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PERIODIC SOLUTION AND GLOBAL EXPONENTIAL STABILITY FOR SHUNTING INHIBITORY DELAYED CELLULAR NEURAL NETWORKS

ANPING CHEN, JINDE CAO, & LIHONG HUANG

Abstract. For a class of neural system with time-varying perturbations in the time-delayed state, this article studies the periodic solution and global robust exponential stability. New criteria concerning the existence of the periodic solution and global robust exponential stability are obtained by employing Young's inequality, Lyapunov functional, and some analysis techniques. At the same time, the global exponential stability of the equilibrium point of the system is obtained. Previous results are improved and generalized. Our results are shown to be more effective than the existing results. In addition, these results can be used for designing globally stable and periodic oscillatory neural networks. Our results are easy to be checked and applied in practice.

1. Introduction

The dynamics of cellular neural networks(CNNs) and delayed cellular neural networks(DCNNs) have been investigated in recent years, due to their great potential in information processing systems. CNNs and DCNNs have been applied in solving problems such as image and signal processing, vision, pattern recognition and optimization. Many important results can be found in the references for this article.

It is known that the neural networks possess possibly three dynamic properties: convergence, oscillation and chaotic behavior. The first dynamic behavior has been widely studied, [1, 2, 8, 9, 10, 11, 12, 13, 14, 15, 16, 22, 25, 26, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40]. However, for the oscillator dynamic behavior, the study has stayed in a lower level. Only a few results have obtained in [7, 9, 10, 11, 12, 13, 14, 15]. As for the chaotic dynamic property, the research advances continue to be slow.

In this paper, we study a class of shunting inhibitory type DCNNs, which was first proposed by Bouzerdoum and Pinter [3]. It has been applied to psychophysics, speech, perception, robotics, adaptive pattern recognition, vision and image processing [3, 4, 5, 6, 24, 25, 29, 30, 34]. So its dynamic behavior research has an important significance for theory and applications.

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We consider a two-dimensional grid of processing cells. Let C_{ij} denote the cell at the (i,j) position of the lattice, and let r-neighborhood $N_r(i,j)$ of C_{ij} be

$$N_r(i,j) = \{C_{kl} | \max |k-i|, |l-j| \le r, 1 \le k \le m; 1 \le l \le n\}.$$

In SICNNs, neighboring cells exert mutual inhibitory interaction of the shunting type. The dynamics of a cell C_{ij} are described by the following nonlinear ordinary differential equation [6],

$$\frac{dx_{ij}}{dt} = -a_{ij}x_{ij}(t) - \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} f(x_{kl}(t))x_{ij} + L_{ij}(t).$$

In the system above, x_{ij} is the activity of the cell C_{ij}, L_{ij} is the external input to C_{ij}, the constant $a_{ij} > 0$ represents a passive delay rate of the cell activity, $c_{ij}^{kl} \ge 0$ is the connection or coupling strength of postsynaptic activity of the cell transmitted to the cell C_{ij}, and the activation function $f(x_{kl})$ is a positive continuous function, representing the output or firing rate of the cell C_{kl}. When L_{ij} is a constant, the power and stability of the system have been researched in [3, 4, 5, 6, 24, 25, 29, 30, 34]. In [13, 40], we have studied the existence and global stability of almost periodic solution for the system above with delays. However, to best our knowledge, the periodic solution and global exponential stability are seldom discussed for the model. In this paper, we introduce the delays into the system, and consider the periodic solution and exponential stability of the SIDCNNs

$$\frac{dx_{ij}}{dt} = -a_{ij}x_{ij}(t) - \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} f_{kl}(x_{kl}(t-\tau_{kl}))x_{ij} + L_{ij}(t), \tag{1.1}$$

where $a_{ij} > 0$, $c_{ij}^{kl} \ge 0$. Here $L_{ij}(t)$ is a continuous periodic functions with period ω; i.e., $L_{ij}(t+\omega) = L_{ij}(t), \forall t \in \mathbb{R}$.

We assume that the nonlinear system (1.1) satisfies the initial conditions

$$x_{ij}(s) = \varphi_{ij}(s), \quad s \in [-\tau, 0], \tag{1.2}$$

where $\tau = \max_{(i,j)}\{\tau_{ij}\}$, $\varphi = (\varphi_{11}, \dots, \varphi_{ij}, \dots, \varphi_{mn})^T \in C([-\tau,0], \mathbb{R}^{m\times n})$. The solution of system (1.1) through $(0,\varphi)$ is denoted by

$$x(t,\varphi) = (x_{11}(t,\varphi), \dots, x_{ij}(t,\varphi), \dots, x_{mn}(t,\varphi))^T.$$

Define $x_t(\varphi) = x(t+\theta, \varphi)$, $\theta \in [-\tau, 0]$, $t \ge 0$. Then $x_t(\varphi)$ is in $C = C([-\tau,0], \mathbb{R}^{m\times n})$ the Banach space of continuous functions which map $[-\tau, 0]$ into $\mathbb{R}^{m\times n}$ with topology of uniformly converge. The norm is defined as

$$\|x_t\|_p = \sup_{-\tau\le\theta\le 0} \Big(\sum_{(i,j)} |x_{ij}(t+\theta)|^p \Big)^{1/p},$$

in which $p \ge 1$. When $p = +\infty$, the ∞-norm is

$$\|x_t\|_\infty = \sup_{-\tau\le\theta\le 0} \max_{(i,j)} [|x_{t_{(i,j)}}(t+\theta)|].$$

We assume that the following conditions are satisfied:

(H1) The functions $f_{ij}(x)$ $(i = 1, 2, \dots, m; j = 1, 2, \dots, n)$ are positive on $\mathbb{R}$.

(H2) There is a constant $\mu_{ij} > 0$ such that

$$|f_{ij}(x) - f_{ij}(y)| \le \mu_{ij}|x-y|, \quad \text{for any } x, y \in \mathbb{R}.$$

For convenience, we set

$$M_f = \max_{(i,j)} \sup_{x\in\mathbb{R}} \{f_{ij}(x)\}, \quad p_{ij} = M_f \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl},$$

$$L_{ij} = \max_{t\in\mathbb{R}} |L_{ij}(t)|, \quad q_{ij} = \begin{cases} |\varphi_{ij}(0)|, & \text{if } |\varphi_{ij}(0)| \geq \frac{L_{ij}}{a_{ij}}, \\ \frac{L_{ij}}{a_{ij}}, & \text{if } |\varphi_{ij}(0)| < \frac{L_{ij}}{a_{ij}}. \end{cases}$$

When $a_{ij} \geq p_{ij}$, we define

$$N_{ij} = \begin{cases} \frac{a_{ij} q_{ij}}{a_{ij} - p_{ij}}, & \text{if } a_{ij} > p_{ij}, \\ q_{ij}, & \text{if } a_{ij} = p_{ij}. \end{cases}$$

The main results of this article are stated in the next theorems. To this end introduce the following assumptions

(H3) For $i = 1, 2, \dots, m; j = 1, 2, \dots, n$,

$$\begin{aligned} &- p a_{ij} + p M_f \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} + (p-1) \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} \mu_{kl} N_{ij} \\ &+ \sum_{(i,j)} \Big(\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} \mu_{kl} N_{ij} \Big) < 0 \,. \end{aligned}$$

Theorem 1.1. *Assume that the hypotheses (H1–(H3) are satisfied. Then system* (1.1) *has a unique ω-periodic solution and all other solution converge globally exponentially to this solution in the p-norm as $t \to +\infty$, where $p \geq 1$.*

(H4) For $i = 1, 2, \dots, m; j = 1, 2, \dots, n$,

$$-a_{ij} + M_f \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} + \sum_{(i,j)} \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} \mu_{kl} N_{ij} < 0 \,.$$

Corollary 1.2. *Assume that the hypotheses (H1), (H2), (H4) are satisfied. Then system* (1.1) *has a unique ω-periodic solution and all other solutions converge globally exponentially to this solution in the 1-norm as $t \to +\infty$.*

Corollary 1.3. *Assume that (H1)-(H3) are satisfied, and $L_{ij}(t) = L_{ij}$ is constant. Then system* (1.1) *has a unique equilibrium x^* and all other solution converge globally exponentially to the equilibrium in the p-norm as $t \to +\infty$.*

(H5) For $i = 1, 2, \dots, m; j = 1, 2, \dots, n$,

$$-a_{ij} + M_f \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} + \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} \mu_{kl} N_{ij} < 0 \,,$$

Theorem 1.4. *Assume (H1), (H2), (H5) are satisfied. Then system* (1.1) *has a unique ω-periodic solution and all other solution converge globally exponentially to this solution on the ∞-norm as $t \to +\infty$.*

Corollary 1.5. *Assume that (H1), (H2), (H5) are satisfied, and $L_{ij}(t) = L_{ij}$ is constant. Then system* (1.1) *has a unique equilibrium x^* and all other solution converge globally exponentially to the equilibrium in the ∞-norm as $t \to +\infty$.*

The organization of this paper is as follows. In section 2, we give some definitions and lemmas. In section 3, we state the proofs of the Theorem 1.1 and Theorem 1.4. In section 4, we shall show an example to illustrate our main results. In section 5, we give some conclusion of the main results.

2. Some definitions and lemmas

In this section, we give some definitions and lemmas. Let $C = C([-\tau, 0], \mathbb{R}^{m\times n})$ be the Banach space of continuous functions which map $[-\tau, 0]$ into $\mathbb{R}^{m\times n}$ with the topology of uniform converge. Let

$$x^*(t) = (x^*_{11}(t), \dots, x^*_{ij}(t), \dots, x^*_{mn}(t))^T$$

is the periodic solution of system (1.1) with the initial conditions ψ^* and $x(t) = (x_{11}(t), \dots, x_{ij}(t), \dots, x_{mn}(t))^T$ be the solution of system (1.1) with the initial conditions φ. We denote

$$\|\varphi - \psi^*\|_p^p = \sup_{-\tau\le\theta\le 0}\Big[\sum_{i=1}^{n}|\varphi_{ij}(\theta) - \psi^*_{ij}|^p\Big], \quad p \ge 1.$$

$$\|\varphi - \psi^*\|_\infty = \sup_{-\tau\le\theta\le 0}\max_{(i,j)}\Big[|\varphi_{(i,j)}(\theta) - \psi^*_{(i,j)}(\theta)|\Big].$$

Definition. The periodic solution $x^*(t)$ of system (1.1) is said to be globally exponentially stable in the p-norm, if there exists a constant $\varepsilon > 0$ and $k \ge 1$ such that for all $t > 0$,

$$\sum_{(i,j)}|x_{ij}(t) - x^*_{ij}(t)|^p \le k\|\varphi - \psi^*\|_p^p e^{-\varepsilon t}\,.$$

Definition. The periodic solution $x^*(t)$ of system (1.1) is said to be globally exponentially stable in ∞-norm, if there exists a constant $\varepsilon > 0$ and $k \ge 1$ such that for all $t > 0$,

$$\max_{(i,j)}|x_{ij}(t) - x^*_{ij}(t)| \le k\|\varphi - \psi^*\|_\infty e^{-\varepsilon t},$$

Definition. Let $F(t) : \mathbb{R} \to \mathbb{R}$ be a continuous function, then the upper right Dini derivate is defined as

$$D^+F(t) = \limsup_{h\to 0^+}\frac{1}{h}(F(t+h) - F(t)).$$

Lemma 2.1 ([13]). *Suppose that f_{ij} is a positive continuous function on $\mathbb{R}$, $L_{ij}(t)$ is a bounded continuous function and $a_{ij} \ge p_{ij}$. Then the solution $x_{ij}(t)$ of system (1.1) is bounded on $\mathbb{R}^+$, and $|x_{ij}(t)| \le N_{ij}$, where*

$$p_{ij} = M_f\sum_{C_{kl}\in N_r(i,j)}c^{kl}_{ij},$$

$$N_{ij} = \begin{cases}\frac{a_{ij}q_{ij}}{a_{ij}-p_{ij}}, & \text{if } a_{ij} > p_{ij},\\ q_{ij}, & \text{if } a_{ij} = p_{ij}.\end{cases}$$

$$q_{ij} = \begin{cases}|\varphi_{ij}(0)|, & \text{if } |\varphi_{ij}(0)| \ge \frac{L_{ij}}{a_{ij}},\\ \frac{L_{ij}}{a_{ij}}, & \text{if } |\varphi_{ij}(0)| < \frac{L_{ij}}{a_{ij}}.\end{cases}$$

$$M_f = \max_{(i,j)}\sup_{x\in\mathbb{R}}\{f_{ij}(x)\}, \quad L_{ij} = \sup_{t\in\mathbb{R}}\{|L_{ij}(t)|\}.$$

The proof of this lemma follows from a minor modification of the proof in [13, Lemma 2].

3. Proofs of the main results

The proof of Theorem 1.1. (1) Case $p > 1$: For any $\varphi, \psi \in C$, let $x(t,\varphi)$ and $x(t,\psi)$ represent the solution of system (1.1) through $(0,\varphi)$ and $(0,\psi)$ respectively. It follows from system (1.1) that

$$\begin{aligned}\frac{d}{dt}\left(x_{ij}(t,\varphi) - x_{ij}(t,\psi)\right) = &- a_{ij}(x_{ij}(t,\varphi) - x_{ij}(t,\psi)) \\ &- \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\big[f_{kl}(x_{kl}(t-\tau_{kl},\varphi))x_{ij}(t,\varphi) \\ &- f_{kl}(x_{kl}(t-\tau_{kl},\psi))x_{ij}(t,\psi)\big],\end{aligned} \tag{3.1}$$

for all $t \ge 0, i = 1,2,\dots,m; j = 1,2,\dots,n$. From (H3), there exists a small $\varepsilon > 0$ such that

$$\begin{aligned}\varepsilon - pa_{ij} + pM_f \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} + (p-1) \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij} \\ +e^{\varepsilon\tau} \sum_{(i,j)} \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij} < 0\,.\end{aligned}$$

Now, we consider the Lyapunov functional

$$\begin{aligned}V(t) =&V_1(t) + V_2(t) \\ =&\sum_{(i,j)} |x_{ij}(t,\varphi) - x_{ij}(t,\psi)|^p e^{\varepsilon t} \\ &+ \sum_{(i,j)} \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij} \int_{t-\tau_{kl}}^{t} |x_{kl}(s,\varphi) - x_{kl}(s,\psi)|^p\, e^{\varepsilon(s+\tau_{kl})} ds,\end{aligned} \tag{3.2}$$

in which

$$V_1(t) = \sum_{(i,j)} |x_{ij}(t,\varphi) - x_{ij}(t,\psi)|^p e^{\varepsilon t},$$

$$V_2(t) = \sum_{(i,j)} \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij} \int_{t-\tau_{kl}}^{t} |x_{kl}(s,\varphi) - x_{kl}(s,\psi)|^p\, e^{\varepsilon(s+\tau_{kl})} ds.$$

Calculating the upper right derivative D^+V_1 of V_1 along the solution of system (3.1), we have

$$\begin{aligned}&D^+V_1\big|_{(3.1)} \\ &\le \sum_{(i,j)} \big\{\varepsilon e^{\varepsilon t}|x_{ij}(t,\varphi) - x_{ij}(t,\psi)|^p + pe^{\varepsilon t}|x_{ij}(t,\varphi) - x_{ij}(t,\psi)|^{p-1} \\ &\quad \times D^+|x_{ij}(t,\varphi) - x_{ij}(t,\psi)|\big\} \\ &= \sum_{(i,j)} \big\{\varepsilon e^{\varepsilon t}|x_{ij}(t,\varphi) - x_{ij}(t,\psi)|^p + pe^{\varepsilon t}|x_{ij}(t,\varphi) - x_{ij}(t,\psi)|^{p-1} \\ &\quad \times \operatorname{sign}(x_{ij}(t,\varphi) - x_{ij}(t,\psi))\big[- a_{ij}(x_{ij}(t,\varphi) - x_{ij}(t,\psi) \\ &\quad - \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\big(f_{kl}(x_{kl}(t-\tau_{kl},\varphi))x_{ij}(t,\varphi) - f_{kl}(x_{kl}(t-\tau_{kl},\psi))x_{ij}(t,\psi)\big)\big]\big\}\,;\end{aligned}$$

i.e.,

$$
\begin{aligned}
&D^+V_1\big|_{(3.1)} \\
&\le e^{\varepsilon t}\sum_{(i,j)}\big\{(\varepsilon - pa_{ij})|x_{ij}(t,\varphi)-x_{ij}(t,\psi)|^p + p|x_{ij}(t,\varphi)-x_{ij}(t,\psi)|^{p-1} \\
&\quad\times \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\big|\,[f_{kl}(x_{kl}(t-\tau_{kl},\varphi)) - f_{kl}(x_{kl}(t-\tau_{kl},\psi))]\,x_{ij}(t,\varphi) \\
&\quad + f_{kl}(x_{kl}(t-\tau_{kl},\psi))\,(x_{ij}(t,\varphi)-x_{ij}(t,\psi))\,\big|\big\} \\
&\le e^{\varepsilon t}\sum_{(i,j)}(\varepsilon - pa_{ij})|x_{ij}(t,\varphi)-x_{ij}(t,\psi)|^p + e^{\varepsilon t}\sum_{(i,j)}\big\{p|x_{ij}(t,\varphi)-x_{ij}(t,\psi)|^{p-1} \\
&\quad\times \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\big[\mu_{kl}\,|x_{kl}(t-\tau_{kl},\varphi)-x_{kl}(t-\tau_{kl},\psi)||x_{ij}(t,\varphi)| \\
&\quad + M_f|x_{ij}(t,\varphi)-x_{ij}(t,\psi)|\big]\big\} \\
&= e^{\varepsilon t}\sum_{(i,j)}\Big(\varepsilon - pa_{ij} + pM_f\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\Big)\,|x_{ij}(t,\varphi)-x_{ij}(t,\psi)|^p \\
&\quad + e^{\varepsilon t}\sum_{(i,j)}\Big\{p\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij}\,|x_{kl}(t-\tau_{kl},\varphi)-x_{kl}(t-\tau_{kl},\psi)| \\
&\quad \times |x_{ij}(t,\varphi)-x_{ij}(t,\psi)|^{p-1}\Big\}.
\end{aligned}
$$

Using the inequality $ab \le \frac{1}{p}a^p + \frac{1}{q}b^q$, $(\frac{1}{p}+\frac{1}{q}=1,\ p>1, a,b\ge 0)$ [37]. We obtain

$$
\begin{aligned}
&D^+V_1(t)\big|_{(3.1)} \\
&\le e^{\varepsilon t}\sum_{(i,j)}\Big(\varepsilon - pa_{ij} + pM_f\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\Big)\,|x_{ij}(t,\varphi)-x_{ij}(t,\psi)|^p \\
&\quad + e^{\varepsilon t}\sum_{(i,j)}\big\{p\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij}\big[\frac{1}{p}|x_{kl}(t-\tau_{kl},\varphi)-x_{kl}(t-\tau_{kl},\psi)|^p \\
&\quad + \frac{p-1}{p}|x_{ij}(t,\varphi)-x_{ij}(t,\psi)|^p\big]\big\} \\
&= e^{\varepsilon t}\sum_{(i,j)}\big(\varepsilon - pa_{ij} + pM_f\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\big)\,|x_{ij}(t,\varphi)-x_{ij}(t,\psi)|^p \\
&\quad + e^{\varepsilon t}\sum_{(i,j)}\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij}|x_{kl}(t-\tau_{kl},\varphi)-x_{kl}(t-\tau_{kl},\psi)|^p \\
&\quad + e^{\varepsilon t}\sum_{(i,j)}(p-1)\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij}\,|x_{ij}(t,\varphi)-x_{ij}(t,\psi)|^p \\
&= e^{\varepsilon t}\sum_{(i,j)}\bigg(\varepsilon - pa_{ij} + pM_f\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} + (p-1)\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij}\bigg) \\
&\quad \times |x_{ij}(t,\varphi)-x_{ij}(t,\psi)|^p \\
&\quad + e^{\varepsilon t}\sum_{(i,j)}\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij}\,|x_{kl}(t-\tau_{kl},\varphi)-x_{kl}(t-\tau_{kl},\psi)|^p\,.
\end{aligned}
\tag{3.3}
$$

Calculating the upper right Dini derivative D^+V_2 of V_2 along the solution of system (3.1), we have

$$
\begin{aligned}
D^+V_2\big|_{(3.1)} &= \sum_{(i,j)} \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij}\big[|x_{kl}(t,\varphi)-x_{kl}(t,\psi)|^p e^{\varepsilon(t+\tau_{kl})} \\
&\quad - |x_{kl}(t-\tau_{kl},\varphi)-x_{kl}(t-\tau_{kl},\psi)|^p\, e^{\varepsilon t}\big] \\
&\le e^{\varepsilon t}e^{\varepsilon\tau}\sum_{(i,j)} \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij}\,|x_{kl}(t,\varphi)-x_{kl}(t,\psi)|^p \\
&\quad - e^{\varepsilon t}\sum_{(i,j)} \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij}\,|x_{kl}(t-\tau_{kl},\varphi)-x_{kl}(t-\tau_{kl},\psi)|^p\,.
\end{aligned}
\tag{3.4}
$$

From (3.3) and (3.4), we can obtain

$$
\begin{aligned}
&D^+V|_{(3.1)} \\
&\le D^+V_1|_{(3.1)} + D^+V_2|_{(3.1)} \\
&\le e^{\varepsilon t}\sum_{(i,j)}\Big(\varepsilon - pa_{ij} + pM_f\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} + (p-1)\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij}\Big) \\
&\quad \times |x_{ij}(t,\varphi)-x_{ij}(t,\psi)|^p + e^{\varepsilon t}e^{\varepsilon\tau}\sum_{(i,j)}\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij}\,|x_{kl}(t,\varphi)-x_{kl}(t,\psi)|^p \\
&\le e^{\varepsilon t}\sum_{(i,j)}\big(\varepsilon - pa_{ij} + pM_f\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} + (p-1)\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij}\big) \\
&\times |x_{ij}(t,\varphi)-x_{ij}(t,\psi)|^p \\
&\quad + e^{\varepsilon t}\Big(e^{\varepsilon\tau}\sum_{(i,j)}\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij}\Big)\sum_{(i,j)}|x_{ij}(t,\varphi)-x_{ij}(t,\psi)|^p \\
&= \sum_{(i,j)}\Big(\varepsilon - pa_{ij} + pM_f\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} + (p-1)\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij} \\
&\quad + e^{\varepsilon\tau}\sum_{(i,j)}\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij}\Big)\,|x_{ij}(t,\varphi)-x_{ij}(t,\psi)|^p \\
&< 0, \quad \text{for all } t\ge 0.
\end{aligned}
$$

i.e. $D^+V(t)\le 0$, Thus, we have

$$
V(t)\le V(0), \quad \text{for all } t\ge 0. \tag{3.5}
$$

where

$$
\begin{aligned}
V(t) &= \sum_{(i,j)}|x_{ij}(t,\varphi)-x_{ij}(t,\psi)|^p e^{\varepsilon t} \\
&\quad + \sum_{(i,j)}\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij}\int_{t-\tau_{kl}}^{t}|x_{kl}(s,\varphi)-x_{kl}(s,\psi)|^p\, e^{\varepsilon(s+\tau_{kl})}ds.
\end{aligned}
$$

Note that

$$
V(t)\ge e^{\varepsilon t}\sum_{(i,j)}|x_{ij}(t,\varphi)-x_{ij}(t,\psi)|^p, \quad \text{for all } t\ge 0. \tag{3.6}
$$

Again,

$$
\begin{aligned}
&V(0)\\
&= \sum_{(i,j)} |x_{ij}(0,\varphi) - x_{ij}(0,\psi)|^p \\
&\quad + \sum_{(i,j)} \sum_{C_{kl} \in N_r(i,j)} c_{ij}^{kl} \mu_{kl} N_{ij} \int_{-\tau_{kl}}^{0} |x_{kl}(s,\varphi) - x_{kl}(s,\psi)|^p \, e^{\varepsilon(s+\tau_{kl})} ds \\
&\le \|\varphi - \psi\|_p^p + \sum_{(i,j)} \sum_{C_{kl} \in N_r(i,j)} c_{ij}^{kl} \mu_{kl} N_{ij} \int_{-\tau}^{0} |x_{kl}(s,\varphi) - x_{kl}(s,\psi)|^p \, e^{\varepsilon(s+\tau)} ds \\
&\le \|\varphi - \psi\|_p^p + \sum_{(i,j)} \sum_{C_{kl} \in N_r(i,j)} c_{ij}^{kl} \mu_{kl} N_{ij} \int_{-\tau}^{0} \sum_{(i,j)} |x_{ij}(s,\varphi) - x_{ij}(s,\psi)|^p \, e^{\varepsilon(s+\tau)} ds \\
&\le \|\varphi - \psi\|_p^p + \tau e^{\varepsilon\tau} \sum_{(i,j)} \sum_{C_{kl} \in N_r(i,j)} c_{ij}^{kl} \mu_{kl} N_{ij} \|\varphi - \psi\|_p^p \\
&= \Big(1 + \tau e^{\varepsilon\tau} \sum_{(i,j)} \sum_{C_{kl} \in N_r(i,j)} c_{ij}^{kl} \mu_{kl} N_{ij}\Big) \|\varphi - \psi\|_p^p .
\end{aligned}
\tag{3.7}
$$

Set

$$
k = 1 + \tau e^{\varepsilon\tau} \sum_{(i,j)} \sum_{C_{kl} \in N_r(i,j)} c_{ij}^{kl} \mu_{kl} N_{ij},
$$

then $k > 1$. Therefore, from (3.5)–(3.7), we have

$$
\sum_{(i,j)} |x_{ij}(t,\varphi) - x_{ij}(t,\psi)|^p \le k e^{-\varepsilon t} \|\varphi - \psi\|_p^p . \tag{3.8}
$$

In addition, from this inequality, we can easily obtain

$$
\|x_t(\varphi) - x_t(\psi)\|_p \le k^{\frac{1}{p}} e^{-\frac{\varepsilon}{p}(t-\tau)} \|\varphi - \psi\|_p . \tag{3.9}
$$

Now, we can choose a positive integer m such that

$$
k^{\frac{1}{p}} e^{-\frac{\varepsilon}{p}(m\omega - \tau)} \le \frac{1}{2}. \tag{3.10}
$$

Define a Poincaré map

$$
P : C([-\tau, 0], \mathbb{R}^{m\times n}) \to C([-\tau, 0], \mathbb{R}^{m\times n})
$$

by $P\varphi = x_\omega(\varphi)$, then we can derive from (3.9) and (3.10) that

$$
\|P^m \varphi - P^m \psi\|_p \le \frac{1}{2} \|\varphi - \psi\|_p .
$$

Therefore, P^m is a contraction map. Then there exists a unique fixed point $x^* \in C([-\tau, 0], \mathbb{R}^{m\times n})$ such that $P^m x^* = x^*$. Note that

$$
P^m(Px^*) = P(P^m x^*) = Px^* .
$$

This implies $Px^* \in C([-\tau, 0], \mathbb{R}^{m\times n})$ is also a fixed point of P^m. So,

$$
Px^* = x^*, \quad \text{i.e.} \quad x_\omega(x^*) = x^* .
$$

Let $x(t,x^*)$ be the solution of system (1.1) through $(0,x^*)$, obviously, $x(t+\omega,x^*)$ is also a solution of system (1.1) and note that

$$x_{t+\omega}(x^*) = x_t(x_\omega(x^*)) = x_t(x^*), \quad \text{for all } t \geq 0.$$

So, $x(t+\omega,x^*) = x(t,x^*)$, for all $t \geq 0$. This shows that $x(t,x^*)$ is exactly one ω-periodic solution of system (1.1) and it is easy to see from (3.8) that all solutions of system (1.1) converge globally exponentially to it on p-norm as $t \to +\infty$. The proof of the case $p > 1$ is completed.

(2) Case $p = 1$: From (H4), there exists a small $\varepsilon > 0$ such that

$$\varepsilon - a_{ij} + M_f \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} + e^{\varepsilon\tau} \sum_{(i,j)} \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij} < 0.$$

we consider the Lyapunov functional

$$\begin{aligned} V(t) &= \sum_{(i,j)} |x_{ij}(t,\varphi) - x_{ij}(t,\psi)| e^{\varepsilon t} \\ &\quad + \sum_{(i,j)} \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij} \int_{t-\tau_{kl}}^{t} |x_{kl}(s,\varphi) - x_{kl}(s,\psi)|\, e^{\varepsilon(s+\tau_{kl})} ds, \end{aligned}$$

by making a minor modification for the proof of the case $p > 1$ above, we can obtain the proof.

Proof of the Theorem 1.4. For any $\varphi, \psi \in C$, let $x(t,\varphi)$ and $x(t,\psi)$ represent the solutions of system (1.1) through $(0,\varphi)$ and $(0,\psi)$ respectively. Let $i_0j_0 = i_0j_0(t)$ is the down index such that

$$\max_{(i,j)} |x_{ij}(t,\varphi) - x_{ij}(t,\psi))| = |x_{i_0j_0}(t,\varphi) - x_{i_0j_0}(t,\psi)|,$$

From (H5), there exists a small $\varepsilon > 0$ such that

$$\varepsilon - a_{i_0j_0} + M_f \sum_{C_{kl}\in N_r(i,j)} c_{i_0j_0}^{kl} + e^{\varepsilon\tau} \sum_{C_{kl}\in N_r(i_0,j_0)} c_{i_0j_0}^{kl}\mu_{kl}N_{i_0j_0} < 0.$$

Now, we construct the Lyapunov functional

$$\begin{aligned} W(t) &= W_1(t) + W_2(t) \\ &= |x_{i_0j_0}(t,\varphi) - x_{i_0j_0}(t,\psi)| e^{\varepsilon t} \\ &\quad + \sum_{C_{kl}\in N_r(i_0,j_0)} c_{i_0j_0}^{kl}\mu_{kl}N_{i_0j_0} \int_{t-\tau_{kl}}^{t} |x_{kl}(s,\varphi) - x_{kl}(s,\psi)|\, e^{\varepsilon(s+\tau_{kl})} ds, \end{aligned} \tag{3.11}$$

in which $W_1(t) = |x_{i_0j_0}(t,\varphi) - x_{i_0j_0}(t,\psi)| e^{\varepsilon t}$ and

$$W_2(t) = \sum_{C_{kl}\in N_r(i_0,j_0)} c_{i_0j_0}^{kl}\mu_{kl}N_{i_0j_0} \int_{t-\tau_{kl}}^{t} |x_{kl}(s,\varphi) - x_{kl}(s,\psi)|\, e^{\varepsilon(s+\tau_{kl})} ds.$$

Calculating the upper right derivative D^+W_1 of W_1 along the solution of system (3.1), we have

$$
\begin{aligned}
&D^+W_1\big|_{(3.1)}\\
&= e^{\varepsilon t}\operatorname{sign}(x_{i_0j_0}(t,\varphi)-x_{i_0j_0}(t,\psi))\frac{d}{dt}(x_{i_0j_0}(t,\varphi)-x_{i_0j_0}(t,\psi)\\
&\quad+\varepsilon e^{\varepsilon t}|x_{i_0j_0}(t,\varphi)-x_{i_0j_0}(t,\psi)|\\
&= e^{\varepsilon t}\operatorname{sign}(x_{i_0j_0}(t,\varphi)-x_{i_0j_0}(t,\psi))\big\{-a_{i_0j_0}(x_{i_0j_0}(t,\varphi)-x_{i_0j_0}(t,\psi))\\
&\quad-\sum_{C_{kl}\in N_r(i_0,j_0)} c_{i_0j_0}^{kl}\big[f_{kl}(x_{kl}(t-\tau_{kl},\varphi))\varphi_{i_0j_0}(t,\varphi)\\
&\quad-f_{kl}(x_{kl}(t-\tau_{kl},\psi))\psi_{i_0j_0}(t,\psi)\big]\big\}+\varepsilon e^{\varepsilon t}|x_{i_0j_0}(t,\varphi)-x_{i_0j_0}(t,\psi)|\,;
\end{aligned}
$$

i. e.,

$$
\begin{aligned}
&D^+W_1\big|_{(3.1)}\\
&\le e^{\varepsilon t}\big\{(\varepsilon-a_{i_0j_0})|x_{i_0j_0}(t,\varphi)-x_{i_0j_0}(t,\psi)|\\
&\quad+\sum_{C_{kl}\in N_r(i_0,j_0)} c_{i_0j_0}^{kl}\big[\big|f_{kl}(x_{kl}(t-\tau_{kl},\varphi))-f_{kl}(x_{kl}(t-\tau_{kl},\psi))\big|\big|x_{i_0j_0}(t,\varphi)\big|\\
&\quad+\big|f_{kl}(x_{kl}(t-\tau_{kl},\psi))\big|\big|x_{i_0j_0}(t,\varphi)-x_{i_0j_0}(t,\psi)\big|\big]\big\}\\
&\le e^{\varepsilon t}\big\{(\varepsilon-a_{i_0j_0})|x_{i_0j_0}(t,\varphi)-x_{i_0j_0}(t,\psi)|\\
&\quad+M_f\sum_{C_{kl}\in N_r(i_0,j_0)} c_{i_0j_0}^{kl}\big|x_{i_0j_0}(t,\varphi)-x_{i_0j_0}(t,\psi)\big|\\
&\quad+\sum_{C_{kl}\in N_r(i_0,j_0)} c_{i_0j_0}^{kl}\mu_{kl}N_{i_0j_0}\big|x_{kl}(t-\tau_{kl},\varphi)-x_{kl}(t-\tau_{kl},\psi)\big|\big\}\\
&= e^{\varepsilon t}\big(\varepsilon-a_{i_0j_0}+M_f\sum_{C_{kl}\in N_r(i_0,j_0)} c_{i_0j_0}^{kl}\big)\,|x_{i_0j_0}(t,\varphi)-x_{i_0j_0}(t,\psi)|\\
&\quad+e^{\varepsilon t}\sum_{C_{kl}\in N_r(i_0,j_0)} c_{i_0j_0}^{kl}\mu_{kl}N_{i_0j_0}\,|x_{kl}(t-\tau_{kl},\varphi)-x_{kl}(t-\tau_{kl},\psi)|\,.
\end{aligned}
\tag{3.12}
$$

Calculating the upper right derivative D^+W_2 of W_2 along the solution of system (3.1), we have

$$
\begin{aligned}
&D^+W_2\big|_{(3.1)}\\
&=\sum_{C_{kl}\in N_r(i_0,j_0)} c_{i_0j_0}^{kl}\mu_{kl}N_{i_0j_0}e^{\varepsilon(t+\tau_{kl})}\,|x_{kl}(t,\varphi)-x_{kl}(t,\psi)|\\
&\quad-e^{\varepsilon t}\sum_{C_{kl}\in N_r(i_0,j_0)} c_{i_0j_0}^{kl}\mu_{kl}N_{i_0j_0}\,|x_{kl}(t-\tau_{kl},\varphi)-x_{kl}(t-\tau_{kl},\psi)|\,.
\end{aligned}
\tag{3.13}
$$

From (3.12) and (3.13), we get

$$\begin{aligned}
&D^+W|_{(3.1)}\\
&\le D^+W_1|_{(3.1)} + D^+W_2|_{(3.1)}\\
&\le e^{\varepsilon t}\Big(\varepsilon - a_{i_0j_0} + M_f \sum_{C_{kl}\in N_r(i_0,j_0)} c^{kl}_{i_0j_0}\Big)\,|x_{i_0j_0}(t,\varphi) - x_{i_0j_0}(t,\psi)|\\
&\quad + e^{\varepsilon t}e^{\varepsilon\tau} \sum_{C_{kl}\in N_r(i_0,j_0)} c^{kl}_{i_0j_0}\mu_{kl}N_{i_0j_0}\,|x_{kl}(t,\varphi) - x_{kl}(t,\psi)|\\
&\le e^{\varepsilon t}\sum_{(i,j)}\Big(\varepsilon - a_{i_0j_0} + M_f \sum_{C_{kl}\in N_r(i,j)} c^{kl}_{i_0j_0}\Big)\max_{(i,j)}|x_{ij}(t,\varphi) - x_{ij}(t,\psi)|\\
&\quad + e^{\varepsilon t}e^{\varepsilon\tau}\Big(\sum_{C_{kl}\in N_r(i_0,j_0)} c^{kl}_{i_0j_0}\mu_{kl}N_{i_0j_0}\Big)\max_{(i,j)}|x_{ij}(t,\varphi) - x_{ij}(t,\psi)|\\
&= e^{\varepsilon t}\Big(\varepsilon - a_{i_0j_0} + M_f \sum_{C_{kl}\in N_r(i_0,j_0)} c^{kl}_{i_0j_0} + e^{\varepsilon\tau}\sum_{C_{kl}\in N_r(i_0,j_0)} c^{kl}_{i_0j_0}\mu_{kl}N_{i_0j_0}\Big)\\
&\quad \times \max_{(i,j)}|x_{ij}(t,\varphi) - x_{ij}(t,\psi)|\\
&< 0\,.
\end{aligned}$$

Therefore, $D^+W(t) \le 0$. Thus $W(t) \le W(0)$. Note that

$$W(t) \ge e^{\varepsilon t}|x_{i_0j_0}(t,\varphi) - x_{i_0j_0}(t,\psi)| = e^{\varepsilon t}\max_{(i,j)}|x_{ij}(t,\varphi) - x_{ij}(t,\psi)|, \quad t \ge 0.$$

Again,

$$\begin{aligned}
W(0) &= |x_{i_0j_0}(0,\varphi) - x_{i_0j_0}(0,\psi)|\\
&\quad + \sum_{C_{kl}\in N_r(i_0,j_0)} c^{kl}_{i_0j_0}\mu_{kl}N_{i_0j_0}\int_{-\tau_{kl}}^0 |x_{kl}(s,\varphi) - x_{kl}(s,\psi)|\, e^{\varepsilon(s+\tau_{kl})}ds\\
&\le \|\varphi-\psi\|_\infty + \tau e^{\varepsilon\tau}\sum_{C_{kl}\in N_r(i_0,j_0)} c^{kl}_{i_0j_0}\mu_{kl}N_{i_0j_0}\|\varphi-\psi\|_\infty\\
&= (1 + \tau e^{\varepsilon\tau}\max_{(i,j)}\sum_{C_{kl}\in N_r(i,j)} c^{kl}_{ij}\mu_{kl}N_{ij})\|\varphi-\psi\|_\infty.
\end{aligned}$$

Set

$$k = 1 + \tau e^{\varepsilon\tau}\max_{(i,j)}\sum_{C_{kl}\in N_r(i,j)} c^{kl}_{ij}\mu_{kl}N_{ij},$$

then $k > 1$. Thus, we have

$$\max_{(i,j)}|x_{ij}(t,\varphi) - x_{ij}(t,\psi)| \le ke^{-\varepsilon t}\|\varphi-\psi\|_\infty. \tag{3.14}$$

This implies

$$\|x_t(\varphi) - x_t(\psi)\|_\infty \le ke^{-\varepsilon(t-\tau)}\|\varphi-\psi\|_\infty. \tag{3.15}$$

We can choose a positive integer m such that $ke^{-\varepsilon(m\omega-\tau)} \le \frac{1}{2}$. Now, define a Poincaré map

$$P : C([-\tau,0],\mathbb{R}^{m\times n}) \to C([-\tau,0],\mathbb{R}^{m\times n})$$

by $P\varphi = x_\omega(\varphi)$, then we can derive from (3.15) that

$$\|P^m\varphi - P^m\psi\|_\infty \le \frac{1}{2}\|\varphi - \psi\|_\infty.$$

So, P^m is a contraction map. Hence, there exists a unique fixed point $x^* \in C([-\tau,0],\mathbb{R}^{m\times n})$ such that $P^m x^* = x^*$. Note that

$$P^m(px^*) = P(P^m x^*) = Px^*.$$

This implies that $Px^* \in C([-\tau,0],\mathbb{R}^{m\times n})$ is also a fixed point of P^m. So,

$$Px^* = x^*, \quad \text{i. e.} \quad x_\omega(x^*) = x^*.$$

Let $x(t,x^*)$ be the solution of system (1.1) through $(0,x^*)$, obviously, $x(t+\omega,x^*)$ is also a solution of system (1.1) and note that

$$x_{t+\omega}(x^*) = x_t(x_\omega(x^*)) = x_t(x^*), \quad \text{for all } t \ge 0.$$

Therefore, $x(t+\omega,x^*) = x(t,x^*)$, for all $t \ge 0$. This shows that $x(t,x^*)$ is exactly one ω-periodic solution of system (1.1) and from (3.14) it is easy to see that all solutions of system (1.1) converge exponentially to it on ∞-norm as $t \to +\infty$. The proof is complete.

4. An example

For $i,j = 1,2,3$, consider the system of SICNNs

$$\frac{dx_{ij}}{dt} = -x_{ij}(t) - \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} f_{kl}(x_{kl}(t-0.1))x_{ij} + \cos t \tag{4.1}$$

with the initial condition $\varphi_{ij}(s) = \sin s, \quad s \in [0.1, 0]$. Let $f_{kl}(x) = f(x) = 0.1(|x+1| + |x-1|)$, then f_{kj} satisfies assumptions (H1) and (H2), and $\mu_{kl} = 0.2$ $(k,l = 1,2,3)$, $M_f = 0.2$, $a_{ij} = 1$, $L_{ij} = 1$, $\varphi_{ij}(0) = 0$, $q_{ij} = 1$ $(i,j = 1,2,3)$. Again we take

$$C = (c_{ij})_{3\times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} = \begin{pmatrix} 0.1 & 0.05 & 0.1 \\ 0.05 & 0.1 & 0.05 \\ 0.1 & 0.05 & 0.1 \end{pmatrix}.$$

Set $r = 1$, then we have

$$\begin{pmatrix} \sum_{C_{kl}\in N_r(1,1)} c_{11}^{kl} & \sum_{C_{kl}\in N_r(1,2)} c_{12}^{kl} & \sum_{C_{kl}\in N_r(1,3)} c_{13}^{kl} \\ \sum_{C_{kl}\in N_r(2,1)} c_{21}^{kl} & \sum_{C_{kl}\in N_r(2,2)} c_{22}^{kl} & \sum_{C_{kl}\in N_r(2,3)} c_{23}^{kl} \\ \sum_{C_{kl}\in N_r(3,1)} c_{31}^{kl} & \sum_{C_{kl}\in N_r(3,2)} c_{32}^{kl} & \sum_{C_{kl}\in N_r(3,3)} c_{33}^{kl} \end{pmatrix} = \begin{pmatrix} 0.3 & 0.45 & 0.3 \\ 0.45 & 0.7 & 0.45 \\ 0.3 & 0.45 & 0.3 \end{pmatrix}.$$

and

$$(p_{ij})_{3\times 3} = (M_f \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl})_{3\times 3} = \begin{pmatrix} 0.06 & 0.09 & 0.06 \\ 0.09 & 0.14 & 0.09 \\ 0.06 & 0.09 & 0.06 \end{pmatrix}.$$

Clearly $p_{ij} < a_{ij}$, for $i, j = 1, 2, 3$.

$$
\begin{gathered}
(N_{ij})_{3\times 3} = (\frac{a_{ij}q_{ij}}{a_{ij}-p_{ij}})_{3\times 3} = \begin{pmatrix} \frac{100}{94} & \frac{100}{91} & \frac{100}{94} \\ \frac{100}{91} & \frac{100}{86} & \frac{100}{91} \\ \frac{100}{94} & \frac{100}{91} & \frac{100}{94} \end{pmatrix}, \\
(\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij})_{3\times 3} = \begin{pmatrix} \frac{6}{94} & \frac{9}{91} & \frac{6}{94} \\ \frac{9}{91} & \frac{14}{86} & \frac{9}{91} \\ \frac{6}{94} & \frac{9}{91} & \frac{6}{94} \end{pmatrix}, \\
\sum_{(i,j)}\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij} = 0.8137142.
\end{gathered}
$$

Taking $p = 2$, we easily check that

$$
\begin{aligned}
&\Big(-pa_{ij} + pM_f \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} + (p-1)\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij} \\
&+ \sum_{(i,j)}(\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij})\Big)_{3\times 3} \\
&= \Big(0.8137142 + 0.4\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} + 0.2\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}N_{ij} - 2\Big)_{3\times 3} \\
&= \begin{pmatrix} -1.002456 & -0.9073847 & -1.002456 \\ -0.9073847 & -0.7434951 & -0.9073847 \\ -1.002456 & -0.9073847 & -1.002456 \end{pmatrix} < 0.
\end{aligned}
$$

Therefore, (H3) holds. Hence, then system (4.1) has a unique 2π-periodic solution and all other solution converge globally exponentially to it on the 2-norm as $t \to +\infty$. Again take $p = 1$, we can easily check that

$$
\begin{aligned}
&\Big(-a_{ij} + M_f \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} + \sum_{(i,j)}\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij}\Big)_{3\times 3} \\
&= \Big(-0.1862858 + 0.2\sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\Big)_{3\times 3} \\
&= \begin{pmatrix} -0.1262858 & -0.0962858 & -0.1262858 \\ -0.0962858 & -0.0462858 & -0.0962858 \\ -0.1262858 & -0.0962858 & -0.1262858 \end{pmatrix} < 0\,.
\end{aligned}
$$

Therefore, (H4) holds. By Corollary 1.2, system (4.1) has a unique 2π-periodic solution and all other solution converge globally exponentially to it on the 1-norm as $t \to +\infty$. We can easily check that

$$
\begin{aligned}
&\Big(-a_{ij} + M_f \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl} + \sum_{C_{kl}\in N_r(i,j)} c_{ij}^{kl}\mu_{kl}N_{ij}\Big)_{3\times 3} \\
&= \begin{pmatrix} -0.8761702 & -0.8110989 & -0.8761702 \\ -0.8110989 & -0.6972093 & -0.8110989 \\ -0.8761702 & -0.8110989 & -0.8761702 \end{pmatrix} < 0.
\end{aligned}
$$

Thus, (H5) is satisfied. By Theorem 1.4, system (4.1) has a unique 2π- solution and all other solution converge globally exponentially to it on the ∞-norm as t approaches $+\infty$.

Conclusion. In this paper, we have derived some simple sufficient conditions in term of systems parameters for periodic solutions and global exponential stability of SIDCNNs. The results possess important significance in some applied fields, and the conditions are easily checked in practice. These play an important role in design and application of SIDCNNS. In addition, the method of this paper may be applied to some other systems such as the systems given in [14, 19, 20, 22, 23].

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