24_01 +1*Z28_08 -1*Z28_10
-1*Z28_13 -1*Z28_14 +3*Z28_15 +2*Z28_16 +1*Z28_17 +1*Z28_18 -2*Z33_01
-2*Z33_02 -1*Z33_04 -1*Z33_05 -1*Z33_08 +2*Z33_09 +1*Z33_10 -1*Z33_13
-1*Z33_14 +1*Z33_15 -2*Z36_00 -1*Z36_01 -1*Z36_02 +1*Z36_03 -3*Z39_00
-2*Z39_01 -2*Z39_02 -1*Z39_04 -1*Z39_05 -2*Z40_00 -1*Z40_01 -1*Z40_02
-2*Z41_00 +1*Y09_00 -1*Y12_01 -1*Y12_02 +1*Y12_03 -1*Y13_00 -1*Y14_00
-4*ZZ5_00 -5*ZZ6_00 -6*ZZ7_00 -7*ZZ8_00 -8*ZZ9_00 -1*X01_00 >= 0,
>
>  +1*Z01_03 +2*Z15_00 +1*Z15_01 +1*Z15_02 +1*Z24_00 +1*Z28_08
+3*Z28_15 +2*Z28_16 +1*Z28_17 +1*Z28_18 -1*Z33_01 -1*Z33_02 +2*Z33_09
+1*Z33_10 +1*Z33_15 -1*Z36_00 +1*Z36_03 -2*Z39_00 -1*Z39_01 -1*Z39_02
-1*Z40_00 -1*Z41_00 +1*Y12_03 -3*ZZ5_00 -4*ZZ6_00 -5*ZZ7_00 -6*ZZ8_00
-7*ZZ9_00 >= 0,
>
>  +1*Z15_00 +1*Z28_08 +3*Z28_15 +2*Z28_16 +1*Z28_17 +1*Z28_18
+2*Z33_09 +1*Z33_10 +1*Z33_15 +1*Z36_03 -1*Z39_00 +1*Y12_03 -2*ZZ5_00
-3*ZZ6_00 -4*ZZ7_00 -5*ZZ8_00 -6*ZZ9_00 >= 0,
>
>  +3*Z01_03 +2*Z01_04 +1*Z01_05 +2*Z01_09 +1*Z01_10 +4*Z15_00
+3*Z15_01 +3*Z15_02 +2*Z15_03 +2*Z15_04 +1*Z15_05 +2*Z15_06 +1*Z15_07
+3*Z24_00 +2*Z24_01 +1*Z24_02 +1*Z28_08 -1*Z28_10 -1*Z28_13 -2*Z28_14
+3*Z28_15 +2*Z28_16 +1*Z28_17 +1*Z28_18 -1*Z28_20 -2*Z33_01 -3*Z33_02
```

```
-1*Z33_04 -2*Z33_05 -1*Z33_08 +2*Z33_09 +1*Z33_10 -1*Z33_13 -2*Z33_14
+1*Z33_15 -1*Z33_17 -3*Z36_00 -1*Z36_01 -2*Z36_02 +1*Z36_03 -1*Z36_05
-4*Z39_00 -2*Z39_01 -3*Z39_02 -1*Z39_04 -2*Z39_05 -3*Z40_00 -1*Z40_01
-2*Z40_02 -3*Z41_00 +2*Y09_00 +1*Y09_01 -1*Y12_01 -2*Y12_02 +1*Y12_03
-1*Y12_05 -2*Y13_00 -1*Y13_02 -2*Y14_00 -5*ZZ5_00 -6*ZZ6_00 -7*ZZ7_00
-8*ZZ8_00 -9*ZZ9_00 -2*X01_00 >= 0,
>
> -1*Z01_03 -1*Z01_04 -1*Z01_09 -2*Z15_00 -1*Z15_01 -1*Z15_02
-1*Z15_03 -1*Z15_04 -1*Z15_06 -1*Z24_00 -1*Z24_01 -1*Z28_08 +1*Z28_14
-2*Z28_15 -1*Z28_16 -1*Z28_17 -1*Z28_18 +1*Z33_01 +1*Z33_02 +1*Z33_05
-1*Z33_09 -1*Z33_10 +1*Z33_14 -1*Z33_15 +1*Z36_00 +1*Z36_02 -1*Z36_03
+2*Z39_00 +1*Z39_01 +1*Z39_02 +1*Z39_05 +1*Z40_00 +1*Z40_02 +1*Z41_00
-1*Y09_00 +1*Y12_02 -1*Y12_03 +1*Y13_00 +1*Y14_00 +2*ZZ5_00 +3*ZZ6_00
+3*ZZ7_00 +4*ZZ8_00 +4*ZZ9_00 +1*X01_00  <= 0,
>
> -1*Z01_03 -1*Z15_00 -1*Z15_01 -1*Z15_02 -1*Z24_00 -1*Z28_08
-1*Z28_15 -1*Z28_16 -1*Z28_17 -1*Z28_18 +1*Z33_02 -1*Z33_09 -1*Z33_10
-1*Z33_15 +1*Z36_00 -1*Z36_03 +1*Z39_00 +1*Z39_02 +1*Z40_00 +1*Z41_00
-1*Y12_03 +1*ZZ5_00 +2*ZZ6_00 +2*ZZ7_00 +2*ZZ8_00 +3*ZZ9_00  <= 0,
>
> -1*Z15_00 -1*Z28_08 -1*Z28_15 -1*Z28_16 -1*Z28_17 -1*Z28_18
-1*Z33_09 -1*Z33_10 -1*Z33_15 -1*Z36_03 +1*Z39_00 -1*Y12_03 +1*ZZ5_00
+1*ZZ6_00 +1*ZZ7_00 +2*ZZ8_00 +2*ZZ9_00  <= 0,
>
> -5*Z01_03 -6*Z01_04 -7*Z01_05 -6*Z01_09 -7*Z01_10 -8*Z01_11
-2*Z15_00 -3*Z15_01 -3*Z15_02 -4*Z15_03 -4*Z15_04 -5*Z15_05 -4*Z15_06
-5*Z15_07 -6*Z15_08 -1*Z24_00 -2*Z24_01 -3*Z24_02 -4*Z24_03 -3*Z28_08
-4*Z28_09 -5*Z28_10 -4*Z28_12 -5*Z28_13 -6*Z28_14 -3*Z28_15 -4*Z28_16
-5*Z28_17 -5*Z28_18 -6*Z28_19 -7*Z28_20 -2*Z33_01 -3*Z33_02 -2*Z33_03
-3*Z33_04 -4*Z33_05 -2*Z33_07 -3*Z33_08 -2*Z33_09 -3*Z33_10 -4*Z33_11
-3*Z33_13 -4*Z33_14 -3*Z33_15 -4*Z33_16 -5*Z33_17 -1*Z36_00 -1*Z36_01
-2*Z36_02 -1*Z36_03 -2*Z36_04 -3*Z36_05 -2*Z39_00 -2*Z39_01 -3*Z39_02
-2*Z39_03 -3*Z39_04 -4*Z39_05 -1*Z40_00 -1*Z40_01 -2*Z40_02 -1*Z41_00
-1*Y09_01 -2*Y09_02 -1*Y12_01 -2*Y12_02 -1*Y12_03 -2*Y12_04 -3*Y12_05
-1*Y13_02 -5*ZZ5_00 -6*ZZ6_00 -7*ZZ7_00 -8*ZZ8_00 -9*ZZ9_00  >= -1,
>
> -2*Z01_03 +2*Z01_05 +2*Z01_10 +4*Z01_11 -6*Z15_00 -4*Z15_01
-4*Z15_02 -2*Z15_03 -2*Z15_04 -2*Z15_06 +2*Z15_08 -6*Z24_00 -4*Z24_01
-2*Z24_02 -3*Z28_08 -1*Z28_09 +1*Z28_10 -1*Z28_12 +1*Z28_13 +3*Z28_14
-4*Z28_15 -2*Z28_16 +2*Z28_19 +4*Z28_20 -2*Z33_01 -3*Z33_03 -1*Z33_04
+1*Z33_05 -3*Z33_07 -1*Z33_08 -4*Z33_09 -2*Z33_10 -1*Z33_13 +1*Z33_14
-2*Z33_15 +2*Z33_17 -2*Z36_00 -3*Z36_01 -1*Z36_02 -4*Z36_03 -2*Z36_04
-1*Z39_01 +1*Z39_02 -2*Z39_03 +2*Z39_05 -1*Z40_00 -2*Z40_01 -5*Y09_00
-3*Y09_01 -1*Y09_02 -2*Y12_01 -3*Y12_03 -1*Y12_04 +1*Y12_05 -2*Y13_00
-3*Y13_01 -1*Y13_02 -1*Y14_00  >= 0,
>
> +3*Z01_03 +2*Z01_04 +1*Z01_05 +2*Z01_09 +1*Z01_10 +4*Z15_00
+3*Z15_01 +3*Z15_02 +2*Z15_03 +2*Z15_04 +1*Z15_05 +2*Z15_06 +1*Z15_07
+3*Z24_00 +2*Z24_01 +1*Z24_02 +1*Z28_08 -1*Z28_10 -1*Z28_13 -2*Z28_14
+3*Z28_15 +2*Z28_16 +1*Z28_17 +1*Z28_18 -1*Z28_20 -2*Z33_01 -3*Z33_02
-1*Z33_04 -2*Z33_05 -1*Z33_08 +2*Z33_09 +1*Z33_10 -1*Z33_13 -2*Z33_14
+1*Z33_15 -1*Z33_17 -3*Z36_00 -1*Z36_01 -2*Z36_02 +1*Z36_03 -1*Z36_05
-4*Z39_00 -2*Z39_01 -3*Z39_02 -1*Z39_04 -2*Z39_05 -3*Z40_00 -1*Z40_01
-2*Z40_02 -3*Z41_00 +2*Y09_00 +1*Y09_01 -1*Y12_01 -2*Y12_02 +1*Y12_03
-1*Y12_05 -2*Y13_00 -1*Y13_02 -2*Y14_00 -5*ZZ5_00 -6*ZZ6_00 -7*ZZ7_00
-8*ZZ8_00 -9*ZZ9_00 -2*X01_00  >= 0};
```

```
cnsts := {0 <= Z15_00 + Z28_08 + 3 Z28_15 + 2 Z28_16 + Z28_17 + Z28_18
     + 2 Z33_09 + Z33_10 + Z33_15 + Z36_03 - Z39_00 + Y12_03 - 2 ZZ5_00
     - 3 ZZ6_00 - 4 ZZ7_00 - 5 ZZ8_00 - 6 ZZ9_00, 0 <= Z01_04 + 2 Z01_03
     + Z01_09 + 3 Z15_00 + 2 Z15_01 + 2 Z15_02 + Z15_03 + Z15_04 + Z15_06
     + 2 Z24_00 + Z24_01 + Z28_08 - Z28_10 - Z28_13 - Z28_14 + 3 Z28_15
     + 2 Z28_16 + Z28_17 + Z28_18 - 2 Z33_01 - 2 Z33_02 - Z33_04 - Z33_05
     - Z33_08 + 2 Z33_09 + Z33_10 - Z33_13 - Z33_14 + Z33_15 - 2 Z36_00
     - Z36_01 - Z36_02 + Z36_03 - 3 Z39_00 - 2 Z39_01 - 2 Z39_02 - Z39_04
     - Z39_05 - 2 Z40_00 - Z40_01 - Z40_02 - 2 Z41_00 + Y09_00 - Y12_01
     - Y12_02 + Y12_03 - Y13_00 - Y14_00 - 4 ZZ5_00 - 5 ZZ6_00 - 6 ZZ7_00
     - 7 ZZ8_00 - 8 ZZ9_00 - X01_00, 0 <= 2 Z01_04 + 3 Z01_03 + 2 Z01_09
     + Z01_05 + Z01_10 + 4 Z15_00 + 3 Z15_01 + 3 Z15_02 + 2 Z15_03 + 2 Z15_04
     + Z15_05 + 2 Z15_06 + Z15_07 + 3 Z24_00 + 2 Z24_01 + Z24_02 + Z28_08
     - Z28_10 - Z28_13 - 2 Z28_14 + 3 Z28_15 + 2 Z28_16 + Z28_17 + Z28_18
     - Z28_20 - 2 Z33_01 - 3 Z33_02 - Z33_04 - 2 Z33_05 - Z33_08 + 2 Z33_09
     + Z33_10 - Z33_13 - 2 Z33_14 + Z33_15 - Z33_17 - 3 Z36_00 - Z36_01
     - 2 Z36_02 + Z36_03 - Z36_05 - 4 Z39_00 - 2 Z39_01 - 3 Z39_02 - Z39_04
     - 2 Z39_05 - 3 Z40_00 - Z40_01 - 2 Z40_02 - 3 Z41_00 + 2 Y09_00 + Y09_01
     - Y12_01 - 2 Y12_02 + Y12_03 - Y12_05 - 2 Y13_00 - Y13_02 - 2 Y14_00
     - 5 ZZ5_00 - 6 ZZ6_00 - 7 ZZ7_00 - 8 ZZ8_00 - 9 ZZ9_00 - 2 X01_00, -Z01_04
     - Z01_03 - Z01_09 - 2 Z15_00 - Z15_01 - Z15_02 - Z15_03 - Z15_04 - Z15_06
     - Z24_00 - Z24_01 - Z28_08 + Z28_14 - 2 Z28_15 - Z28_16 - Z28_17 - Z28_18
     + Z33_01 + Z33_02 + Z33_05 - Z33_09 - Z33_10 + Z33_14 - Z33_15 + Z36_00
     + Z36_02 - Z36_03 + 2 Z39_00 + Z39_01 + Z39_02 + Z39_05 + Z40_00 + Z40_02
     + Z41_00 - Y09_00 + Y12_02 - Y12_03 + Y13_00 + Y14_00 + 2 ZZ5_00
     + 3 ZZ6_00 + 3 ZZ7_00 + 4 ZZ8_00 + 4 ZZ9_00 + X01_00 <= 0, 0 <= Z01_03
     + 2 Z15_00 + Z15_01 + Z15_02 + Z24_00 + Z28_08 + 3 Z28_15 + 2 Z28_16
     + Z28_17 + Z28_18 - Z33_01 - Z33_02 + 2 Z33_09 + Z33_10 + Z33_15 - Z36_00
     + Z36_03 - 2 Z39_00 - Z39_01 - Z39_02 - Z40_00 - Z41_00 + Y12_03
     - 3 ZZ5_00 - 4 ZZ6_00 - 5 ZZ7_00 - 6 ZZ8_00 - 7 ZZ9_00, -Z15_00 - Z28_08
     - Z28_15 - Z28_16 - Z28_17 - Z28_18 - Z33_09 - Z33_10 - Z33_15 - Z36_03
     + Z39_00 - Y12_03 + ZZ5_00 + ZZ6_00 + ZZ7_00 + 2 ZZ8_00 + 2 ZZ9_00 <= 0, 0
     <= -2 Z01_03 + 2 Z01_05 + 2 Z01_10 + 4 Z01_11 - 6 Z15_00 - 4 Z15_01
     - 4 Z15_02 - 2 Z15_03 - 2 Z15_04 - 2 Z15_06 + 2 Z15_08 - 6 Z24_00
     - 4 Z24_01 - 2 Z24_02 - 3 Z28_08 - Z28_09 + Z28_10 - Z28_12 + Z28_13
     + 3 Z28_14 - 4 Z28_15 - 2 Z28_16 + 2 Z28_19 + 4 Z28_20 - 2 Z33_01
     - 3 Z33_03 - Z33_04 + Z33_05 - 3 Z33_07 - Z33_08 - 4 Z33_09 - 2 Z33_10
     - Z33_13 + Z33_14 - 2 Z33_15 + 2 Z33_17 - 2 Z36_00 - 3 Z36_01 - Z36_02
     - 4 Z36_03 - 2 Z36_04 - Z39_01 + Z39_02 - 2 Z39_03 + 2 Z39_05 - Z40_00
     - 2 Z40_01 - 5 Y09_00 - 3 Y09_01 - Y09_02 - 2 Y12_01 - 3 Y12_03 - Y12_04
     + Y12_05 - 2 Y13_00 - 3 Y13_01 - Y13_02 - Y14_00, -1 <= -6 Z01_04
     - 5 Z01_03 - 6 Z01_09 - 7 Z01_05 - 7 Z01_10 - 8 Z01_11 - 2 Z15_00
     - 3 Z15_01 - 3 Z15_02 - 4 Z15_03 - 4 Z15_04 - 5 Z15_05 - 4 Z15_06
     - 5 Z15_07 - 6 Z15_08 - Z24_00 - 2 Z24_01 - 3 Z24_02 - 4 Z24_03 - 3 Z28_08
     - 4 Z28_09 - 5 Z28_10 - 4 Z28_12 - 5 Z28_13 - 6 Z28_14 - 3 Z28_15
     - 4 Z28_16 - 5 Z28_17 - 5 Z28_18 - 6 Z28_19 - 7 Z28_20 - 2 Z33_01
     - 3 Z33_02 - 2 Z33_03 - 3 Z33_04 - 4 Z33_05 - 2 Z33_07 - 3 Z33_08
     - 2 Z33_09 - 3 Z33_10 - 4 Z33_11 - 3 Z33_13 - 4 Z33_14 - 3 Z33_15
     - 4 Z33_16 - 5 Z33_17 - Z36_00 - Z36_01 - 2 Z36_02 - Z36_03 - 2 Z36_04
     - 3 Z36_05 - 2 Z39_00 - 2 Z39_01 - 3 Z39_02 - 2 Z39_03 - 3 Z39_04
     - 4 Z39_05 - Z40_00 - Z40_01 - 2 Z40_02 - Z41_00 - Y09_01 - 2 Y09_02
     - Y12_01 - 2 Y12_02 - Y12_03 - 2 Y12_04 - 3 Y12_05 - Y13_02 - 5 ZZ5_00
     - 6 ZZ6_00 - 7 ZZ7_00 - 8 ZZ8_00 - 9 ZZ9_00, -Z01_03 - Z15_00 - Z15_01
     - Z15_02 - Z24_00 - Z28_08 - Z28_15 - Z28_16 - Z28_17 - Z28_18 + Z33_02
     - Z33_09 - Z33_10 - Z33_15 + Z36_00 - Z36_03 + Z39_00 + Z39_02 + Z40_00
     + Z41_00 - Y12_03 + ZZ5_00 + 2 ZZ6_00 + 2 ZZ7_00 + 2 ZZ8_00 + 3 ZZ9_00 <=
    0}
>
```

```
> maximize(obj,cnsts,NONNEGATIVE);
>
{ZZ7_00 = 0, ZZ8_00 = 0, ZZ9_00 = 0, X01_00 = 0, Z01_03 = 0, Z28_14 = 0,
    Z28_09 = 1/6, Z01_11 = 1/24, Z33_03 = 0, Z33_04 = 0, Z33_05 = 0, Z33_07 = 0,
    Z33_08 = 0, Z33_09 = 0, Z33_10 = 0, Z33_11 = 0, Z33_13 = 0, Z33_14 = 0,
    Z33_15 = 0, Z33_16 = 0, Z33_17 = 0, Z36_00 = 0, Z36_01 = 0, Z36_02 = 0,
    Z36_03 = 0, Z36_04 = 0, Z36_05 = 0, Z39_00 = 0, Z39_01 = 0, Z39_02 = 0,
    Z39_03 = 0, Z39_04 = 0, Z39_05 = 0, Z40_00 = 0, Z40_01 = 0, Z40_02 = 0,
    Z41_00 = 0, Y09_00 = 0, Y09_01 = 0, Y09_02 = 0, Y12_01 = 0, Y12_02 = 0,
    Y12_03 = 0, Y12_04 = 0, Y12_05 = 0, Y13_00 = 0, Y13_01 = 0, Y13_02 = 0,
    Y14_00 = 0, ZZ5_00 = 0, ZZ6_00 = 0, Z15_04 = 0, Z15_05 = 0, Z15_06 = 0,
    Z15_07 = 0, Z15_08 = 0, Z24_00 = 0, Z24_01 = 0, Z24_02 = 0, Z24_03 = 0,
    Z28_08 = 0, Z28_10 = 0, Z28_12 = 0, Z28_13 = 0, Z28_15 = 0, Z28_16 = 0,
    Z28_17 = 0, Z28_18 = 0, Z28_19 = 0, Z28_20 = 0, Z33_01 = 0, Z33_02 = 0,
    Z01_04 = 0, Z01_09 = 0, Z01_05 = 0, Z01_10 = 0, Z15_00 = 0, Z15_01 = 0,
    Z15_02 = 0, Z15_03 = 0}
>
> subs(%,obj);
                                        3/8
>
> (dualobj,dualcnsts):=dual(obj,cnsts,y);
>
dualobj, dualcnsts := y3, {1 <= -4 y1 + 7 y3 + y9,
    1 <= 4 y1 - y2 + 3 y3 - y4 - y6 - 2 y8 - 3 y9,
    1 <= 6 y1 - 2 y2 + 2 y3 - 2 y4 - y5 - y6 - y7 - 3 y8 - 4 y9,
    2 <= -y1 + 4 y3 + y4 + y8 + 2 y9, 2 <= y1 + 3 y3 + y8 + y9,
    2 <= 3 y1 + 2 y3, 3 <= y2 + 3 y3 + y4 + y6 + 2 y8 + 3 y9, 1 <= 5 y3 - y9,
    3 <= 2 y1 + y2 + 2 y3 + y4 + 2 y8 + 2 y9, 1 <= 2 y1 + 4 y3 - y4 - y8 - 2 y9,
    1 <= 4 y3, 1 <= 2 y1 + 3 y3 - y9, 1 <= 4 y1 + 2 y3 - y4 - y8 - 2 y9,
    1 <= 2 y1 - y2 + 3 y3 - y4 - y5 - y6 - y7 - y8 - y9,
    1 <= 4 y1 - 2 y2 + 2 y3 - y4 - y5 - y6 - 2 y7 - 2 y8 - 2 y9,
    1 <= -2 y1 + 6 y3, 1 <= 6 y1 - y2 + y3 - y4 - y6 - 2 y8 - 3 y9,
    2 <= -y1 + 5 y3 + y8 + y9, 1 <= 3 y3 + y9,
    1 <= 4 y1 - y2 + y3 - y4 - y5 - y6 - y7 - y8 - y9,
    2 <= y1 + 2 y3 + y4 + y8 + 2 y9, 2 <= 3 y1 + y3 + y8 + y9, 2 <= y1 + 4 y3,
    3 <= 2 y1 + y2 + y3 + y4 + y6 + 2 y8 + 3 y9, 1 <= -2 y1 + 5 y3 + y9,
    2 <= 3 y1 - y2 + 3 y3 - y4 - y5 - y6 - y7 - y8 - y9,
    2 <= -y1 + y2 + 3 y3 + y4 + y6 + 2 y8 + 3 y9,
    2 <= y1 + y2 + 2 y3 + y4 + 2 y8 + 2 y9,
    3 <= 2 y2 + 2 y3 + 2 y4 + y5 + y6 + y7 + 3 y8 + 4 y9, 1 <= 2 y1 + 2 y3,
    1 <= 4 y1 - 3 y2 + 3 y3 - 2 y4 - y5 - y6 - 3 y7 - 3 y8 - 3 y9,
    1 <= y2 + y3 + y4 + y6 + 2 y8 + 3 y9, 1 <= 2 y1 + y3 + y8 + y9,
    1 <= 2 y3 + y4 + y8 + 2 y9, 2 <= y1 + y2 + y3 + y4 + y6 + 2 y8 + 3 y9,
    1 <= -2 y1 + 4 y3 + y4 + y8 + 2 y9, 2 <= -3 y1 + 6 y3 + y4 + y8 + 2 y9,
    1 <= 3 y3 + y8 + y9, 1 <= 3 y1 - y2 + y3 - y4 - y5 - y6 - y7 - y8 - y9,
    2 <= 2 y3 + y4 + y8 + 2 y9, 2 <= 2 y1 + y3 + y8 + y9, 1 <= y1 + 2 y3,
    1 <= 3 y1 + y3 - y9, 1 <= 5 y1 - y4 - y8 - 2 y9,
    6 <= 4 y2 + 6 y3 + 3 y4 + y5 + 2 y6 + 3 y7 + 5 y8 + 6 y9,
    1 <= y1 + y4 + y8 + 2 y9,
    5 <= 3 y2 + 5 y3 + 2 y4 + y5 + y6 + 2 y7 + 4 y8 + 5 y9, 1 <= y1 + y3 + y9,
    2 <= 2 y1 + y4 + y8 + 2 y9, 1 <= 3 y1, 1 <= -y1 + 3 y3 + y9,
    1 <= y4 + y8 + 2 y9, 1 <= 6 y3 - y4 - y8 - 2 y9,
    9 <= 7 y2 + 9 y3 + 4 y4 + 2 y5 + 3 y6 + 6 y7 + 8 y8 + 9 y9,
    8 <= 6 y2 + 8 y3 + 4 y4 + 2 y5 + 2 y6 + 5 y7 + 7 y8 + 8 y9,
    7 <= 5 y2 + 7 y3 + 3 y4 + y5 + 2 y6 + 4 y7 + 6 y8 + 7 y9,
    1 <= 2 y1 - 2 y2 + 4 y3 - y4 - y5 - y6 - 2 y7 - 2 y8 - 2 y9,
    1 <= 2 y1 - y2 + 5 y3 - y4 - y6 - 2 y8 - 3 y9,
    1 <= -y2 + 5 y3 - y4 - y5 - y6 - y7 - y8 - y9, 1 <= -2 y1 + 7 y3 - y9,
    1 <= -4 y1 + 8 y3}
```

```
> minimize(dualobj,dualcnsts,NONNEGATIVE);
>
 {y6 = 0, y9 = 5/8, y1 = 1/2, y2 = 0, y3 = 3/8, y5 = 0, y8 = 0, y7 = 0, y4 = 0}
>
> quit
```