Electronic Journal of Differential Equations, Vol. 2005(2005), No. 78, pp. 1–8.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu (login: ftp)

POSITIVE SOLUTIONS FOR THE BEAM EQUATION UNDER CERTAIN BOUNDARY CONDITIONS

BO YANG

Abstract. We consider a boundary-value problem for the beam equation, in which the boundary conditions mean that the beam is embedded at one end and fastened with a sliding clamp at the other end. Some priori estimates to the positive solutions for the boundary-value problem are obtained. Sufficient conditions for the existence and nonexistence of positive solutions for the boundary-value problem are established.

1. Introduction

In this paper, we consider the fourth order beam equation

$$u''''(t) = g(t)f(u(t)), \quad 0 \le t \le 1, \tag{1.1}$$

together with boundary conditions

$$u(0) = u'(0) = u'(1) = u'''(1) = 0. \tag{1.2}$$

Throughout this paper, we assume that

(H1) $f : [0, \infty) \to [0, \infty)$ is continuous

(H2) $g : [0, 1] \to [0, \infty)$ is a continuous function such that $\int_0^1 g(t)\,dt > 0$.

Equation (1.1) and the boundary conditions (1.2) arise from the study of elasticity and have definite physical meanings. Equation (1.1) describes the deflection or deformation of an elastic beam under a certain force. The boundary conditions (1.2) mean that the beam is embedded at the end $t = 0$, and fastened with a sliding clamp at the end $t = 1$.

In 1989, Gupta [12] considered the boundary-value problem

$$u''''(t) + f(t)u(t) = e(t), \quad 0 < t < \pi, \tag{1.3}$$

$$u'(0) = u'''(0) = u'''(\pi) = u'(\pi) = 0, \tag{1.4}$$

where (1.4) means that the beam is fastened with sliding clamps at both ends $t = 0$ and $t = \pi$. In 2004, Kosmatov [16] considered (1.1) together with the boundary conditions

$$u(0) = u'(0) = u'(1) = u(1) = 0, \tag{1.5}$$

2000 *Mathematics Subject Classification.* 34B18.
Key words and phrases. Fixed point theorem; cone; nonlinear boundary-value problem; positive solution.

Submitted April 29, 2005. Published July 8, 2005.

and obtained sufficient conditions for existence of infinitely many solutions to the problem (1.1)-(1.5). Note that the boundary conditions (1.5) mean that the beam is embedded at both ends $t = 0$ and $t = 1$.

In fact, (1.1) has been studied by many authors under various boundary conditions and by different approaches. For some other results on boundary-value problems of the beam equation, we refer the reader to the papers of Agarwal [1], Bai and Wang [4], Davis and Henderson [6], Dalmasso [5], Dunninger [7], Elgindi and Guan [8], Eloe, Henderson, and Kosmatov [9], Graef and B. Yang [11], Gupta [13], Ma [17, 18], Ma and Wang [19], B. Yang [20], and Y. Yang [21].

In this paper, we will study the positive solutions of the problem (1.1)-(1.2). By positive solution, we mean a solution $u(t)$ such that $u(t) > 0$ for $t \in (0, 1)$. A beam can have different shapes under different boundary constraints. One of the purposes of this paper is to make some estimates to the shape of the beam under boundary conditions (1.2).

This paper is organized as follows. In Section 2, we give the Green's function for the problem (1.1)-(1.2), state the Krasnosel'skii's fixed point theorem, and fix some notations. In Section 3, we present some priori estimates to positive solutions to the problem (1.1)-(1.2). In Sections 4 and 5, we establish some existence and nonexistence results for positive solutions to the problem (1.1)-(1.2).

2. Preliminaries

The Green's function $G : [0, 1] \times [0, 1] \to [0, \infty)$ for the problem (1.1)-(1.2) is

$$G(t,s) = \begin{cases} \frac{1}{12} t^2 (6s - 3s^2 - 2t), & \text{if } 0 \le t \le s \le 1, \\ \frac{1}{12} s^2 (6t - 3t^2 - 2s), & \text{if } 0 \le s \le t \le 1. \end{cases}$$

Then problem (1.1)-(1.2) is equivalent to the integral equation

$$u(t) = \int_0^1 G(t,s) g(s) f(u(s))\, ds, \quad 0 \le t \le 1. \tag{2.1}$$

It is easy to verify that G is a continuous function, and $G(t, s) > 0$ if $t, s \in (0, 1)$. We will need the following fixed point theorem, which is due to Krasnosel'skii [15], to prove some of our results.

Theorem 2.1. *Let $(X, \|\cdot\|)$ be Banach space over the reals, and let $P \subset X$ be a cone in X. Let H_1 and H_2 be real numbers such that $H_2 > H_1 > 0$, and let*

$$\Omega_i = \{v \in X \mid \|v\| < H_i\}, \quad i = 1, 2.$$

If $L : P \cap (\overline{\Omega_2} - \Omega_1) \to P$ is a completely continuous operator such that, either

(K1) *$\|Lv\| \le \|v\|$ if $v \in P \cap \partial\Omega_1$, and $\|Lv\| \ge \|v\|$ if $v \in P \cap \partial\Omega_2$, or*
(K2) *$\|Lv\| \ge \|v\|$ if $v \in P \cap \partial\Omega_1$, and $\|Lv\| \le \|v\|$ if $v \in P \cap \partial\Omega_2$.*

Then L has a fixed point in $P \cap (\overline{\Omega_2} - \Omega_1)$.

For the rest of this paper, we let $X = C[0, 1]$ be with norm

$$\|v\| = \max_{t \in [0,1]} |v(t)|, \quad \forall v \in X.$$

Clearly X is a Banach space. We define $Y = \{v \in X \mid v(t) \ge 0 \text{ for } 0 \le t \le 1\}$, and define the operator $T : Y \to X$ by

$$(Tu)(t) = \int_0^1 G(t,s) g(s) f(u(s))\, ds, \quad 0 \le t \le 1. \tag{2.2}$$

It is clear that if (H1) and (H2) hold, then $T : Y \to Y$ is a completely continuous operator. We also define the constants

$$F_0 = \limsup_{x\to 0^+} \frac{f(x)}{x}, \quad f_0 = \liminf_{x\to 0^+} \frac{f(x)}{x},$$
$$F_\infty = \limsup_{x\to +\infty} \frac{f(x)}{x}, \quad f_\infty = \liminf_{x\to +\infty} \frac{f(x)}{x}.$$

These constants, which are associated with the function f, will be used in Sections 4 and 5.

3. Estimates for Positive Solutions

In this section, we shall give some estimates for positive solutions of the problem (1.1)-(1.2). To this purpose, we define the functions $a : [0,1] \to [0,1]$, $b : [0,1] \to [0,1]$, and $c : [0,1] \to [0,1]$ by

$$a(t) = 3t^2 - 2t^3, \quad b(t) = 2t - t^2, \quad c(t) = 4t^2 - 4t^3 + t^4.$$

It is easy to see that $b(t) \ge c(t) \ge a(t) \ge t^2$ for $0 \le t \le 1$.

Lemma 3.1. *If $u \in C^4[0,1]$ satisfies the boundary conditions (1.2), and*

$$u''''(t) \ge 0 \quad \text{for} \quad 0 \le t \le 1, \tag{3.1}$$

then

$$u'''(t) \le 0, \quad u'(t) \ge 0, \quad u(t) \ge 0 \quad \text{for} \quad 0 \le t \le 1. \tag{3.2}$$

Proof. Note that (3.1) implies that u''' is nondecreasing. Since $u'''(1) = 0$, we have $u'''(t) \le 0$ on $[0,1]$, which means that u' is concave downward on $[0,1]$. Since $u'(0) = u'(1) = 0$, we have $u'(t) \ge 0$ on $[0,1]$. Since $u(0) = 0$, we have $u(t) \ge 0$ on $[0,1]$. The proof is complete. □

Lemma 3.2. *If $u \in C^4[0,1]$ satisfies (1.2) and (3.1), then*

$$u(t) \ge a(t)u(1) \quad \text{for} \quad 0 \le t \le 1. \tag{3.3}$$

Proof. If $u \in C^4[0,1]$ satisfies (1.2) and (3.1), then $u(0) = 0$, $u(1) \ge 0$, and $u'(t) \ge 0$ for $0 \le t \le 1$. If $u(1) = 0$, then $u(t) \equiv 0$, and it is easy to see that (3.3) is true in this case.

Now we prove (3.3) when $u(1) > 0$. Without loss of generality, we assume that $u(1) = 1$. If we define

$$h(t) = u(t) - a(t)u(1) = u(t) - (3t^2 - 2t^3), \quad 0 \le t \le 1,$$

then

$$h'(t) = u'(t) - (6t - 6t^2), \quad h''(t) = u''(t) - (6 - 12t),$$
$$h'''(t) = u'''(t) + 12,$$
$$h''''(t) = u''''(t) \ge 0, \quad 0 \le t \le 1. \tag{3.4}$$

To prove the lemma, it suffices to show that $h(t) \ge 0$ on $[0,1]$. It is easy to see that $h(0) = h(1) = 0$. By mean value theorem, there exists $r_1 \in (0,1)$ such that $h'(r_1) = 0$. It is also easy to see that $h'(0) = h'(1) = 0$. Since $h'(0) = h'(r_1) = h'(1) = 0$, there exist $r_2 \in (0, r_1)$ and $t_2 \in (r_1, 1)$ such that $h''(r_2) = h''(t_2) = 0$.

Note that (3.4) implies that h'' is concave upward. Since $h''(r_2) = h''(t_2) = 0$, we have

$$h''(t) \ge 0 \ \text{on} \ (0, r_2), \quad h''(t) \le 0 \ \text{on} \ (r_2, t_2), \quad \text{and} \quad h''(t) \ge 0 \ \text{on} \ (t_2, 1).$$

These inequalities, together with the fact that $h'(0) = h'(r_1) = h'(1) = 0$, imply that

$$h'(t) \geq 0 \text{ on } (0, r_1), \quad h'(t) \leq 0 \text{ on } (r_1, 1).$$

Since $h(0) = h(1) = 0$, we have $h(t) \geq 0$ on $(0, 1)$. The proof is complete. $\square$

Lemma 3.3. *If $u \in C^4[0,1]$ satisfies (1.2) and (3.1), then*

$$u(t) \leq u(1)b(t) \quad \textit{for} \quad t \in [0,1]. \tag{3.5}$$

Proof. Without loss of generality, we assume that $u(1) = 1$. If we define

$$h(t) = b(t)u(1) - u(t) = 2t - t^2 - u(t), \quad 0 \leq t \leq 1,$$

then

$$h'(t) = 2 - 2t - u'(t), \quad h''(t) = -2 - u''(t),$$

$$h'''(t) = -u'''(t), \quad 0 \leq t \leq 1. \tag{3.6}$$

It is easy to see that $h(0) = h(1) = h'(1) = 0$. By mean value theorem, because $h(0) = h(1) = 0$, there exists $r_1 \in (0, 1)$ such that $h'(r_1) = 0$. We see from (3.6) and (3.2) that $h'''(t) \geq 0$ on $[0, 1]$, which implies that h' is concave upward on $[0, 1]$. Since $h'(r_1) = h'(1) = 0$, we have

$$h'(t) \geq 0 \text{ on } (0, r_1), \quad h'(t) \leq 0 \text{ on } (r_1, 1).$$

Since $h(0) = h(1) = 0$, we have $h(t) \geq 0$ on $(0, 1)$. The proof is complete. $\square$

Lemma 3.4. *If $u \in C^4[0,1]$ satisfies (1.2) and (3.1), and $u''''(t)$ is nondecreasing on $[0,1]$, then*

$$u(t) \leq u(1)c(t) \quad \textit{for} \quad t \in [0,1]. \tag{3.7}$$

Proof. Without loss of generality, we assume that $u(1) = 1$. If we define

$$h(t) = c(t)u(1) - u(t) = 4t^2 - 4t^3 + t^4 - u(t), \quad 0 \leq t \leq 1,$$

then

$$h'(t) = 8t - 12t^2 + 4t^3 - u'(t), \quad h''(t) = 8 - 24t + 12t^2 - u''(t),$$

$$h'''(t) = -24 + 24t - u'''(t),$$

$$h''''(t) = 24 - u''''(t), \quad 0 \leq t \leq 1. \tag{3.8}$$

It is easily seen that $h(0) = h(1) = h'(0) = h'(1) = 0$. By the mean value theorem, because $h(0) = h(1) = 0$, there exists $r_1 \in (0, 1)$ such that $h'(r_1) = 0$. Since $h'(0) = h'(r_1) = h'(1) = 0$, there exist $r_2 \in (0, r_1)$ and $t_2 \in (r_1, 1)$ such that $h''(r_2) = h''(t_2) = 0$. As a consequence, there exists $r_3 \in (r_2, t_2)$ such that $h'''(r_3) = 0$.

Note that u'''' is nondecreasing by assumption. It follows from (3.8) that $h'''(t)$ is concave downward. It is easy to see that $h'''(1) = 0$. Because $h'''(r_3) = h'''(1) = 0$, we have

$$h'''(t) \leq 0 \text{ on } (0, r_3), \quad \text{and} \quad h'''(t) \geq 0 \text{ on } (r_3, 1).$$

Since $h''(r_2) = h''(t_2) = 0$, we have

$$h''(t) \geq 0 \text{ on } (0, r_2), \quad h''(t) \leq 0 \text{ on } (r_2, t_2), \quad h''(t) \geq 0 \text{ on } (t_2, 1).$$

Because $h'(0) = h'(r_1) = h'(1) = 0$, we have

$$h'(t) \geq 0 \text{ for } t \in (0, r_1), \quad h'(t) \leq 0 \text{ for } t \in (r_1, 1).$$

Hence, $h(t) \geq 0$ for $0 \leq t \leq 1$. The proof is complete. $\square$

Theorem 3.5. *Suppose that (H1) and (H2) hold. If $u(t)$ is a nonnegative solution to the problem* (1.1)-(1.2)*, then $u(t)$ satisfies* (3.2)*,* (3.3)*, and* (3.5)*.*

Proof. If $u(t)$ is a nonnegative solution to the problem (1.1)-(1.2), then $u(t)$ satisfies the boundary conditions (1.2), and

$$u''''(t) = g(t)f(u(t)) \geq 0, \quad 0 \leq t \leq 1.$$

Now Theorem 3.5 follows directly from Lemmas 3.1, 3.2, and 3.3. The proof is complete. ☐

Theorem 3.6. *Suppose that (H1), (H2), and the following condition hold.*

(H3) *Both f and g are nondecreasing functions.*

If $u(t)$ is a nonnegative solution to the problem (1.1)-(1.2)*, then $u(t)$ satisfies* (3.2)*,* (3.3)*, and* (3.7)*.*

Proof. By Theorem 3.5, $u(t)$ satisfies (3.2) and (3.3). Therefore $u(t)$ is nondecreasing on $[0,1]$. It is obvious that $u''''(t) = g(t)f(u(t)) \geq 0$. By (H3), we have that $u''''(t) = g(t)f(u(t))$ is nondecreasing on the interval $[0,1]$. It follows directly from Lemma 3.4 that $u(t)$ satisfies (3.7). The proof is complete. ☐

4. Existence and Nonexistence Results

First, we define some important constants:

$$A = \int_0^1 G(1,s)g(s)a(s)\,ds, \quad B = \int_0^1 G(1,s)g(s)b(s)\,ds.$$

We also define

$$\begin{aligned}P = \big\{v \in X : v(1) \geq 0,\ v(t) \text{ is nondecreasing on } [0,1], \\ a(t)v(1) \leq v(t) \leq b(t)v(1) \text{ on } [0,1]\big\}.\end{aligned}$$

Clearly P is a positive cone in X. It is obvious that if $u \in P$, then $u(1) = \|u\|$. We see from Theorem 3.5 that if $u(t)$ is a nonnegative solution to the problem (1.1)-(1.2), then $u \in P$. In a similar fashion to Theorem 3.5, we can show that $T(P) \subset P$. To find a positive solution to the problem (1.1)-(1.2), we need only to find a fixed point u of T such that $u \in P$ and $u(1) = \|u\| > 0$.

The next two theorems provide sufficient conditions for the existence of at least one positive solution for the problem (1.1)-(1.2).

Theorem 4.1. *Suppose that (H1) and (H2) hold. If $BF_0 < 1 < Af_\infty$, then problem* (1.1)-(1.2) *has at least one positive solution.*

Proof. First, we choose $\varepsilon > 0$ such that $(F_0 + \varepsilon)B \leq 1$. By the definition of F_0, there exists $H_1 > 0$ such that $f(x) \leq (F_0 + \varepsilon)x$ for $0 < x \leq H_1$. Now for each $u \in P$ with $\|u\| = H_1$, we have

$$\begin{aligned}(Tu)(1) &= \int_0^1 G(1,s)g(s)f(u(s))\,ds \\ &\leq \int_0^1 G(1,s)g(s)(F_0+\varepsilon)u(s)\,ds \\ &\leq (F_0+\varepsilon)\|u\| \int_0^1 G(1,s)g(s)b(s)\,ds \\ &= (F_0+\varepsilon)\|u\|B \leq \|u\|,\end{aligned}$$

which means $\|Tu\| \le \|u\|$. Thus, if we let $\Omega_1 = \{u \in X \mid \|u\| < H_1\}$, then

$$\|Tu\| \le \|u\| \quad \text{for} \quad u \in P \cap \partial\Omega_1.$$

To construct Ω_2, we choose $\delta > 0$ and $\tau \in (0, 1/4)$ such that

$$\int_\tau^1 G(1,s)g(s)a(s)\,ds \cdot (f_\infty - \delta) \ge 1.$$

There exists $H_3 > 2H_1$ such that $f(x) \ge (f_\infty - \delta)x$ for $x \ge H_3$. Let $H_2 = H_3/\tau^2$. If $u \in P$ such that $\|u\| = H_2$, then for each $t \in [\tau, 1]$, we have

$$u(t) \ge H_2 a(t) \ge H_2 t^2 \ge H_2 \tau^2 \ge H_3.$$

Therefore, for each $u \in P$ with $\|u\| = H_2$, we have

$$\begin{aligned}
(Tu)(1) &= \int_0^1 G(1,s)g(s)f(u(s))\,ds \\
&\ge \int_\tau^1 G(1,s)g(s)f(u(s))\,ds \\
&\ge \int_\tau^1 G(1,s)g(s)(f_\infty - \delta)u(s)\,ds \\
&\ge \int_\tau^1 G(1,s)g(s)a(s)\,ds \cdot (f_\infty - \delta)\|u\| \ge \|u\|,
\end{aligned}$$

which means $\|Tu\| \ge \|u\|$. Thus, if we let $\Omega_2 = \{u \in X \mid \|u\| < H_2\}$, then $\overline{\Omega_1} \subset \Omega_2$, and

$$\|Tu\| \ge \|u\| \quad \text{for} \quad u \in P \cap \partial\Omega_2.$$

Now that the condition (K1) of Theorem 2.1 is satisfied, there exists a fixed point of T in $P \cap (\overline{\Omega_2} - \Omega_1)$. The proof is now complete. □

Theorem 4.2. *Suppose that (H1) and (H2) hold. If $BF_\infty < 1 < Af_0$, then the problem* (1.1)-(1.2) *has at least one positive solution.*

The proof of Theorem 4.2 is very similar to that of Theorem 4.1 and therefore omitted. The next two theorems provide sufficient conditions for the nonexistence of positive solutions to the problem (1.1)-(1.2).

Theorem 4.3. *Suppose (H1) and (H2) hold. If $Bf(x) < x$ for all $x > 0$, then the problem* (1.1)-(1.2) *has no positive solutions.*

Proof. Assume the contrary that $u(t)$ is a positive solution of the problem (1.1)-(1.2). Then $u \in P$, $u(t) > 0$ for $0 < t \le 1$, and

$$\begin{aligned}
u(1) &= \int_0^1 G(1,s)g(s)f(u(s))\,ds \\
&< B^{-1}\int_0^1 G(1,s)g(s)u(s)\,ds \\
&\le B^{-1}\int_0^1 G(1,s)g(s)b(s)\,ds \cdot u(1) \\
&= B^{-1}Bu(1) = u(1),
\end{aligned}$$

which is a contradiction. The proof is complete. □

Theorem 4.4. *Suppose (H1) and (H2) hold. If $Af(x) > x$ for all $x > 0$, then the problem* (1.1)-(1.2) *has no positive solutions.*

5. More Existence and Nonexistence Results

In this section, we define a new constant

$$C = \int_0^1 G(1,s)g(s)c(s)\,ds,$$

and define the positive cone Q of X by

$$\begin{aligned} Q = \big\{ v \in X : v(1) \geq 0,\ v(t) \text{ is nondecreasing on } [0,1], \\ a(t)v(1) \leq v(t) \leq c(t)v(1) \text{ on } [0,1] \big\}. \end{aligned}$$

It is obvious that if $u \in Q$, then $u(1) = \|u\|$. We see from Theorem 3.6 that if (H1), (H2), and (H3) hold, and $u(t)$ is a nonnegative solution to the problem (1.1)-(1.2), then $u \in Q$. In a similar fashion to Theorem 3.6, we can show that if (H1), (H2), and (H3) hold, then $T(Q) \subset Q$.

Theorem 5.1. *Suppose that (H1), (H2), and (H3) hold. If either $CF_0 < 1 < Af_\infty$ or $CF_\infty < 1 < Af_0$, then problem* (1.1)-(1.2) *has at least one positive solution.*

The proof of the above theorem is omitted, because it is very similar to that of Theorem 4.1. The only difference is that we use the positive cone Q, instead of P, in the proof of Theorem 5.1.

Theorem 5.2. *Suppose (H1), (H2), and (H3) hold. If $Cf(x) < x$ for all $x > 0$, then problem* (1.1)-(1.2) *has no positive solutions.*

The proof of the above theorem is quite similar to that of Theorem 4.3 and therefore omitted.

Example 5.3. Consider the beam equation

$$u''''(t) = \lambda(t + 2t^2)u(t)(1 + 2u(t))/(1 + u(t)), \quad 0 \leq t \leq 1, \tag{5.1}$$

where $\lambda > 0$ is a parameter, together with the boundary conditions (1.2). In this example, $g(t) = t + 2t^2$ and $f(u) = \lambda u(1 + 2u)/(1 + u)$. It is easy to see that $f_0 = F_0 = \lambda$, $f_\infty = F_\infty = 2\lambda$, and

$$\lambda x < f(x) < 2\lambda x \quad \text{for} \quad x > 0.$$

Calculations indicate that $A = 47/756$, $B = 43/630$, and $C = 1937/30240$. By Theorem 4.1, if

$$8.04 \approx 1/(2A) < \lambda < 1/B \approx 14.56,$$

then the problem (5.1)-(1.2) has at least one positive solution. From Theorems 4.3 and 4.4 we see that if

$$\lambda \leq 1/(2B) \approx 7.326 \quad \text{or} \quad \lambda \geq 1/A \approx 16.085,$$

then the problem (5.1)-(1.2) has no positive solutions.

Note that $g(t)$ is increasing on $[0, 1]$, and $f(u)$ is increasing on $[0, +\infty)$. Therefore Theorems 5.1 and 5.2 apply. From Theorem 5.1 we see that if

$$8.04 \approx 1/(2A) < \lambda < 1/C \approx 15.612,$$

then the problem (5.1)-(1.2) has at least one positive solution. From Theorem 5.2 we see that if

$$\lambda \le 1/(2C) \approx 7.806,$$

then the problem (5.1)-(1.2) has no positive solutions. This example shows that our existence and nonexistence results are quite sharp indeed.

Acknowledgment. The author is grateful to the anonymous referee for his/her valuable comments and suggestions.

References

[1] R. P. Agarwal; *On fourth-order boundary-value problems arising in beam analysis, Differential Integral Equations*, 2 (1989), 91–110.

[2] P. R. Agarwal, D. O'Regan, and V. Lakshmikantham; *Singular $(p, n-p)$ focal and (n, p) higher order boundary-value problems*, Nonlinear Anal., 42 (2000), no. 2, Ser. A: Theory Methods, 215–228.

[3] D. R. Anderson and J. M. Davis; *Multiple solutions and eigenvalues for third-order right focal boundary-value problems*, J. Math. Anal. Appl., 267 (2002), no. 1, 135–157.

[4] Z. Bai and H. Wang; *On positive solutions of some nonlinear fourth-order beam equations*, J. Math. Anal. Appl., 270 (2002), no. 2, 357–368.

[5] R. Dalmasso; *Uniqueness of positive solutions for some nonlinear fourth-order equations*, J. Math. Anal. Appl., 201 (1996), no. 1, 152–168.

[6] J. Davis and J. Henderson; *Uniqueness implies existence for fourth-order Lidstone boundary-value problems*, Panamer. Math. J., 8 (1998), no. 4, 23–35.

[7] D. R. Dunninger; *Multiplicity of positive solutions for a nonlinear fourth order equation*, Ann. Polon. Math., 77 (2001), no. 2, 161–168.

[8] M. B. M. Elgindi and Z. Guan; *On the global solvability of a class of fourth-order nonlinear boundary-value problems*, Internat. J. Math. Math. Sci., 20 (1997), no. 2, 257-262.

[9] P. W. Eloe, J. L. Henderson, and N. Kosmatov; *Countable positive solutions of a conjugate type boundary-value problem*, Commun. Appl. Nonlinear Anal. 7 (2000), 47–55.

[10] J. R. Graef and B. Yang; *Existence and nonexistence of positive solutions of fourth order nonlinear boundary-value problems*, Appl. Anal., 74 (2000), no. 1–2, 201–214.

[11] J. R. Graef and B. Yang; *On a nonlinear boundary-value problem for fourth order equations*, Appl. Anal., 72 (1999), no. 3–4, 439–448.

[12] C. P. Gupta; *A nonlinear boundary-value problem associated with the static equilibrium of an elastic beam supported by sliding clamps*, Internat. J. Math. Math. Sci. 12 (1989), no. 4, 697–711.

[13] C. P. Gupta; *Existence and uniqueness theorems for the bending of an elastics beam equation*, Appl. Anal., 26 (1988), no. 4, 289–304.

[14] J. Henderson and H. Wang; *Positive solutions for nonlinear eigenvalue problems*, J. Math. Anal. Appl., 208 (1997), no. 1, 252–259.

[15] M.A. Krasnosel'skii; *Positive Solutions of Operator Equations*, Noordhoff, Groningen, 1964.

[16] N. Kosmatov; *Countably many solutions of a fourth order boundary-value problem*, Electronic J. of Qualitative Theory of Differential Equations, 2004, no. 12, 1–15.

[17] R. Ma; *Multiple positive solutions for a semipositone fourth-order bounadry value problem*, Hiroshima Math. J., 33 (2003), 217–227.

[18] R. Ma; *Existence and uniqueness theorems for some fourth-order nonlinear boundary-value problems*, Int. J. Math. Math. Sci., 23 (2000), no. 11, 783–788.

[19] R. Ma and H. Wang; *On the existence of positive solutions of fourth-order ordinary differential equations*, Applicable Analysis, 59 (1995), no. 1–4, 225–231.

[20] B. Yang; *Positive solutions for a fourth order boundary-value problem*, Electronic J. of Qualitative Theory of Differential Equations, 2005, no. 3, pp. 1–17.

[21] Y. Yang; *Fourth-order two-point boundary-value problems*, Proc. Amer. Math. Soc., 104 (1988), no. 1, 175–180.

Bo Yang

Department of Mathematics, Kennesaw State University, Kennesaw, GA 30144, USA

E-mail address: byang@kennesaw.edu