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BOUNDEDNESS OF SOLUTIONS TO FOURTH-ORDER DIFFERENTIAL EQUATION WITH OSCILLATORY RESTORING AND FORCING TERMS

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Abstract. This paper concerns the fourth order differential equation

$$x'''' + ax''' + f(x'') + g(x') + h(x) = p(t).$$

Using the Cauchy formula for the particular solution of non-homogeneous linear differential equations with constant coefficients, we prove that the solution and its derivatives up to order three are bounded.

1. Introduction

This paper studies the boundedness of solutions of the fourth-order nonlinear differential equation

$$x'''' + ax''' + f(x'') + g(x') + h(x) = p(t), \tag{1.1}$$

where $a > 0$, f, g, h and p, and their first derivatives are continuous functions depending on the arguments shown. In addition, h and p are oscillatory in the following sense: For each argument u, there exist numbers $\beta_1 > \alpha_1 > u > \alpha_{-1} > \beta_{-1}$ such that

$$\phi(\alpha_1) < 0, \quad \phi(\beta_1) > 0, \quad \phi(\alpha_{-1}) < 0, \quad \phi(\beta_{-1}) > 0,$$

where ϕ is either $h(x)$ or $p(t)$, u is either x or t and all roots of the restoring term $h(x)$ are isolated.

There has been a lot of work concerning the boundedness of the solutions of nonlinear ordinary differential equations; see the references in this article and the references cited therein. We can mention in this direction, for fourth order nonlinear ordinary differential equations, the works of Afuwape and Adesina [1] where the frequency-domain approach was used, while Tiryaki and Tunc [12, 13, 14, 15, 16] have used the Lyapunov second method. All these results generalize in one way or another some results on third order nonlinear differential equations, see for instance [9, 10]. Equation (1.1) for which $f(x'') = bx''$ and $g(x') = cx'$, that is,

$$x'''' + ax''' + bx'' + cx' + h(x) = p(t),$$

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was studied by Omeike [8], recently, for the existence of bounded solutions, where a, b and c are assumed to satisfy conditions which ensure that the auxiliary equation,

$$\lambda^3 + a\lambda^2 + b\lambda + c = 0,$$

possesses negative real roots. Moreover, $a^2 > 4b$. Also, recently, Ogundare [7], studied (1.1) for which $f(x'') = bx''$, that is

$$x'''' + ax''' + bx'' + g(x') + h(x) = p(t),$$

and obtained results which ensure existence of a bounded solution.

Following the approach in [2, 7, 8], we shall use the Cauchy formula for the particular solution of the nonhomogeneous linear part of (1.1), to prove that the solution $x(t)$ and its derivatives $x'(t), x''(t)$ and $x'''(t)$ are bounded.

2. Preliminaries

In this section, we shall state and prove certain results useful in the proof of our main result in §3.

Lemma 2.1. *Assume there exist positive constants a, b, c, H, P, ($a^2 > 4b$) such that for all $x \in \mathbb{R}$ and $t \geq 0$ the following inequalities hold:*

(i) $|h(x)| \leq H$
(ii) $|p(t)| \leq P$
(iii) $0 < \frac{f(x'')}{x''} \leq b < \infty, f(0) = 0$
(iv) $0 < \frac{g(x')}{x'} \leq c < \infty, g(0) = 0.$

Then each solution $x(t)$ of (1.1) satisfies

$$\limsup_{t\to\infty} |x'''(t)| \leq \frac{4(H+P)}{a} := D''',$$

provided

$$\limsup_{t\to\infty} |x'(t)| \leq \frac{(H+P)}{c} := D', \tag{2.1}$$

$$\limsup_{t\to\infty} |x''(t)| \leq \frac{2(H+P)}{b} := D''. \tag{2.2}$$

Note that a, b and c satisfy conditions ensuring that the auxiliary equation

$$\lambda^3 + a\lambda^2 + b\lambda + c = 0$$

have negative real roots.

Proof of Lemma 2.1. Substituting $w := x'''$, from (1.1), we obtain the equation

$$w' + aw = p(t) - f(x''(t)) - g(x'(t)) - h(x(t)),$$

with solutions of the form

$$\begin{aligned} x'''(t) &= w(t) \\ &= Ce^{-at} + \int_{T_x}^{t} e^{-a(t-\tau)}[p(\tau) - f(x''(\tau)) - g(x'(\tau)) - h(x(\tau))]d\tau, \end{aligned}$$

where C is an arbitrary constant and T_x is a great enough number. Let (2.1) and (2.2) hold. Thus, by virtue of (i),(ii),(iii) and (iv), for $t \geq T_x$, we have not only that

$$
\begin{aligned}
&\Big|\int_{T_x}^{t} e^{-a(t-\tau)}[p(\tau) - f(x''(\tau)) - g(x'(\tau)) - h(x(\tau))]d\tau\Big| \\
&= \Big|\int_{T_x}^{t} e^{-a(t-\tau)}[p(\tau) - \frac{f(x''(\tau))}{x''(\tau)}x''(\tau) - \frac{g(x'(\tau))}{x'(\tau)}x'(\tau) - h(x(\tau))]d\tau\Big| \\
&\leq \int_{T_x}^{t} \Big|p(\tau) - \frac{f(x''(\tau))}{x''(\tau)}x''(\tau) - \frac{g(x'(\tau))}{x'(\tau)}x'(\tau) - h(x(\tau))\Big|e^{-a(t-\tau)}d\tau \\
&\leq \int_{T_x}^{t} \Big(|p(\tau)| + \Big|\frac{f(x''(\tau))}{x''(\tau)}\Big||x''(\tau)| + \Big|\frac{g(x'(\tau))}{x'(\tau)}\Big||x'(\tau)| + |h(x(\tau))|\Big)e^{-a(t-\tau)}d\tau \\
&\leq \int_{T_x}^{t} \big(P + b|x''(\tau)| + c|x'(\tau)| + H\big)e^{-a(t-\tau)}d\tau \\
&\leq \frac{4(H+P)}{a}\big(1 - e^{-a(t-T_x)}\big)
\end{aligned}
$$

but also that

$$
\limsup_{t\to\infty} |x'''(t)| \leq \frac{4(H+P)}{a}.
$$

□

Lemma 2.2. *Under the assumptions of Lemma 2.1, if*

(i) $|h'(x)| \leq H'$ *for all* $x \in \mathbb{R}$, *and*
(ii) $\big|\int_0^\infty p(t)dt\big| < \infty$,

where H' *is a suitable constant, then every bounded solution* $x(t)$ *of* (1.1) *either satisfies the relation*

$$
\lim_{t\to\infty} x(t) = \bar{x}, \quad \lim_{t\to\infty} x'(t) = \lim_{t\to\infty} x''(t) = \lim_{t\to\infty} x'''(t) = 0, \quad (h(\bar{x}) = 0) \tag{2.3}
$$

or there exists a root $\bar{x}$ *of* $h(x)$ *such that* $(x(t) - \bar{x})$ *oscillates.*

Proof. Substituting a fixed bounded solution $x(t)$ of (1.1) into (1.1) and integrating the result from T_x to t (T_x - a large enough number, whose magnitude will be specified later), we get the identity

$$
\begin{aligned}
\int_{T_x}^{t} h(x(\tau))d\tau = &-\{x'''(t) - x'''(T_x) + a[x''(t) - x''(T_x)]\} \\
&- \int_{T_x}^{t} f(x''(\tau))d\tau - \int_{T_x}^{t} g(x'(\tau))d\tau + \int_{T_x}^{t} p(\tau)d \\
&=: I(t).
\end{aligned}
$$

$$
\begin{aligned}
\Big|\int_{T_x}^t h(x(\tau))d\tau\Big| &\le |x'''(t)-x'''(T_x)| + a|x''(t)-x''(T_x)| + \Big|\int_{T_x}^t \frac{f(x''(\tau))}{x''(\tau)}x''(\tau)d\tau\Big| \\
&\quad + \Big|\int_{T_x}^t \frac{g(x'(\tau))}{x'(\tau)}x'(\tau)d\tau\Big| + \Big|\int_{T_x}^t p(\tau)d\tau\Big| \\
&\le |x'''(t)-x'''(T_x)| + a|x''(t)-x''(T_x)| \\
&\quad + \Big|\int_{T_x}^t b dx'(\tau)\Big| + \Big|\int_{T_x}^t c dx(\tau)\Big| + \Big|\int_{T_x}^t p(\tau)d\tau\Big| \\
&\le |x'''(t)-x'''(T_x)| + a|x''(t)-x''(T_x)| \\
&\quad + b|x'(t)-x'(T_x)| + c|x(t)-x(T_x)| + \Big|\int_{T_x}^t p(\tau)d\tau\Big|.
\end{aligned}
$$

Therefore, by virtue of condition (ii), the assertion of Lemma 2.1 and the boundedness of $x(t)$, there exists a constant M_x such that for $t \ge T_x$ the relation

$$|I(t)| \le M_x; \quad \text{i.e.,} \quad \Big|\int_{T_x}^t h(x(\tau))d\tau\Big| \le M_x\,. \tag{2.4}$$

Now, let us assume that $x(t)$ does not converge to any root $\bar{x}$ of $h(x)$: i.e.,

$$\limsup_{t\to\infty} |x(t)-\bar{x}| > 0 \tag{2.5}$$

and simultaneously, for $t \ge T_x$,

$$h(x(t)) \ge 0 \quad \text{or} \quad h(x(t)) \le 0. \tag{2.6}$$

Then

$$H(t) := \int_{T_x}^t h(x(\tau))d\tau \quad \text{for } t \ge T_x$$

evidently is a composed monotone function with a finite or infinite limit for $t \to \infty$. Since (2.4) implies that the "divergent case" can be disregarded, it follows from (2.6) that not only

$$\lim_{t\to\infty} \int_{T_x}^t |h(x(\tau))|d\tau = \lim_{t\to\infty} \Big|\int_{T_x}^t h(x(\tau))d\tau\Big| \le M_x \tag{2.7}$$

but also

$$\liminf_{t\to\infty} |x(t)-\bar{x}| = 0 \tag{2.8}$$

holds, because otherwise (i.e. if $\liminf_{t\to\infty} |x(t)-\bar{x}| > 0$) (2.6) together with the fact that the roots of $h(x)$ are isolated would yield

$$\liminf_{t\to\infty} |h(x(t))| = \liminf_{t\to\infty} |h(x(t)) - h(\bar{x})| > 0,$$

which is a contradiction to (2.7). Thus (2.5) and (2.8) imply

$$\limsup_{t\to\infty} |h(x(t))| = \limsup_{t\to\infty} |h(x(t)) - h(\bar{x})| > 0 = \liminf_{t\to\infty} |h(x(t))|$$

and consequently there exists a sequence $\{t_i\} \ge T_x$ and a positive constant $\tilde{H}$ such that

(a) $\liminf_{i\to\infty \Rightarrow t_i\to\infty} d(t_i, t_{i-1}) > 0$
(b) $|h(x(t_i))| \ge \tilde{H}$;

here and in what follows, $d(x,y)$ denotes the distance between x and y. Hence

$$M_x \ge \lim_{t\to\infty}\int_{t_1}^{t}|h(x(\tau))|d\tau = \sum_{i=2}^{\infty}\int_{t_{i-1}}^{t_i}|h(x(\tau))|d\tau$$

implies

$$\limsup_{i\to\infty\Rightarrow t_i\to\infty}\int_{t_{i-1}}^{t_i}|h(x(t))|dt = 0$$

or (cf. (a),(b)),

$$H'\limsup_{t\to\infty}|x'(t)| \ge \limsup_{t\to\infty}\Big|\frac{dh(x(t))}{dx(t)}x'(t)\Big| = \limsup_{t\to\infty}\Big|\frac{dh(x(t))}{dt}\Big| = \infty.$$

According to the assertion of Lemma 2.1, this is impossible and that is why $(x(t)-\bar{x})$ necessarily oscillates.

The remaining part of our lemma follows immediately from the assertion

$$\begin{gathered} x(t)\in C^{(n)}[0,\infty), \quad \limsup_{t\to\infty}|x^{(n)}(t)|<\infty, \quad \lim_{t\to\infty}|x(t)|<\infty \\ \Rightarrow \lim_{t\to\infty}x^{(k)}(t)=0, \end{gathered} \tag{2.9}$$

where n is a natural number greater than or equal to 3, and $k=1,\dots,n-1$. The proof of this statement can be found in [6, p.161]. This completes the proof. □

Lemma 2.3. *Under the assumptions of Lemma 2.2 and if*

(i) $|p'(t)|\le P'$ *for all* $t\ge 0$,
(ii) $\limsup_{t\to\infty}|p(t)|>0$
(iii) $|f'(x'')|\le b_0$
(iv) $|g'(x')|\le c_0$

where b_0, c_0, P' *are suitable constants, then for every bounded solution* $x(t)$ *of* (1.1) *there exists a root* $\bar{x}$ *of* $h(x)$ *such that* $(x(t)-\bar{x})$ *oscillates.*

Proof. If Lemma 2.3 does not hold, then according to Lemma 2.2, (2.3) holds and the derivatives of $x(t)$ satisfy

$$\begin{aligned} x^{(v)}(t) &= p'(t)-ax''''(t)-f'(x''(t))x'''(t)-g'(x'(t))x''(t)-h'(x(t))x'(t),\\ |x^{(v)}(t)| &= |p'(t)-ax''''(t)-f'(x''(t))x'''(t)-g'(x'(t))x''(t)-h'(x(t))x'(t)|\\ &\le |p'(t)|+a|x''''(t)|+|f'(x''(t))||x'''(t)|+|g'(x'(t))||x''(t)|\\ &\quad +|h'(x(t))||x'(t)|. \end{aligned}$$

Thus, by part (i) of Lemma 2.2 and parts (i), (iii) of Lemma 2.3, we have

$$|x^{(v)}(t)| \le P'+a|x''''(t)|+b_0|x'''(t)|+c_0|x''(t)|+H'|x'(t)|.$$

Hence by the boundedness of $x'(t), x''(t), x'''(t), x''''(t)$, there exists a constant K such that

$$\limsup_{t\to\infty}|x^{(v)}(t)|\le K,$$

which according to (2.9) gives

$$\lim_{t\to\infty}x(t)=\bar{x} \Longrightarrow \lim_{t\to\infty}h(x(t))=h(\bar{x})=0,\ \lim_{t\to\infty}x^{(j)}(t)=0,\ \ j=1,2,3$$

or

$$\limsup_{t\to\infty}|p(t)| = \limsup_{t\to\infty}|x''''(t)+ax'''(t)+bx''(t)+g(x'(t))+h(x(t))| = 0$$

a contradiction to $\limsup_{t\to\infty}|p(t)|>0$. □

3. Main Result

Now we can give the principal result of our paper.

Theorem 3.1. *If there exist positive constants H, H', P, P', P_0, R such that for $|x|>R$ and $t\geq 0$ the following conditions are satisfied:*

1. $|h(x)|\leq H$, $|h'(x)|\leq H'$
2. $0<\frac{f(x'')}{x''}\leq b<\infty$, $f(0)=0$,
3. $0<\frac{g(x')}{x'}\leq c<\infty$, $g(0)=0$,
4. $|p(t)|\leq P$, $|p'(t)|\leq P'$, $\left|\int_0^t p(\tau)d\tau\right|\leq P_0$, $\limsup_{t\to\infty}|p(t)|>0$,
5. $\min[d(\bar{x}_k,\bar{x}_{k+1}), d(\bar{x}_k,\bar{x}_{k-1})]>\frac{(H+P)}{c_1}\left(\frac{4}{a}+\frac{2a}{b}+\frac{b}{c}\right)+\frac{P_0}{c_1}$,

where $\bar{x}_k$ are roots of $h(x)$ with $h'(\bar{x}_k)>0$ and $\bar{x}_{k-1},\bar{x}_{k+1}$ denote the couple of adjacent roots of $\bar{x}_k$ ($k=0,\pm2,\pm4,\dots$), then all solutions $x(t)$ of (1.1) are bounded and for each of them there exists a root $\bar{x}$ of $h(x)$ such that $(x(t)-\bar{x})$ oscillates.

Proof. Let us assume, on the contrary, that $x(t)$ is an unbounded solution of (1.1); i.e., for example, $\limsup_{t\to\infty}x(t)=\infty$. Lemma 2.1 implies the existence of a number $T_0\geq 0$ great enough such that for $t\geq T_0$,

$$|x'(t)|\leq D'+\epsilon_1,\quad |x''(t)|\leq D''+\epsilon_2,\quad |x'''(t)|\leq D'''+\epsilon_3$$

with $\epsilon_1>0$, $\epsilon_2>0$, $\epsilon_3>0$ small enough. Let $T_1\geq T_0$ be the last point with $x(T_1)=x_k(k-\text{even})$ and $T_2>T_1$ be the first point with $x(T_2)=\bar{x}_{k+1}$. If we integrate (1.1) from T_1 to $t, T_1\leq t\leq T_2$, we come to

$$\begin{aligned}
&[x'''(t)-x'''(T_1)]+a[x''(t)-x''(T_1)]\\
&+\int_{T_1}^t f(x''(\tau))d\tau+\int_{T_1}^t g(x'(\tau))d\tau+\int_{T_1}^t h(x(\tau))d\tau\\
&=\int_{T_1}^t p(\tau)d\tau.
\end{aligned}$$

Thus,

$$\begin{aligned}
\int_{T_1}^t \frac{g(x'(\tau))}{x'(\tau)}dx(\tau)&=\int_{T_1}^t p(\tau)d\tau+x'''(T_1)-x'''(t)+a[x''(T_1)-x''(t)]\\
&\quad-\int_{T_1}^t\frac{f(x''(\tau))}{x''(\tau)}dx'(\tau)-\int_{T_1}^t h(x(\tau))d\tau.
\end{aligned}$$

Since by (3), $0<\frac{g(x'(\tau))}{x'(\tau)}\leq c$, there is a constant c_1, small enough such that

$$0<c_1\leq\frac{g(x'(\tau))}{x'(\tau)}\leq c.$$

Therefore,

$$\begin{aligned}
c_1|x(t)-x(T_1)|&\leq|x'''(t)|+|x'''(T_1)|+a[|x''(t)|+|x''(T_1)|]+b[|x'(t)|+|x'(T_1)|]\\
&\quad+\Big|\int_{T_1}^t h(x(\tau))d\tau\Big|+\Big|\int_{T_1}^t p(\tau)d\tau\Big|.
\end{aligned}$$

Thus,

$$|x(t)| \le |x(T_1)| + \frac{2}{c_1}\big(D''' + aD'' + bD' + \frac{P_0}{2}\big) + \epsilon,$$

where ϵ is an arbitrary small positive constant. This is a contradiction to $x(T_2) = \bar{x}_{k+1}$ with respect to (4).

Since the remaining part of our theorem follows immediately from Lemma 2.3, the proof is complete. □

4. Example

Consider the equation

$$\begin{aligned} &x''''(t) + \frac{385}{16}x'''(t) + \frac{259x''(t)}{2\,(1+(x''(t))^2)} \\ &+ \big(7x'(t) + \frac{x'(t)}{1+(x'(t))^2}\big) + \frac{1}{10}\sin x(t) \\ &= \frac{1}{10}\cos t, \end{aligned} \tag{4.1}$$

where

$$a = \frac{385}{16}, \quad f(x''(t)) = \frac{259x''(t)}{2\big(1+(x''(t))^2\big)}, \quad g(x'(t)) = 7x'(t) + \frac{x'(t)}{1+(x'(t))^2},$$

$h(x(t)) = \frac{1}{10}\sin x(t)$ and $p(t) = \frac{1}{10}\cos t$, with $\sin x(t)$ and $\cos t$ being oscillatory. A simple calculation (with the earlier notation) gives $H = 0.1$, $H' = 0.1$, $P = 0.1$, $P' = 0.1$, $P_0 = 0.1$, $b = \frac{259}{2}$, $c = 8$ and $c_1 = 7$. It is obvious that the conditions (1)–(4) of Theorem 3.1 are satisfied. For condition (5), since $h(x(t)) = \frac{1}{10}\sin x(t)$ the roots of $h(x(t))$ are

$$\bar{x}_{k-1} = (k-1)\pi, \bar{x}_k = k\pi, \quad \bar{x}_{k+1} = (k+1)\pi, \quad (k = 0, \pm 2, \pm 4, \dots),$$

where $\bar{x}_{k-1}$ and $\bar{x}_{k+1}$ are the couple of adjacent roots of $\bar{x}_k = k\pi$. Thus,

$$\min\big\{d(\bar{x}_k, \bar{x}_{k+1}), d(\bar{x}_k, \bar{x}_{k-1})\big\} = \pi$$

and

$$\frac{(H+P)}{c_1}\big(\frac{4}{a} + \frac{2a}{b} + \frac{b}{c}\big) + \frac{P_0}{c_1} = \frac{5684041880}{11574192000} < 1.$$

Since $\pi > 1$, then all the conditions of Theorem 3.1 are satisfied, thus all solutions $x(t)$ of (4.1) are bounded and for each of them there exists a root $\bar{x}$ of $h(x(t))$ such that $(x(t) - \bar{x})$ oscillates.

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