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A CLASSICAL SOLUTION WEAKENED ON THE AXIS FOR A MIXED PROBLEM OF INHOMOGENEOUS HYPERBOLIC EQUATIONS

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Abstract. The main purpose of this paper is to present sufficient conditions, on the forcing term of a mixed problem for three dimensional hyperbolic equations of any even order, for the existence of axially weakened classical solutions.

1. Introduction

Let $r=\sqrt{x_1^2+x_2^2+x_3^2}$ and $G=\{x\in\mathbb{R}^3 : 0<r<R<\infty\}$. On the cylinder $\overline{Q}=\overline{G}\times[0,T]$, we consider the mixed problem

$$\prod_{k=1}^{m}\big(\frac{\partial^2}{\partial t^2}-a_k^2\Delta\big)u+F(r,t;u)=f(r,t) \tag{1.1}$$

$$\frac{\partial^p u}{\partial t^p}\big|_{t=0}=\phi_p(r),\quad 0\le p\le 2m-1 \tag{1.2}$$

$$\big(\frac{\partial\Delta^k u}{\partial\nu}+\frac{1}{r}\Delta^k u\big)\big|_{\Gamma}=0,\quad 0\le k\le m-1\,, \tag{1.3}$$

where Γ denotes the lateral surface of the cylinder Q, $\frac{\partial}{\partial\nu}$ is the partial derivative with respect to the normal ν which is outward to the lateral surface Γ of the cylinder Q, the reals $a_1,\dots,a_n$ are different, $\Delta=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}$ is the Laplacian, $\Delta_r\varphi=\frac{\partial^2\varphi}{\partial r^2}+\frac{2}{r}+\frac{\partial\varphi}{\partial r}$, and

$$\begin{aligned} F(r,t;u)=\sum_{p+q<m}\big[b_{p,q}(r,t)\sum_{i=1}^{3}x_i\frac{\partial^{2p+1}\Delta^q u}{\partial t^{2p}\partial x_i}\\ +C_{p+q}(r,t)\frac{\partial^{2p+1}\Delta^q u}{\partial t^{2p+1}}+d_{p,q}(r,t)\frac{\partial^{2p}\Delta^q u}{\partial t^{2p}}\big].\end{aligned} \tag{1.4}$$

We assume that

$$b_{p,q}(R,t)=0 \tag{1.5}$$

which is the consistency condition. We also assume that the coefficients $b_{p,q}$, $C_{p,q}$, $d_{p,q}$ are continuous in $\overline{Q}$ and their derivatives $\frac{\partial b_{p,q}}{\partial r}$, $\frac{\partial C_{p,q}}{\partial r}$, $\frac{\partial d_{p,q}}{\partial r}$ are bounded in $\overline{Q}$.

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The central symmetry of the problem with respect to the variable x is reflected by the dependence of the data on it, in the invariance of the Laplacian with respect to rotation, and in the geometry of G. Before we state our results, let us dwell a moment on the existing literature concerning similar problems. First, we recall known facts concerning hyperbolic equations; as it is well known, hyperbolic equations do not keep smoothness. So, the space of continuous functions is badly adapted for investigating them; it is not suitable in the sense that for more than one space variable, we can not establish the agreement of necessary and sufficient conditions imposed on the initial functions for the existence of the classical solutions as it was shown by Sobolev [10] for the Cauchy problem of the wave equation. In contrast, Sobolev spaces are more adapted for investigating hyperbolic equations. But if the dimension is greater than the order of the operator, then solutions in Sobolev spaces may be discontinuous even at any point. Therefore, it arises the question of how to use the good quality of the spaces of continuous functions. Naturally, arises the question to localize the set of irregular points of the solutions. We will then make some compromise, we weaken the requirements on the sought for solutions; As, in our situation, they are classical except on the axis of the cylinder, we will impose some conditions there, and call the new defined solutions, *weakened on the axis*.

Classical solution of mixed problems for hyperbolic equations has been the subject of extensive research. However, their common drawback, is that the sufficient conditions, imposed on the initial functions for the existence of a classical solution, disagree with the necessary ones, see, for example, the works of Steklov [11] and Petrovsky [9].

This limitation has been eliminated for one-dimensional hyperbolic equations; an agreement between necessary and sufficient conditions for the initial functions has been established in the case of mixed problems for the telegraph equation by Chernyatin [5,6] and Egorov [7]. Baranowskaya [2] extended this result to more general one-dimensional hyperbolic equations of second order. Also, it has been extended to one-dimensional hyperbolic equations of any even order in the work of Koku and Yurchuk [4].

For multidimensional hyperbolic equations, the situation is much more delicate. All the known sufficient conditions for the existence of classical solutions contain additional conditions for the initial functions. Moreover, Sobolev [10] proved that the requirements for the initial-value (Cauchy) problem for the wave equation could not be reduced by one derivative. The solution at the point $r = 0$ requires that $\Phi \in C^3$ and $\Psi \in C^2$ in the case of central symmetry. Besov [3] arrived at similar results in terms of what is called now Besov spaces. An agreement between necessary and sufficient conditions for the initial functions has been established in the case of mixed problems for homogeneous three-dimensional hyperbolic equations of any even order in the case of central symmetry by Al-Momani [1] and Al-Momani and Yurchuk [12].

By $C^{2m}_{\{|x|=0\}}(\overline{Q})$ we denote the set of functions $u \in C^1(\overline{Q}) \cap C^{2m}(\overline{Q}\backslash\{0\} \times [0,T])$ satisfying the conditions

$$\frac{\partial^p \Delta^n u}{\partial t^p} \in C^1(\overline{Q}), \quad p+2n < 2m-1 \tag{1.6}$$

$$\frac{\partial^{p+2} \Delta^n u}{\partial t^p \partial r^2} \in C(\overline{Q}), \quad p+2n < 2m-2 \tag{1.7}$$

$$\lim_{r\to 0} \sum_{i=1}^{3} x_i \frac{\partial^{p+1} \Delta^n u(x,t)}{\partial t^p \partial x_i} = 0, \quad p+2n = 2m-1 \tag{1.8}$$

$$\lim_{r\to 0} r \frac{\partial^p \Delta^n u(x,t)}{\partial t^p}, \quad p+2n = 2m\,. \tag{1.9}$$

Definition. The *weakened on the axis* $r=0$ classical solution in $\overline{Q}$ of the problem (1.1)-(1.3) is a function in $C^{2m}_{\{|x|=0\}}(\overline{Q})$, which satisfies equation (1.1) pointwise in $\overline{Q}\backslash\{0\} \times [0,T]$ and satisfies the conditions (1.2), (1.3) in the usual sense.

Next, we state a result from [1] which provides the existence of the *weakened on the axis* classical solution in $\overline{Q}$ of our problem (1.1)-(1.3) with $f(r,t)=0$ in (1.1).

Theorem 1.1. *Let the coefficients $b_{p,q}$, $C_{p,q}$, $d_{p,q}$ be continuous in $\overline{Q}$ and have bounded first partial derivatives in $\overline{Q}$ with respect to r and the condition (1.5) holds. The necessary and sufficient conditions for the existence of the weakened on the axis classical solution in $\overline{Q}$ of the problem*

$$\prod_{k=1}^{m} \big(\frac{\partial^2}{\partial t^2} - a_k^2 \Delta\big) u + F(r,t;u) = 0$$

with (1.2) and (1.3) are the following conditions on the initial functions ϕ_p:

$$\phi_p \in C^{2m-p}(0,R] \tag{1.10}$$

$$\Delta_r^n \phi_p \in C^2[0,R], \quad p+2n < 2m-2 \tag{1.11}$$

$$\Delta_r^n \phi_p \in C^1(0,R], \quad p+2n < 2m-1 \tag{1.12}$$

$$\lim_{r\to 0} r \frac{d}{dr} \Delta_r^n \phi_p(r) = 0, \quad p+2n < 2m-1 \tag{1.13}$$

$$\lim_{r\to 0} r \Delta_r^n \phi_p(r) = 0, \quad p+2n = 2m \tag{1.14}$$

$$\big[\frac{d}{dr} \Delta_r^n \phi_p(r) + \frac{1}{r} \Delta_r^n \phi_p(r)\big]\big|_{r=R} = 0, \quad p+2n < 2m \tag{1.15}$$

2. Main result

Theorem 2.1. *Sufficient conditions for the existence of the weakened on the axis classical solution in $\overline{Q}$ of problem* (1.1)-(1.3) *are given by* (1.10)-(1.15) *on the initial functions ϕ_p, and the following conditions on the function f:*

$$f \in C(\overline{Q}\backslash\{0\} \times [0,T]), \tag{2.1}$$

$$\lim_{r\to 0} r f(r,t) = 0, \quad 0 \le t \le T \tag{2.2}$$

$$\int_0^R r^2 \big|\frac{\partial f(r,t)}{\partial r}\big|^2 dr \in C[0,T]. \tag{2.3}$$

Proof. We transform (1.1)-(1.3) to the spherical coordinates and set $\nu(r,t) = ru(r,t)$ and use the equality

$$\Delta_r \frac{\nu(r,t)}{r} = \frac{1}{r}\frac{\partial^2 \nu(r,t)}{\partial r^2}.$$

Then the function $\nu(r,t)$ satisfies

$$\prod_{k=1}^{m} \big(\frac{\partial^2}{\partial t^2} - a_k^2 \frac{\partial^2}{\partial r^2}\big)\nu + F(r,t;\nu) = f_1(r,t) \tag{2.4}$$

with the initial and boundary conditions

$$\frac{\partial^p u}{\partial t^p}|_{t=0} = \Phi_p(r), \Phi_p(r) = r\phi_p(r), \quad 0 \le p \le 2m-1 \tag{2.5}$$

$$\frac{\partial^{2k}\nu}{\partial r^{2k}}|_{r=0} = \frac{\partial^{2k+1}\nu}{\partial r^{2k+1}}|_{r=R} = 0, \quad 0 \le k \le m-1\,, \tag{2.6}$$

where

$$\begin{aligned} F(r,t;\nu) = \sum_{p+q<m} & b_{p,q}(r,t) r \frac{\partial^{2p+2q+1}\nu}{\partial t^{2p}\partial r^{2q+1}} \\ & + C_{p,q}(r,t)\frac{\partial^{2p+2q+1}\nu}{\partial t^{2p+1}\partial r^{2q}} + \big(d_{p,q}(r,t) - b_{p,q}(r,t)\big)\frac{\partial^{2p+2q}\nu}{\partial t^{2p}\partial r^{2q}}\big] \end{aligned}$$

and the functions $\Phi_p(r) = r\phi_p(r)$ satisfy the conditions

$$\begin{gathered} \Phi_p \in C^{2m-p}[0,R], \quad \Phi_p^{(2n)}(0) = 0, \quad 0 \le 2n \le 2m-p \\ \Phi_p^{(2k+1)}(R) = 0, \quad 0 \le 2k+1 \le 2m-p\,. \end{gathered}$$

The function $f_1(r,t) = rf(r,t)$ satisfies the conditions

$$\begin{gathered} f_1 \in C\left([0,R]\times[0,T]\right), \quad f_1(0,t) = 0 \\ \int_0^R |\frac{\partial f_1(r,t)}{\partial r}|^2 dr \in C[0,T]\,. \end{gathered}$$

As is shown in [4], under the conditions (2.1)-(2.3), problem (2.4)-(2.6) has a classical solution $\nu \in C^{2m}(\overline{Q})$ which satisfies

$$\begin{aligned} &\nu(r,t) \\ &= \frac{1}{2}\sum_{1\le k\le m} a_k^p \Big\{ \sum_{0\le i\le 2m-1} \alpha_{i,k}\big[\overline{\Phi}_i^{(-i)}(r+a_k t) + (-1)^i \overline{\Phi}_i^{(-i)}(r-a_k t)\big] \\ &\quad - \alpha_{2m-1,k}\int_0^t \big[\overline{F}^{(1-2m)}(r+a_k(t-\tau),\tau;\nu) - \overline{F}^{(1-2m)}(r+a_k(t-\tau),\tau;\nu)\big]d\tau \\ &\quad + \alpha_{2m-1,k}\int_0^t \big[\overline{f}^{(1-2m)}(r+a_k(t-\tau),\tau) - \overline{f}_1^{(1-2m)}(r-a_k(t-\tau),\tau)\big]d\tau \Big\}\,. \end{aligned} \tag{2.7}$$

Here,

$$\overline{f}^{(-k)}(r,\tau) = \int_0^r \frac{(r-\zeta)^{k-1}}{(k-1)!}\overline{f}_1(\zeta,\tau)d\tau \tag{2.8}$$

$$\overline{f}_1^{(k)}(r,F) = \frac{\partial^k}{\partial r^k}\overline{f}_1(r,\tau), \quad k \ge 0 \tag{2.9}$$

we denote by $\overline{f}_1$ the $4R$-periodic continuation with respect to r of the function $f_1(r,t)$ from $\overline{Q}$ to $R^1 \times [0,T]$ which is constructed in the same way as $\overline{\Phi}$ and $\overline{F}$ in [1]. Let

$$\alpha_{i,k} = \begin{cases} 1, & \text{if } i = k \\ 0, & \text{if } i \neq k\,. \end{cases}$$

Using (2.7), we will show that the function $u(x,t) = \frac{\nu(r,t)}{r}$ is the *weakened on the axis* classical solution in $\overline{Q}$ of the problem (1.1)-(1.3). By substitution we can check that the function u satisfies at $r = 0$ the equation (1.1), initial conditions (1.2) and boundary conditions (1.3). Therefore, to complete the proof of our theorem it remains to prove that

$$u(x,t) = \frac{\nu(r,t)}{r}, \quad u \in C^{2m}_{\{r=0\}}(\overline{Q})$$

since $\nu \in C^{2m}(\overline{Q})$ the function $u = \frac{\nu}{r} \in C^{2m}(\overline{Q}\backslash\{0\} \times [0,T])$. Now we must show that the function u satisfies the conditions (1.6)-(1.9).

Equation (2.7) implies that $\frac{\partial^{p+q}\nu}{\partial t^p \partial r^q}$ satisfies

$$\begin{aligned} \frac{\partial^{p+q}\nu}{\partial t^p \partial r^q} = \frac{1}{2} \sum_{1\le k\le m} a_k^p \Big\{ &\sum_{0\le i\le 2m-1} \alpha_{i,k} \big[\overline{\Phi}_i^{(p+q-i)}(r+a_k t) \\ &+ (-1)^{p+i}\overline{\Phi}_i^{(p+q-i)}(r-a_k t)\big] \\ &- \alpha_{2m-1,k} \int_0^t \big[\overline{F}^{(p+q+1-2m)}(r+a_k(t-\tau),\tau;\overline{\nu}) \\ &+ (-1)^{p+1}\overline{F}^{(p+q+1-2m)}(r-a_k(t-\tau),\tau;\overline{\nu})\big]d\tau \\ &+ \alpha_{2m-1,k} \int_0^t [\overline{f}_1^{(p+q+1-2m)}(r+a_k(t-\tau),\tau) \\ &+ (-1)^{p+1}\overline{f}_1^{(p+q+1-2m)}(r-a_k(t-\tau),\tau)]d\tau \Big\}\,. \end{aligned}$$

The continuations $\overline{\Phi}_i$, $\overline{\nu}$, $\overline{F}$ and $\overline{f}_1$ satisfy the following conditions:

$$\overline{\Phi}_i^{(p+q-i)}(r \pm a_k t) = \sum_{j=0}^{2m-p-q} \overline{\Phi}_i^{(p+q-i+j)}(\pm a_k t)\frac{r^j}{j!} + \theta(r^{2m-p-q}) \tag{2.10}$$

where $\theta(r^{2m-p-q})$ means a function $g(r)$ such that $\lim_{r\to 0} g(r)/(r^{2m-p-q}) = 0$,

$$\begin{aligned} &\int_0^t \overline{F}^{(p+q+1-2m)}(r \pm a_k(t-\tau),\tau;\overline{\nu})d\tau \\ &= \sum_{j=0}^{2m-p-q} \int_0^t \overline{F}^{p+q+1-2m+j}(\pm a_k(t-\tau),\tau;\overline{\nu})d\tau\frac{r^j}{j!} + \theta(r^{2m-p-q})\,, \end{aligned} \tag{2.11}$$

and

$$\overline{f}_1 \in C(R^1 \times [0,T]), \quad \int_0^t \overline{f}_1(r \pm a_k(t-\tau),\tau)d\tau \in C^1(R^1 \times [0,T])\,. \tag{2.12}$$

By the Taylor formula with remainder in Peano form [8],

$$\begin{aligned}&\int_0^t [\overline{f}_1^{(p+q+1-2m)}(r \pm a_k(t-\tau),\tau)d\tau \\ &= \sum_{j=0}^{2m-p-q} \int_0^t \overline{f}_1^{(p+q+1-2m)}(\pm a_k(t-\tau),\tau)\frac{r^j}{j!} + \theta(r^{2m-p-q})\end{aligned} \tag{2.13}$$

and

$$\overline{f}_1^{(-k)}(r,\tau) = (-1)^{1+k}\overline{f}_1^{(-k)}(r,t). \tag{2.14}$$

From (2.3) and

$$\frac{\partial u(x,t)}{\partial x_i} = \frac{x_i}{r}\frac{\partial u(r,t)}{\partial r}$$

and on the basis of (2.10)-(2.14), we have

$$\begin{aligned}\frac{\partial^p \Delta^n u}{\partial t^p} &= \frac{1}{2r}\sum_{1\le k\le m} a_k^p\Big\{\sum_{j=0}^{2m-2n-p}(1-(-1)^j)\Big[\sum_{0\le i\le 2m-1}\alpha_{i,k}\overline{\Phi}_i^{(2n+p-i+j)}(a_k t) \\ &- \alpha_{2m-1,k}\int_0^t \overline{F}^{(2n+p+1-2m+j)}(a_k(t-\tau),\tau;\overline{\nu})d\tau \\ &+ \alpha_{2m-1,k}\int_0^t \overline{f}_1^{(2n+p+1-2m+j)}(a_k(t-\tau),\tau)d\tau\Big]\frac{r^j}{j!}\Big\} + \theta(r^{2m-2n-p}),\end{aligned} \tag{2.15}$$

$$\begin{aligned}\frac{\partial^{p+1}\Delta^n u}{\partial t^p \partial x_i} &= \frac{x_i}{2r^2}\sum_{1\le k\le m}^{p} a_k^p\Big\{\sum_{j=0}^{2m-2n-p-1}(1+(-1)^j)(\frac{1}{j!}-\frac{1}{(j+1)!}) \\ &\times\Big[\sum_{0\le i\le 2m-1}\alpha_{i,k}\overline{\Phi}_i^{(2n+p+1-i+j)}(a_k t) \\ &- \alpha_{2m-1,k}\int_0^t \overline{F}^{(2n+p+2-2m+j)}(a_k(t-\tau),\tau;\overline{\nu})d\tau \\ &+ \alpha_{2m-1,k}\int_0^t \overline{f}_1^{(2n+p+2-2m+j)}(a_k(t-\tau),\tau)d\tau\Big]\frac{r^j}{j!}\Big\} + \theta(r^{2m-2n-p-1})\end{aligned} \tag{2.16}$$

Equalities (2.15) and (2.16) show that

$$\begin{aligned}\lim_{r\to 0}\frac{\partial^p \Delta^n u}{\partial t^p} &= \sum_{1\le k\le m} a_k^p\Big[\sum_{0\le i\le 2m-1}\alpha_{i,k}\overline{\Phi}_i^{(2n+p-i+1)}(a_k t) \\ &- \alpha_{2m-1,k}\int_0^t \overline{F}^{(2n+p+2-2m)}(a_k(t-\tau),\tau,\overline{\nu})d\tau \\ &+ \alpha_{2m-1,k}\int_0^t \overline{f}_1^{(2n+p+2-2m)}(a_k(t-\tau),\tau)d\tau\Big]\end{aligned} \tag{2.17}$$

and

$$\lim_{r\to 0}\frac{\partial^{p+1}\Delta^n u}{\partial t^p \partial x_i} = 0\,.$$

Therefore, condition (1.6) holds. Equation (2.17) also implies that

$$\begin{aligned}\lim_{r\to 0}\frac{1}{r}\frac{\partial^{p+1}\Delta^n u}{\partial t^p\partial r} &= \lim_{r\to 0}\frac{1}{r^2}\sum_{i=1}^{3}x_i\frac{\partial^{p+1}\Delta^n u}{\partial t^p\partial x_i}\\ &= \frac{1}{3}\sum_{1\le k\le m}a_k^p\Big[\sum_{0\le i\le 2m-1}\alpha_{i,k}\overline{\Phi}_i^{(2n+p+3-i)}(a_kt)\\ &\quad-\alpha_{2m-1,k}\int_0^t\overline{F}^{(2n+p+4-2m)}(a_k(t-\tau),\tau,\overline{\nu})d\tau\\ &\quad+\alpha_{2m-1,k}\int_0^t\overline{f}_1^{(2n+p+4-2m)}(a_k(t-\tau),\tau)d\tau\Big]\,.\end{aligned}$$

Replacing n by $n+1$ in equations (2.17) and (1.4) and taking into account that

$$\frac{\partial^{p+2}\Delta^n u}{\partial t^p\partial r^2}=\frac{\partial^p\Delta^{n+1}u}{\partial t^p}-\frac{2}{r}\frac{\partial^{p+1}\Delta^n u}{\partial t^p\partial r}$$

we obtain

$$\begin{aligned}\lim_{r\to 0}\frac{\partial^{p+2}\Delta^n u}{\partial t^p\partial r^2} &= \frac{1}{3}\sum_{1\le k\le m}a_k^p\Big[\sum_{0\le i\le 2m-1}\alpha_{i,k}\overline{\Phi}_i^{(2n+p+3-i)}(a_kt)\\ &\quad-\alpha_{2m-1,k}\int_0^t\overline{F}^{(2n+p+4-2m)}(a_k(t-\tau),\tau,\overline{\nu})d\tau\\ &\quad+\alpha_{2m-1,k}\int_0^t\overline{f}_1^{(2n+p+4-2m)}(a_k(t-\tau),\tau)d\tau\Big]\end{aligned}$$

and consequently condition (1.7) holds. From (2.16) with $p+2n=2m-1$, we have

$$\sum_{i=1}^{3}x_i\frac{\partial^{p+1}\Delta^n u}{\partial t^p\partial x_i}=\theta(r^o) \tag{2.18}$$

and from (2.15) with $p+2n=2m$, we have

$$r\frac{\partial^p\Delta^n u}{\partial t^p}=\theta(r^o)\,. \tag{2.19}$$

Since $\lim_{r\to 0}\theta(r^o)=0$, so passing in (2.18) and (2.19) to the limit as $r\to 0$, we show that the conditions (1.8) and (1.9) consequently hold. □

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