Electronic Journal of Differential Equations, Vol. 2006(2006), No. 119, pp. 1–7. ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu ftp ejde.math.txstate.edu (login: ftp)

# DISSIPATIVITY OF NEURAL NETWORKS WITH CONTINUOUSLY DISTRIBUTED DELAYS

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ABSTRACT. In this paper, we study the dissipativity and existence of a global attracting set for neural networks models with continuously distributed delays. We use nonnegative matrix and differential inequality techniques to obtain results under general conditions. Also, we give an example to illustrate our results.

### 1. INTRODUCTION

In recent years, the neural networks models have received much attention in the literature and have been applied in many fields such as control, image processing and optimization because of its good properties of controlling. Many results about dynamical behavior of neural networks systems without delays had been produced. As is well known, from a practical point of view, both in biological and manmade neural networks, the delays arise because of the processing of information. More specifically, in the electronic implementation of analog neural networks, the delays occur in the communication and response of the neurons owing to the finite switching speed of amplifiers. Thus, studying of neural networks dynamics with consideration of the delayed problem becomes very important to manufacture high quality neural networks models. In practice, although the use of constant discrete delays in the models serve as a good approximation in simple circuits consisting of a small number of neurons, neural networks usually have a spatial extent due to the presences of a multitude of parallel pathway with a variety of axon sizes and lengths. Therefore there will be a distribution of conduction velocities along these pathways and a distribution of propagation be designed with discrete delays and a more appropriate way is to incorporate continuously distributed delays. Then the study for dynamical behaviors of neural networks models with continuously distributed delays is more important and appropriate. In general application, people pay much attention to the stability of those neural networks models. But at the same time, the dissipativity is also an important concept which is more general than stability in chaos, synchronization, system norm estimation and robust control. There are some results about stability of neural networks models with continuously distributed

<sup>2000</sup> Mathematics Subject Classification. 34D40, 34D45, 34K12.

Key words and phrases. Differential inequality; dissipativity; stability;

global attracting set; spectral radius.

<sup>©2006</sup> Texas State University - San Marcos.

Submitted April 22, 2006. Published September 28, 2006.

The work is supported by National Natural Science Foundation of China under Grant 10671133.

delay and some results about dissipativity of those with or without discrete delays [6, 10]. To the best of my knowledge, few authors have discussed disspativity of the neural networks models with continuously distributed delays. We establish a method for dissipativity of neural networks models with continuously distributed delays. This method based on the properties of nonnegative matrix [4, 5] and differential inequality technique [9, 10], yields some new criterions for dissipativity and global attracting set. Moreover, these criterions are easy to check and apply in practice.

# 2. Preliminaries

Throughout this paper,  $\mathbb{R}^n$  denotes the *n*-dimensional Euclidean space,  $\mathbb{R}_n^+ = [0, +\infty) \times \cdots \times [0, +\infty)$  and C[X, Y] is the class of continuous mapping from the topological space X to the topological space Y.

Consider the neural networks system with continuously distributed delays as follows:

$$\dot{x}_{i}(t) = -\mu_{i}x_{i}(t) + \sum_{j=1}^{n} a_{ij}f_{j}(x_{j}) + \sum_{j=1}^{n} b_{ij} \int_{-\infty}^{t} K_{ij}(t-s)g_{j}(x_{j}(s))ds + P_{i}(t), \quad t \ge 0$$

$$x_{i}(t) = \phi_{i}(t), \quad -\infty < t \le 0, \ i = 1, \dots, n$$

$$(2.1)$$

where *n* denotes the number of neurons in the neural network,  $x_i(t)$  corresponds to the state of the *i*th neuron.  $f_j$  and  $g_j$  denote the activation functions of the *j*th neuron.  $a_{ij}$  and  $b_{ij}$  represent the constant connection weight of the *j*th neuron to the *i*th neuron.  $P_i(t)$  is the external input bias on the *i*th neuron.  $\mu_i > 0$  represents the rate with which the *i*th neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs.  $K_{ij}$  denotes the refractoriness of the *i*, *j*th neuron after it has fired or responded. The initial functions  $\phi_i \in C[(-\infty, 0], \mathbb{R}^n]$  (i = 1, ..., n) are bounded. The delay kernels  $K_{ij}$ with  $\int_0^\infty K_{ij}(s) ds = 1$  are real valued nonnegative continuous functions defined on  $[0, \infty)$ .  $f_j, g_j$  and  $P_i$  are continuous functions.

Clearly, the system above is a basic frame of neural networks. For example, we can obtain models with discrete delays in [2,3,5-7] when function  $K_{ij}$  is  $\delta$ -function. In addition, we can obtain Hopfield neural networks [3] when  $b_{ij} = 0$ , for  $i, j = 1, \ldots, n$ . Meanwhile, if  $P_i(t)$  is constant, we will obtain the system in [2].

For convenience, in the following we shall rewrite (2.1) in the form

$$\dot{x}(t) = -\mu x(t) + Af(x) + \int_{-\infty}^{t} K_0(t-s)g(x(s))ds + P(t), \quad t \ge 0$$
  
$$x_0(s) = \phi(s), \quad -\infty < s \le 0,$$
(2.2)

in which  $x(t) = col\{x_1(t), ..., x_n(t)\}, \mu = diag\{\mu_1, ..., \mu_n\},\$ 

$$f(x) = \operatorname{col}\{f_1(x_1), \dots, f_n(x_n)\}, \quad g(x) = \operatorname{col}\{g_1(x_1), \dots, g_n(x_n)\};$$
$$P(t) = \operatorname{col}\{P_1(t), \dots, P_n(t)\}, \quad A = (a_{ij})_{n \times n}, \quad K_0(\cdot) = (b_{ij}K_{ij}(\cdot))_{n \times n}.$$

Let  $x_t(s) = x(t+s), -\infty < s \le 0$ , then  $x_0(s) = x(s) = \phi(s), -\infty < s \le 0$ .

We always assume that system (2.2) has a continuous solution denoted by  $x(t, 0, \phi)$  or simply by x(t) if no confusion should occur. The inequalities between

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matrices or vectors such as  $A \leq B$  (A > B) means that each pair of corresponding elements of A and B satisfies the inequality. Especially, A is called a nonnegative matrix if  $A \geq 0$ .

Let C be the set of all functions  $\phi \in C[(-\infty,0],\mathbb{R}^n]$ , in which each  $\phi(s) = \operatorname{col}\{\phi_1(s),\ldots,\phi_n(s)\}$  satisfies that  $\sup_{-\infty < s \le 0} |\phi_i(s)|$  always exists as a finite number. For  $x \in \mathbb{R}^n$ , we define  $[x]^+ = \operatorname{col}\{|x_1|,\ldots,|x_n|\}$ . For  $\phi(s) \in C$ ,  $[\phi(s)]^+_{\infty} = \operatorname{col}\{|\phi_1(s)||_{\infty},\ldots,\|\phi_n(s)||_{\infty}\}$ , where  $\|\phi_i(s)\|_{\infty} = \sup_{-\infty < s < 0} |\phi_i(s)|$ .

In the following, we shall give the same definitions as those for the networks with discrete delays given in [1, 6, 7, 8].

**Definition 2.1.** A set  $S \subset C$  is called a positive invariant set of (2.2), if for any initial value  $\phi \in S$ , we have the solution of (2.2)  $x_t(s, 0, \phi) \in S$ , for  $t \geq 0$ ,  $-\infty < s \leq 0$ .

**Definition 2.2.** The system (2.2) is called dissipativity, if there exists a constant vector L > 0, such that for any initial value  $\phi \in C$ , there is a  $T(0, \phi)$ , when  $t > T(0, \phi)$ , the solution  $x(t, 0, \phi)$  of system (2.2) satisfies  $[x(t, 0, \phi)]^+ \leq L$ . In this case, the set  $\Omega = \{\phi \in C | [\phi(s)]_{\infty}^+ \leq L\}$  is said to be the global attracting set of (2.2).

Before we discuss the system (2.2) in detail, we need two Lemmas as follows.

**Lemma 2.3** ([5]). If  $M \ge 0$  and  $\rho(M) < 1$ , then  $(I - M)^{-1} \ge 0$ .

**Lemma 2.4** ([4]). Let  $M \ge 0$  and  $\tilde{M}$  is any principal sub-matrix of M, then  $\rho(\tilde{M}) \le \rho(M)$ .

The symbol  $\rho(M)$  and the matrix I denote the spectral radius of a square matrix M and a unit matrix, respectively. In our analysis, we always suppose that:

(A1) There are matrices  $\alpha = \text{diag}\{\alpha_1, \dots, \alpha_n\}$  and  $\beta = \text{diag}\{\beta_1, \dots, \beta_n\}$  with  $\alpha_j > 0$  and  $\beta_j > 0$ , such that for any  $x \in \mathbb{R}^n$ 

 $[f(x)]^+ \le \alpha[x]^+, \quad [g(x)]^+ \le \beta[x]^+.$ 

- (A2)  $\rho(M) < 1$ , where  $M = \mu^{-1}A^+\alpha + \mu^{-1}B^+\beta$ ,  $\mu = \text{diag}\{\mu_1, \dots, \mu_n\}, A^+ = (|a_{ij}|)_{n \times n}, B^+ = (|b_{ij}|)_{n \times n}$ .
- (A3)  $[R(t)]^+ \leq R$ , where  $R(t) = \mu^{-1}P(t)$  and  $R = \operatorname{col}\{R_1, \dots, R_n\}$  with  $R_i \geq 0$ .

# 3. Dissipativity analysis

In this section, combining the inequality technique [9, 10] with properties of nonnegative matrices [4, 5], we introduce some new results for the dissipativity of system (2.2).

**Theorem 3.1.** If (A1)–(A3) hold, then the set  $\Omega = \{\phi \in C | [\phi]_{\infty}^+ \leq L\}$  is a positive invariant set of system (2.2), where  $L = (I - M)^{-1}R = \operatorname{col}\{L_1, \ldots, L_n\}$ .

*Proof.* According to Definition 2.1, we need to prove that for any  $\phi \in C$  and  $[\phi]^+_{\infty} \leq L$ , the solution  $x(t) \stackrel{\Delta}{=} x(t, 0, \phi)$  of system (2.2) satisfies

$$[x(t)]^+ \le L, \quad \text{for } t \ge 0.$$
 (3.1)

For the proof of this inequality, we first prove, for any given  $\gamma > 1$ , when  $[\phi]_{\infty}^+ < \gamma L$ , the solution x(t) satisfies

$$[x(t)]^+ < \gamma L, \quad \text{for } t \ge 0. \tag{3.2}$$

$$|x_i(t_1)| = \gamma L_i, \quad |x_i(t)| < \gamma L_i \quad \text{for } 0 \le t < t_1;$$
(3.3)

$$[x(t)]^+ < \gamma L, \quad \text{for } 0 \le t < t_1.$$
 (3.4)

where  $L_i$  is the *i*th component of vector L.

Note that  $L = (I - M)^{-1}R$ , i.e., L = LM + R. Then, it follows from (2.2), (3.3), (3.4) that

$$\begin{split} [x(t_1)]^+ &\leq e^{-\mu t_1} [\phi]_{\infty}^+ + \int_0^{t_1} e^{-\mu (t_1 - s)} [Af(x(s))]^+ ds \\ &+ \int_0^{t_1} e^{-\mu (t_1 - s)} \{ \int_{-\infty}^s [K_0(s - \theta)]^+ [g(x(\theta))]^+ d\theta + [P(s)]^+ \} ds \\ &\leq e^{-\mu t_1} [\phi]_{\infty}^+ + \int_0^{t_1} e^{-\mu (t_1 - s)} (A^+ \alpha [x(s)]^+) ds \\ &+ \int_0^{t_1} e^{-\mu (t_1 - s)} \{ \int_{-\infty}^s [K_0(s - \theta)]^+ (\beta [x(\theta)]^+) d\theta + [P(s)]^+ \} ds \\ &< e^{-\mu t_1} [\phi]_{\infty}^+ + (I - e^{-\mu t_1}) [\mu^{-1} (A^+ \alpha + B^+ \beta) \gamma L + R] \\ &\leq e^{-\mu t_1} \gamma L + (I - e^{-\mu t_1}) (M \gamma L + R) < \gamma L \end{split}$$

This inequality implies  $|x_i(t_1)| < \gamma L_i$  (i = 1, ..., n), which contradicts with (3.3). Therefore, (3.2) holds. Let  $\gamma \to 1^+$ , we obtain that

$$[x(t)]^+ \le L, \quad \text{for } t \ge 0$$

The proof is complete.

**Theorem 3.2.** If (A1)–(A3) hold, then the system (2.2) is dissipativity and the set  $\Omega = \{\phi \in C | [\phi(s)]_{\infty}^+ \leq L\}$  is the global attracting set of (2.2).

*Proof.* Without losing generality, we assume L > 0. First, we prove that for any initial value  $\phi \in C$ , there exists a number  $\Gamma > 0$ , large enough, such that the solution x(t) satisfies

$$[x(t)]^+ < \Gamma L, \quad \text{for } t \ge 0.$$
 (3.5)

For a given  $\phi \in C$ , there must be a large enough positive number  $\Gamma$ , such that  $[\phi(t)]^+_{\infty} < \Gamma L$ .

If (3.5) is not true, then there must be some *i* and  $t_2 > 0$ , such that

$$|x_i(t_2)| = \Gamma L_i, |x_i(t)| < \Gamma L_i \text{ for } 0 \le t < t_2;$$
(3.6)

$$[x(t)]^+ < \Gamma L, \quad \text{for } 0 \le t < t_2.$$
 (3.7)

From (2.2), (3.6), (3.7), and L = LM + R, we obtain

- **t** -

$$\begin{aligned} [x(t_2)]^+ &\leq e^{-\mu t_2} [\phi]_{\infty}^+ + \int_0^{t_2} e^{-\mu (t_2 - s)} [Af(x(s))]^+ ds \\ &+ \int_0^{t_2} e^{-\mu (t_2 - s)} \{ \int_{-\infty}^s [K_0(s - \theta)]^+ [g(x(\theta))]^+ d\theta + [P(s)]^+ \} ds \\ &\leq e^{-\mu t_2} [\phi]_{\infty}^+ + \int_0^{t_2} e^{-\mu (t_2 - s)} (A^+ \alpha [x(s)]^+) ds \\ &+ \int_0^{t_2} e^{-\mu (t_2 - s)} \{ \int_{-\infty}^s [K_0(s - \theta)]^+ (\beta [x(\theta)]^+) d\theta + [P(s)]^+ \} ds \end{aligned}$$

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$$< e^{-\mu t_2} [\phi]_{\infty}^+ + (I - e^{-\mu t_2}) [\mu^{-1} (A^+ \alpha + B^+ \beta) \Gamma L + R]$$
  
 
$$\le e^{-\mu t_2} \Gamma L + (I - e^{-\mu t_2}) (M \Gamma L + R) < \Gamma L$$

From the above inequality, it follows that  $|x_i(t_2)| < \Gamma L_i$  (i = 1, ..., n), which contradicts with (3.6), and so (3.5) is true.

In view of Definition 2.2, for the proof of Theorem 3.2, we need to prove that for the above positive vector L, any solution x(t) of the system (2.2) satisfies

$$[x(t)]^+ \le L, \quad \text{as } t \to +\infty. \tag{3.8}$$

To prove (3.8), we first verify that

$$\limsup_{t \to +\infty} [x(t)]^+ \le L. \tag{3.9}$$

It is equivalent to prove that

$$\limsup_{t \to +\infty} ([x(t)]^+ - L) = \sigma \le 0.$$
(3.10)

If (3.10) is false, then there must exist some  $\sigma_i > 0$ . Without losing generality, we denote such components of  $\sigma$  by  $\sigma_{i_1}, \sigma_{i_2}, \ldots, \sigma_{i_m}$ , where  $\sigma_{i_j} > 0, j = 1, \ldots, m$ . By the definition of superior limit and (3.10), for sufficient small constant  $\varepsilon > 0$ , there is  $t_3 > 0$ , such that

$$[x(t)]^+ \le L + \sigma + \epsilon, \quad \text{for } t \ge t_3. \tag{3.11}$$

where  $\epsilon = \operatorname{col}\{\varepsilon, \ldots, \varepsilon\}$ . Meanwhile, since  $\int_0^\infty K_{ij} ds = 1$   $(i, j = 1, \ldots, n)$ , then for the above  $\epsilon$  and  $\Gamma L$  in (3.5), there must be a T > 0, such that, for any t > T,

$$\int_{T}^{\infty} K_{0}(t)\beta\Gamma Ldt \le \epsilon$$
(3.12)

When  $t > t^* \stackrel{\Delta}{=} t_3 + T$ , combining (2.2) (3.11) with (3.12), we can obtain  $D^+[x(t)]^+ + \mu[x(t)]^+$ 

$$\leq A^{+}[f(x(t))]^{+} + \int_{-\infty}^{t} K_{0}(t-s)[g(x(s))]^{+}ds + [P(t)]^{+}$$

$$= A^{+}[f(x(t))]^{+} + \int_{-\infty}^{t-T} K_{0}(t-s)[g(x(s))]^{+}ds$$

$$+ \int_{t-T}^{t} K_{0}(t-s)[g(x(s))]^{+}ds + [P(t)]^{+}$$

$$\leq A^{+}[f(x(t))]^{+} + \int_{T}^{+\infty} K_{0}(s)\beta\Gamma Lds + \int_{t-T}^{t} K_{0}(t-s)\beta(L+\sigma+\epsilon)ds + [P(t)]^{+}$$

$$\leq A^{+}(\alpha[x(t)]^{+}) + \epsilon + B^{+}\beta(L+\sigma+\epsilon) + [P(t)]^{+}$$
(3.13)

in which  $D^+[x(t)]^+$  denotes the Dini derivative of the positive vector  $[x(t)]^+$ . Let both sides of (3.13) integrate from  $t^*$  to t after they multiply  $e^{-\mu(t-s)}$ . Thus,

$$[x(t)]^{+} \leq e^{-\mu(t-t^{*})} [x(t^{*})]^{+} + (I - e^{-\mu(t-t^{*})}) \\ \times \left[\mu^{-1}A^{+}\alpha(L+\sigma+\epsilon) + \mu^{-1}B^{+}\beta(L+\sigma+\epsilon) + R + \mu^{-1}\epsilon\right]$$
(3.14)

Since  $M = \mu^{-1}A^+\alpha + \mu^{-1}B^+\beta$ , it follows from (3.5) and (3.14) that

$$[x(t)]^{+} \le e^{-\mu(t-t^{*})}\Gamma L + \left(I - e^{-\mu(t-t^{*})}\right) \left[ML + M\epsilon + M\sigma + R + \mu^{-1}\epsilon\right]$$
(3.15)

By the definition of superior limit and (3.10), there are  $t_k \to +\infty$ , such that

$$\lim_{k \to +\infty} [x(t_k)]^+ = L + \sigma.$$
(3.16)

In (3.15), let  $t = t_k \to \infty$ ,  $\varepsilon \to 0^+$ . Then, from  $L = (I - M)^{-1}R$  and (3.16), it follows that

$$\sigma \le M\sigma \tag{3.17}$$

Let  $\tilde{\sigma} = \operatorname{col}\{\sigma_{i_1}, \ldots, \sigma_{i_m}\}$ , then from (3.17) follows that

$$\tilde{\sigma} \le \tilde{M}\tilde{\sigma} \tag{3.18}$$

where  $\tilde{M}$  is the *m*-by-*m* principal sub-matrix of *M* corresponding to the positive vector  $\tilde{\sigma}$ , i.e.,  $\tilde{M} = (m_{i_j, i_u}), j, u = 1, \dots, m$ .

By Lemma 2.3, we obtain that  $\rho(\tilde{M}) \ge 1$ . According to Lemma 2.4,  $\rho(M) \ge 1$  which contradict  $\rho(M) < 1$  in (A2). Then, for any  $i, \sigma_i \le 0$ , (3.9) holds. Farther, (3.8) holds. The proof is complete.

**corollary 3.3.** If (A1)-(A3) hold and R = 0 in (A3), then the system (2.2) has an unique equilibrium point  $x^* = 0$  which is global asymptotically stable.

By comparing this Corollary with [11, Theorem 4], we obtain the following remark.

**Remark 3.4.** When f = g,  $P_i(t) = I_i$ , the system (2.2) becomes the model studied by Zhao [11]. To obtain the global asymptotically stability, Zhao assumed (A2), that  $f_j$  satisfies  $xf_j(x) > 0(x \neq 0)$ , and that there exist a positive constant  $\lambda_j$ , such that  $\lambda_j = \sup_{x\neq 0} \frac{f_j(x)}{x}$ . These assumptions imply (A1); so that [11, Theorem 4] is a special case of the Corollary in this paper.

## 4. Illustrative example

We consider the neural networks model, for  $t \ge 0$ ,

$$\dot{x}_{1}(t) = -2x_{1}(t) + \frac{1}{2}\sin x_{1} + \frac{1}{2}\int_{-\infty}^{t} \frac{2}{\pi[1 + (t - s)^{2}]}|x_{1}(s)|ds + r\cos t$$

$$\dot{x}_{2}(t) = -2x_{2}(t) + \frac{1}{2}\sin x_{2} + \frac{1}{2}\int_{-\infty}^{t} \frac{2}{\pi[1 + (t - s)^{2}]}|x_{2}(s)|ds + r\sin t,$$
(4.1)

It can be obtained easily that  $\mu = \text{diag}\{2,2\}, A = \text{diag}\{\frac{1}{2},\frac{1}{2}\}, B = \text{diag}\{\frac{1}{2},\frac{1}{2}\}.$ The delay kernels functions  $K_{ij}(s) = \frac{2}{\pi[1+s^2]}$  (i, j = 1, 2) satisfy  $\int_0^\infty K_{ij}(s)ds = 1$ . Meanwhile, since  $f_1(x_1) = \sin x_1, f_2(x_2) = \sin x_2, g_1(x_1) = |x_1|, g_2(x_2) = |x_2|,$ the condition (A1) holds. With the  $P(t) = \text{col}\{r \cos t, r \sin t\}$ , we can get  $R = \text{col}\{\frac{1}{2}|r|, \frac{1}{2}|r|\}$ . By calculating,  $\rho(M) = \frac{1}{2} < 1$ , then (A2) holds.

By Theorem 3.2, when  $r \neq 0$ , the system (4.1) is dissipative and the set  $\{(x_1, x_2) : |x_1| \leq |r|, |x_2| \leq |r|\}$  is the global attracting set of (4.1). When r = 0, in view of the Corollary, the system (4.1) has a equilibrium point  $x^* = \operatorname{col}\{0, 0\}$  which is global asymptotically stable.

**Remark 4.1.** In this Example, when  $r \neq 0$ , we can not solve the problem on global attracting set with those results in [1, 6, 7, 8, 10] because of the continuously distributed delays. When r = 0, the delay kernels  $K_{ij}$  do not possess properties such as  $\int_0^\infty s K_{ij}(s) ds < \infty$  in [2]. Then it can not be considered with the methods in [2].

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**Conclusions.** In this paper, we research the dissipativity and global attracting set of neural networks models with continuously distributed delays which is the basic frame of neural networks. Combining the differential inequality technique with properties of nonnegative matrices, we introduce some sufficient criterions for dissipativity and a method to calculate the global attracting set of a general class of neural networks models with continuously distributed delays. Through the comparison and illustration of an example, we can see that the model studied in this paper is more general and our method can apply in more general neural networks models than those in the references.

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