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Homoclinic Orbits for a Class of Symmetric Hamiltonian Systems *

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Abstract

We study existence of homoclinic orbits for a class of Hamiltonian systems that are symmetric with respect to independent variable (time). For the scalar case we prove existence and uniqueness of a positive homoclinic solution. For the system case we prove existence of symmetric homoclinic orbits. We use variational approach.

1 Introduction

Recently variational techniques have been applied to obtain existence of homoclinic orbits of the Hamiltonian systems

$$u'' - L(t)u + V_u(t,u) = 0\,, \tag{1.1}$$

see e.g. P.H. Rabinowitz [4] and W. Omana and M. Willem [3]. Here $L(t)$ is a positive definite $n \times n$ matrix, V is assumed to be superquadratic at infinity and subquadratic at zero in u, and the solution $u(t) \in H^1(R, R^n)$ is homoclinic at zero, i.e., $\lim_{t\to\pm\infty} u(t) = 0$. The technical difficulty in applying the mountain pass lemma on the infinite interval $t \in (-\infty, \infty)$, is in verifying the Palais-Smale or "compactness" condition. In [3] a new compact embedding theorem was used to verify the Palais-Smale condition. However, one had to assume that the smallest eigenvalue of $L(t)$ tends to ∞ as $|t| \to \infty$, which is a rather restrictive and not very natural condition, as it excludes e.g. the case of constant L. In this paper we show that this assumption is not necessary in case $L(t)$ and $V(t,u)$ are even in t. For the scalar case we also show existence and uniqueness of a positive homoclinic orbit.

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We begin by considering the equation (1.1) on a finite interval $(-T,T)$ together with the boundary conditions

$$u(-T) = u(T) = 0\,. \tag{1.2}$$

Using the mountain pass lemma we show that the problem (1.1-1.2) has a nontrivial solution. Moreover, using variational approach, we derive uniform in T estimate of H^1 norm of the solution. This is the crucial step, which allows us to obtain homoclinic orbits by letting $T \to \infty$.

2 Positive homoclinics for a scalar equation

In this section we will prove existence and uniqueness of positive homoclinics for a model problem with a cubic nonlinearity. Namely, we are looking for a positive solution of

$$u'' - a(x)u + b(x)u^3 = 0\,, \quad -\infty < x < \infty\,, \tag{2.1}$$

$$u(-\infty) = u(\infty) = u'(-\infty) = u'(\infty) = 0\,. \tag{2.2}$$

We assume that the functions $a(x), b(x) \in C^1(-\infty,\infty)$ are strictly positive on $(-\infty,\infty)$, i.e. $a(x) \geq a_0 > 0$ and $b(x) \geq b_0 > 0$ and moreover we assume that $a(x)$ and $b(x)$ are even with respect to some real number c. Without loss of generality we will assume that $c = 0$, i.e. $a(x)$ and $b(x)$ are even functions. We assume additionally that $xa'(x) > 0$ and $xb'(x) < 0$ for all $x \neq 0$.

We shall obtain the solution of (2.1-2.2) as the limit as $T \to \infty$ of the solutions of

$$u'' - a(x)u + b(x)u^3 = 0 \quad \text{for} \quad x \in (-T,T), \quad u(-T) = u(T) = 0\,. \tag{2.3}$$

We shall need the following lemma from P. Korman and T. Ouyang [2]. (Except for the last assertion, this lemma is also included in B. Gidas, W.-M. Ni and L. Nirenberg [1]).

Lemma 2.1 *Consider the problem*

$$u'' + f(x,u) = 0 \quad \textit{for} \quad x \in (-T,T), \quad u(-T) = u(T) = 0\,. \tag{2.4}$$

Assume that the function $f \in C^1([-T,T] \times R_+)$ is such that

$$f(-x,u) = f(x,u) \quad \textit{for} \quad x \in (-T,T) \quad \textit{and} \quad u > 0\,, \tag{2.5}$$

$$xf_x(x,u) < 0 \quad \textit{for} \quad x \in (-T,T)\backslash\{0\} \quad \textit{and} \quad u > 0\,. \tag{2.6}$$

Then any positive solution of (2.4) is an even function with $u'(x) < 0$ on $(0,T)$. Moreover any two positive solutions of (2.4) cannot intersect on $(-T,T)$ (and hence they are strictly ordered on $(-T,T)$).

Clearly, under our conditions, the Lemma 2.1 applies to the problem (2.3). We are looking for solution of (2.3) which is strictly positive on $(-T,T)$, and so we can work with an equivalent problem

$$u'' - a(x)u + b(x)(u^+)^3 = 0 \quad \text{for } x \in (-T,T)\ ,\ u(-T) = u(T) = 0\,, \tag{2.7}$$

where $u^+ = \max(u,0)$.

Lemma 2.2 *The problem (2.3) has under our conditions a unique positive solution for any $T \geq 1$. Moreover, for this solution we have an estimate*

$$\int_{-T}^{T} \left(u'^2 + u^2\right)\, dx \leq c \quad \textit{uniformly in} \quad T \geq 1\,. \tag{2.8}$$

Proof. We shall work with the problem (2.7) using the space $H_0^1[-T,T]$ of absolutely continuous functions, which vanish at $\pm T$, with the norm $\|u\|^2 = \int_{-T}^{T}(u'^2 + u^2)\, dx$. We consider a functional $f : H_0^1[-T,T] \to R$, defined by

$$f(u) = \int_{-T}^{T} \left[\frac{u'^2}{2} + a(x)\frac{u^2}{2} - b(x)\frac{(u^+)^4}{4}\right] dx\,.$$

Clearly $f(0) = 0$, and it is standard to verify that $f(u)$ satisfies the Palais-Smale condition, and that $f(u)$ has a strict local minimum at $u = 0$. Starting with any function $u_0(x) \in H_0^1[-1,1]$, such that $u_0^+ \not\equiv 0$, we define $u^* \in H_0^1[-T,T]$ as follows: $u^*(x) = \lambda u_0(x)$ for $x \in [-1,1]$, and $u^*(x) = 0$ for $x \in [-T,T]\backslash[-1,1]$. Then $f(u^*) < 0$ for λ sufficiently large, and we conclude by the well-known mountain pass theorem that $f(u)$ has a nontrivial critical point $u(x) \in H_0^1[-T,T]$, which is then easily seen to be a strictly positive solution of (2.3).

The variational approach also allows us to derive the estimate (2.8). Indeed, if we define the set of paths

$$\Gamma_T = \{g(\tau) : [0,1] \to H_0^1[-T,T] \mid g(0) = 0, \quad g(1) = u^*\}\,,$$

then the solution of $u(x)$ of (2.3) is the point where

$$\inf_{g \in \Gamma_T} \max_{\tau \in [0,1]} f(g(\tau)) \equiv c_T\,, \tag{2.9}$$

is achieved. Let now $T_1 > T$. Then $\Gamma_T \subset \Gamma_{T_1}$, since any function in $H_0^1[-T,T]$ can be regarded as belonging to $H_0^1[-T_1,T_1]$, if one extends it by zero in $[-T_1,T_1]\backslash[-T,T]$. Hence for T_1 the set of competing paths in (2.9) is greater than that for T, which implies that

$$c_{T_1} \leq c_T \leq c_1 \quad (T_1 > T \geq 1)\,. \tag{2.10}$$

So that for the positive solution of (2.3),

$$\int_{-T}^{T} \left(\frac{u'^2}{2} + a(x)\frac{u^2}{2} - b(x)\frac{u^4}{4} \right) dx \le c_1 \quad \text{uniformly in} \quad T \ge 1 . \tag{2.11}$$

Multiply (2.3) by u and integrate,

$$\frac{1}{4} \int_{-T}^{T} \left(u'^2 + a(x)u^2 - b(x)u^4 \right) dx = 0 . \tag{2.12}$$

Subtracting (2.12) from (2.11), we establish the estimate (2.8).

Turning to the uniqueness, we notice that any two different solutions $u(x)$ and $v(x)$ of (2.3) would have to be ordered by Lemma 2.1, i.e. $u(x) < v(x)$ for all $x \in (-T, T)$, which leads to a contradiction by an application of Sturm comparison theorem.

Theorem 2.1 *The problem (2.1-2.2) has under our conditions exactly one positive solution. Moreover this solution is an even function with $u'(x) < 0$ for $x > 0$.*

Proof. Take a sequence $T_n \to \infty$, and consider the problem (2.3) on the interval $(-T_n, T_n)$, i.e. consider

$$u'' - a(x)u + b(x)u^3 = 0 \quad \text{on} \quad (-T_n, T_n), \quad u(-T_n) = u(T_n) = 0 . \tag{2.13}$$

By Lemma 2.2 the problem (2.13) has a unique positive solution $u_n(x)$, and

$$\int_{-T_n}^{T_n} \left(u_n'^2 + u_n^2 \right) dx \le c \quad \text{uniformly in} \quad n . \tag{2.14}$$

By Lemma 2.1, $u_n(x)$ takes its maximum at $x = 0$, which implies that $u_n''(0) \le 0$, and then from the equation (2.13),

$$u_n(0) \ge \sqrt{\frac{a(0)}{b(0)}} . \tag{2.15}$$

Writing

$$u_n(x_1) - u_n(x_2) = \int_{x_1}^{x_2} u_n' \, dx \le \sqrt{x_2 - x_1} \left(\int_{x_1}^{x_2} u_n'^2 \, dx \right)^{1/2} ,$$

we conclude that the sequence $\{u_n(x)\}$ is equicontinuous and uniformly bounded on every interval $[-T_n, T_n]$. Hence it has a uniformly convergent subsequence on every $[-T_n, T_n]$.

So let $\{u_{n_k}^1\}$ be a subsequence of $\{u_n\}$ that converges on $[-T_1, T_1]$. Consider this subsequence on $[-T_2, T_2]$ and select a further subsequence $\{u_{n_k}^2\}$ of $\{u_{n_k}^1\}$

that converges uniformly on $[-T_2, T_2]$. Repeat this procedure for all n, and then take a diagonal sequence $\{u_{n_k}\}$, which consists of $u^1_{n_1}, u^2_{n_2}, u^3_{n_3}, \dots$. Since the diagonal sequence is a subsequence of $\{u^p_{n_k}\}$ for any $p \geq 1$, it follows that it converges uniformly on any bounded interval to a function $u(x)$. Expressing u''_{n_k} from the equation (2.13), we conclude that the sequence $\{u''_{n_k}\}$, and then also $\{u'_{n_k}\}$, converge uniformly on bounded intervals. Writing

$$u_{n_k}(x) = \int_a^x (x-\xi) u''_{n_k}(\xi)\, d\xi \quad \text{with} \quad a = -T_{n_k} - 1\,,$$

we conclude that $u(x) \in C^2(-\infty, \infty)$, and that $u''_{n_k} \to u''$ uniformly on bounded intervals. Hence, we can pass to the limit in the equation (2.13), and we conclude that $u(x)$ solves (2.1). By (2.15) $u(x)$ is not identically zero.

Writing

$$u^2(x) = \int_0^x 2uu'\, dx + u'^2(0)\,,$$

we conclude that the limits of $u(x)$ as $x \to \pm\infty$ exist ($uu' \in L^1(-\infty, \infty)$). The only possibility is $u(\pm\infty) = 0$. Next we notice from (2.1) that

$$|u''(x)| < c \quad \text{for all real } x \text{ and some } c > 0 \quad . \tag{2.16}$$

We claim that $u'(\pm\infty) = 0$. If not, there is an $\varepsilon > 0$ and a sequence of $x_n \to \infty$, such that

$$|u'(x_n)| \geq \varepsilon \quad \text{for all} \quad n\,.$$

By (2.16) we can find a δ, such that

$$|u'(x)| \geq \frac{\varepsilon}{2} \quad \text{for all } x \in (x_n - \delta, x_n + \delta) \text{ and all } n\,.$$

This implies that $\int_{-T_{n_k}}^{T_{n_k}} u'^2(x)\, dx$ becomes large with k, say, $\int_{-T_{n_k}}^{T_{n_k}} u'^2(x)\, dx \geq 2c$ for some k, where c is the constant from (2.14). Then by fixing $j > k$ so large that $u_{n_j}(x)$ and $u'_{n_j}(x)$ are uniformly close to $u(x)$ and $u'(x)$ respectively on the interval $(-T_{n_k}, T_{n_k})$ we get, using (2.14),

$$\int_{-T_{n_k}}^{T_{n_k}} u'^2\, dx \simeq \int_{-T_{n_k}}^{T_{n_k}} u'^2_{n_j}\, dx \leq \int_{-T_{n_j}}^{T_{n_j}} u'^2_{n_j}\, dx \leq c\,,$$

which is a contradiction. So that $u(x)$ is a solution of our problem (2.1-2.2).

Since by Lemma 2.1 the functions $u_{n_k}(x)$ are even, with the only maximum at $x = 0$, the same is true for their limit $u(x)$. That $u'(x) < 0$ for $x > 0$ is easily seen by differentiating (2.1) (a similar argument can be found in [2]).

Turning to uniqueness, let $v(x)$ be another positive solution of (2.1), (2.2) (which is also an even function with the only maximum at $x = 0$, as follows by an easy modification of the proof of lemma 1 in [2]). Since

$$\int_{-\infty}^{\infty} b(x) uv(u^2 - v^2)\, dx = 0\,,$$

it follows that the solutions $u(x)$ and $v(x)$ cannot be ordered, and so have to intersect. Two cases are possible: either $u(x)$ and $v(x)$ have at least two positive points of intersection, or only one positive point of intersection. Assume first $\xi_1 > 0$ is the smallest positive point of intersection and $\xi_2 > \xi_1$ the next one, and $u(x) < v(x)$ on (ξ_1, ξ_2). Multiply the equation (2.1) by u' and integrate from ξ_1 to ξ_2. Denoting by $x = x_1(u)$ the inverse function of $u(x)$ on (ξ_1, ξ_2), we obtain denoting $f(x,u) = -a(x)u + b(x)u^3$, and $u_1 = u(\xi_1) = v(\xi_1)$, $u_2 = u(\xi_2) = v(\xi_2)$,

$$\frac{1}{2}u'^2(\xi_2) - \frac{1}{2}u'^2(\xi_1) + \int_{u_1}^{u_2} f(x_1(u), u)\, du = 0\,. \tag{2.17}$$

Doing the same for $v(x)$, and denoting its inverse on (ξ_1, ξ_2) by $x = x_2(v)$, we obtain

$$\frac{1}{2}v'^2(\xi_2) - \frac{1}{2}v'^2(\xi_1) + \int_{u_1}^{u_2} f(x_2(v), v)\, dv = 0\,. \tag{2.18}$$

Subtracting (2.18) from (2.17),

$$\begin{aligned} &\frac{1}{2}\left(u'^2(\xi_2) - v'^2(\xi_2)\right) + \frac{1}{2}\left(v'^2(\xi_1) - u'^2(\xi_1)\right) \\ &\quad + \int_{u_2}^{u_1} \left[f(x_2(u), u) - f(x_1(u), u)\right] du = 0\,. \end{aligned} \tag{2.19}$$

Notice, $u_2 < u_1$ and $x_2(u) > x_1(u)$ for all $u \in (u_2, u_1)$. Keeping in mind that $f(x,u)$ is decreasing in x, and using uniqueness theorem for initial value problems, we conclude that all three terms on the left in (2.19) are negative. This is a contradiction, which rules out the case of two positive intersection points. If ξ_1 is the only intersection point, we integrate from ξ_1 to ∞, obtaining a similar contradiction. Uniqueness of solution follows, completing the proof of the theorem.

3 Homoclinic orbits for a class of Hamiltonian systems

We are looking for nontrivial solutions $u \in H^1(R^n, R)$ of the system

$$u'' - L(t)u + \nabla V(u) = 0 \tag{3.1}$$

$$u(\pm\infty) = u'(\pm\infty) = 0\,. \tag{3.2}$$

We assume that $V \in C^1(R^n, R)$, and the following conditions

$$\begin{aligned} &L(t) \text{ is a positive definite matrix with entries of class } C^1(R), \\ &\text{and } L(-t) = L(t) \text{ for all } t\,, \end{aligned} \tag{3.3}$$

$$(L'(t)\xi,\xi) \geq 0 \quad \text{for all } \xi \in R^n \text{ and } t \geq 0\,, \tag{3.4}$$

$$0 < \gamma V(\xi) \leq (\nabla V(\xi),\xi) \quad \text{for some constant } \gamma > 2 \text{ and all } \xi \in R^n\,. \tag{3.5}$$

Theorem 3.1 *Under assumptions (3.3-3.5) the problem (3.1-3.2) has a non-trivial solution $u(t)$, with $u(-t) = u(t)$ for all t.*

We postpone the proof of the theorem, and present two examples, which show that our result on scalar equations cannot be expected to carry over to systems.

Example 1 On some interval (a,b) consider a system

$$\begin{aligned} u'' - 2u + u(u^2+v^2) = 0 \quad &\text{for} \quad x \in (a,b), \quad u(a) = u(b) = 0 \\ v'' - v + v(u^2+v^2) = 0 \quad &\text{for} \quad x \in (a,b), \quad v(a) = v(b) = 0\,. \end{aligned} \tag{3.6}$$

This system has no positive solution (i.e. solution with $u > 0$ and $v > 0$ on (a,b)). Indeed, we can regard the first equation in (3.6) as a linear equation of the form $u'' + c(x)u = 0$, and the second one as $v'' + d(x)v = 0$. Since $d(x) > c(x)$, the claim follows by the Sturm's comparison theorem. This system is of type (3.1) with $V = \frac{1}{4}\left(u^4+v^4\right) + \frac{1}{2}u^2v^2$.

Example 2 The problem

$$\begin{aligned} u'' - u + u(u^2+v^2) = 0 \quad &\text{for} \quad x \in (a,b), \quad u(a) = u(b) = 0 \\ v'' - v + v(u^2+v^2) = 0 \quad &\text{for} \quad x \in (a,b), \quad v(a) = v(b) = 0\,, \end{aligned} \tag{3.7}$$

has infinitely many positive solutions, all of the form $u = \alpha v$, where α is an arbitrary positive constant. Indeed, regarding u^2+v^2 as a known function, we see that u and v are positive solutions of the same linear equation, and so have to be multiples of one another. Setting $u = \alpha v$, we find u to be the unique (in view of Lemma 2.2) positive solution of

$$u'' - u + u^3\left(1 + \frac{1}{\alpha^2}\right) = 0 \quad \text{for} \quad x \in (a,b), \quad u(a) = u(b) = 0\,, \tag{3.8}$$

while v is the unique positive solution of

$$v'' - v + v^3(\alpha^2+1) = 0 \quad \text{for} \quad x \in (a,b), \quad v(a) = v(b) = 0\,. \tag{3.9}$$

Setting $u = \alpha v$ in (3.8), we obtain (3.9), so that the pair (u,v) is indeed a solution of (3.7).

Proof of the Theorem 3.1. We begin by showing that our condition (3.5) implies that

$$\frac{V(\xi)}{|\xi|^2} \to 0 \quad \text{as} \quad |\xi| \to 0\,. \tag{3.10}$$

Indeed, write (3.5) at $r\xi$ with some constant $r > 0$,

$$r(\nabla V(r\xi), \xi) \geq \gamma V(r\xi)$$

or

$$\frac{d}{dr}V(r\xi) - \frac{\gamma}{r}V(r\xi) \geq 0\,.$$

Multiplying by $r^{-\gamma}$ and integrating over $(\epsilon, 1)$, $0 < \epsilon < 1$,

$$V(\xi) - \frac{V(\epsilon\xi)}{\epsilon^{\gamma}} \geq 0\,.$$

Let now $|\xi| = 1$ and set $\eta = \epsilon\xi$, $|\eta| = \epsilon$. Then

$$\frac{|V(\eta)|}{|\eta|^{\gamma}} \leq c\,,$$

and (3.10) follows.

As in the previous section, we approximate the solution of (3.1-3.2) by the problem

$$u'' - L(t)u + \nabla V(u) = 0 \text{ for } t \in (-T, T),\ u(-T) = u(T) = 0\,, \tag{3.11}$$

$$u(-t) = u(t) \quad \text{for all real } t\,. \tag{3.12}$$

The key step is to show the existence of $\delta > 0$, such that any nontrivial solution of (3.11), (3.12) satisfies

$$|u(0)| > \delta \quad \text{independently of } T > 0\,. \tag{3.13}$$

To prove (3.13) we introduce the "energy" function for the solution $u(t)$ of (3.11),

$$E(t) = \frac{1}{2}|u'(t)|^2 - \frac{1}{2}(L(t)u, u) + V(u(t))\,.$$

Differentiating $E(t)$, and using the equation (3.11) and the condition (3.4), we express

$$E'(t) = -\frac{1}{2}(L'(t)u, u) \leq 0 \quad \text{for all} \quad 0 \leq t \leq T\,,$$

and hence

$$E(0) \geq E(T) = \frac{|u'(T)|^2}{2} \geq 0\,.$$

Since $u(t)$ is even, $u'(0) = 0$, and then

$$E(0) = V(u(0)) - \frac{1}{2}(L(0)u(0), u(0)) \geq 0\,,$$

or

$$V(u(0)) \geq \frac{1}{2}(L(0)u(0), u(0)) \geq c|u(0)|^2\,,$$

$$\frac{V(u(0))}{|u(0)|^2} \geq c\,. \tag{3.14}$$

Comparing (3.14) with (3.10), we conclude the estimate (3.13).

The rest of the proof is the same as that of the Theorem 2.1, except that we use (3.13) instead of (2.15), so we only sketch it. We take a sequence $\{T_k\} \to \infty$ as $k \to \infty$, $0 < T_k < T_{k+1}$ for all k. By E_k we denote the subspace of $H_0^1(-T_k, T_k)$, consisting of even functions. By taking zero extensions, we see that $E_k \subset E_{k+1}$. We consider the functionals $f_k : E_k \to R$, defined by

$$f_k(u) = \int_{-T_k}^{T_k} \left[\frac{1}{2}|u'|^2 + \frac{1}{2}(L(t)u, u) - V(u)\right] dt\,.$$

The mountain pass theorem applies to $f_k(u)$, producing $u_k \in E_k$, which is an even nontrivial solution of

$$u_k'' - L(t)u_k + \nabla V(u_k) = 0 \quad \text{for} \quad t \in (-T_k, T_k) \tag{3.15}$$

$$u_k(-T) = u_k(T) = 0\,. \tag{3.16}$$

The critical values $c_k = f_k(u_k) > 0$ are non-increasing in k. And $u_k(0) > \delta$ uniformly in k, in view of (3.13).

We show next that the H^1 norm of the solution of (3.15-3.16) is bounded uniformly in k. Multiply the equation (3.15) by u_k and integrate,

$$\int_{-T_k}^{T_k} \left[|u_k'|^2 + (L(t)u_k, u_k) - (\nabla V(u_k), u_k)\right] dt \;= 0\,. \tag{3.17}$$

On the other hand by the definition of c_k and (3.5),

$$\begin{aligned}\int_{-T_k}^{T_k} \left[\frac{1}{2}|u_k'|^2 + \frac{1}{2}(L(t)u_k, u_k)\right] dt \;&= \int_{-T_k}^{T_k} V(u_k)\, dt \;+ c_k \\ &\leq \frac{1}{\gamma}\int_{-T_k}^{T_k} (\nabla V(u_k), u_k)\, dt \;+ c_k\,.\end{aligned} \tag{3.18}$$

Using (3.17) in (3.18), we obtain

$$\left(\frac{1}{2} - \frac{1}{\gamma}\right)\int_{-T_k}^{T_k} \left[|u_k'|^2 + (L(t)u_k, u_k)\right] dt \;\leq c_k \leq c_1\,. \tag{3.19}$$

Since $\frac{1}{2} - \frac{1}{\gamma} > 0$, and

$$(L(t)u_k, u_k) \geq (L(0)u_k, u_k) \geq c|u_k|^2\,,$$

we conclude from (3.19) that

$$\int_{-T_k}^{T_k} \left(|u_k'|^2 + |u_k|^2\right) dt \;\leq c \quad \text{uniformly in } k\,.$$

The rest of the proof is the same as in the Theorem 2.1.

Remark. We remark that we can prove a similar result with V depending on t, provided the condition (3.5) is uniform in t, and (3.4) is replaced by

$$(L'(t)\xi, \xi) - V_t(t, \xi) \geq 0 \text{ for all } \xi \in R^n \text{ and } t \geq 0 .$$

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