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OSCILLATION CRITERIA FOR IMPULSIVE DYNAMIC EQUATIONS ON TIME SCALES

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Abstract. Oscillation criteria for impulsive dynamic equations on time scales are obtained via impulsive inequality. An example is given to show that the impulses play a dominant part in the oscillations of dynamic equations on time scales.

1. Introduction

In this paper, we are interested in obtaining oscillation criteria for solutions of the second-order nonlinear impulsive dynamic equation on time scales,

$$\begin{gathered} y^{\Delta\Delta}(t) + f(t, y^\sigma(t)) = 0, \quad t \in \mathbb{J}_\mathbb{T} := [0,\infty) \cap \mathbb{T},\ t \neq t_k,\ k = 1, 2, \dots, \\ y(t_k^+) = g_k(y(t_k^-)), \quad y^\Delta(t_k^+) = h_k(y^\Delta(t_k^-)), \quad k = 1, 2, \dots, \\ y(t_0^+) = y_0, \quad y^\Delta(t_0^+) = y_0^\Delta, \end{gathered} \tag{1.1}$$

where $\mathbb{T}$ is a unbounded-above time scale , with $0 \in \mathbb{T}$, $t_k \in \mathbb{T}, 0 \le t_0 < t_1 < t_2 < \dots < t_k < \dots$ and $\lim_{k\to\infty} t_k = \infty$.

$$y(t_k^+) = \lim_{h\to 0^+} y(t_k + h), \quad y^\Delta(t_k^+) = \lim_{h\to 0^+} y^\Delta(t_k + h), \tag{1.2}$$

which represent right limits of $y(t)$ at $t = t_k$ in the sense of time scales, and in addition, if t_k is right scattered, then $y(t_k^+) = y(t_k), y^\Delta(t_k^+) = y^\Delta(t_k)$. We can defined $y(t_k^-), y^\Delta(t_k^-)$ similar to (1.2).

We suppose that the following conditions hold:

- (H1) $f \in C_{rd}(\mathbb{T} \times \mathbb{R}, \mathbb{R})$, $xf(t, x) > 0$ $(x \neq 0)$ and $f(t, x)/\varphi(x) \ge p(t)$ $(x \neq 0)$, where $p(t) \in C_{rd}(\mathbb{T}, \mathbb{R}_+)$ and $x\varphi(x) > 0$ $(x \neq 0)$, $\varphi'(x) \ge 0$.
- (H2) $g_k, h_k \in C(\mathbb{R}, \mathbb{R})$ and there exist positive constants a_k, a_k^*, b_k, b_k^* such that
$$a_k^* \le \frac{g_k(x)}{x} \le a_k, \quad b_k^* \le \frac{h_k(x)}{x} \le b_k.$$

We note that the theory of dynamic equations on time scales are an adequate mathematical apparatus for the simulation of processes and phenomena observed in biotechnology, chemical technology, economic, neural networks, physics, social sciences etc. For further applications and questions concerning solutions of dynamic equations on time scales, see [3, 5, 6]

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Recently, impulsive dynamic equations on time scales have been investigated by Agarwal et al. [2], Belarbi et al. [7], Benchohra et al. [8, 9, 10, 11], Chang et al. [12] and so forth. In [11], Benchohra et al. considered the existence of extremal solutions for a class of second order impulsive dynamic equations on time scales, we can see that the existence of global solutions can be guaranteed by some simple conditions.

Based on the oscillatory behavior of the impulsive dynamic equations on time scales, Benchohra et al. [8] discuss the existence of oscillatory and nonoscillatory solutions by lower and upper solutions method for the first order impulsive dynamic equations on certain time scales

$$\begin{gathered} y^{\Delta}(t)=f(t,y(t)), \quad t\in \mathbb{J}_{\mathbb{T}}:=[0,\infty)\bigcap \mathbb{T},\ t\neq t_k,\ k=1,\dots, \\ y(t_k^+)=I_k(y(t_k^-)), \quad k=1,\dots . \end{gathered} \tag{1.3}$$

On the other hand, Huang et al. [14] considered the second order nonlinear impulsive dynamic equations on time scales

$$\begin{gathered} y^{\Delta\Delta}(t)+f(t,y^{\sigma}(t))=0, \quad t\in \mathbb{J}_{\mathbb{T}}:=[0,\infty)\cap \mathbb{T},\ t\neq t_k,\ k=1,2,\dots, \\ y(t_k^+)=g_k(y(t_k^-)), y^{\Delta}(t_k^+)=h_k(y^{\Delta}(t_k^-)), \quad k=1,2,\dots, \\ y(t_0^+)=y_0, \quad y^{\Delta}(t_0^+)=y_0^{\Delta}, \end{gathered} \tag{1.4}$$

extend the well-known results of Chen et al. [13] for the impulsive differential equations to (1.4).

Motivated by the ideas in [15], we establish the sufficient conditions for the oscillation of all solutions of (1.1), which utilize Riccati transformation techniques and impulsive inequality. Those results extend some well-known impulsive inequality on differential equations to impulsive dynamic equations. Our method is different from most existing ones. An example is given to show that though a dynamic equation on time scales is nonoscillatory, it may become oscillatory if some impulses are added to it. That is, in some cases, impulses play a dominating part in oscillations of dynamic equations on time scales.

For the remainder of the paper, we assume that, for each $k=1,2,\dots$, the points of impulses t_k are right dense (rd for short). In order to define the solutions of the problem (1.1), we introduce the two spaces:

$$\begin{aligned} AC^i=\{&y:\mathbb{J}_{\mathbb{T}}\to\mathbb{R} \text{ which is } i\text{-times } \Delta\text{-differentiable, and its } i\text{-th} \\ &\text{delta-derivative } y^{\Delta^{(i)}} \text{ is absolutely continuous}\}; \\ PC=\{&y:\mathbb{J}_{\mathbb{T}}\to\mathbb{R} \text{ which is rd-continuous expect at } t_k, \text{ for which} \\ &y(t_k^-),y(t_k^+),y^{\Delta}(t_k^-),y^{\Delta}(t_k^+) \text{ exist with } y(t_k^-)=y(t_k),\ y^{\Delta}(t_k^-)=y^{\Delta}(t_k)\}. \end{aligned}$$

A function $y\in PC\bigcap AC^2(\mathbb{J}_{\mathbb{T}}\backslash\{t_1,\dots\},\mathbb{R})$ is said to be a solution of (1.1), if it satisfies $y^{\Delta\Delta}(t)+f(t,y^{\sigma}(t))=0$ a.e. on $\mathbb{J}_{\mathbb{T}}\backslash\{t_k\}, k=1,2,\dots$, and for each $k=1,2,\dots,y$ satisfies the impulsive condition $y(t_k^+)=g_k(y(t_k)),y^{\Delta}(t_k^+)=h_k(y^{\Delta}(t_k))$ and the initial conditions $y(t_0^+)=y_0,y^{\Delta}(t_0^+)=y_0^{\Delta}$.

A solution y of (1.1) is called oscillatory if it is neither eventually positive nor eventually negative; otherwise it is called nonoscillatory. Equation (1.1) is called oscillatory if all solutions are oscillatory.

2. Preliminary Results

We will briefly recall some basic definitions and facts from the time scales calculus that we will use in the sequel. For more details see [1, 5, 6].

On any time scale $\mathbb{T}$, we define the forward and backward jump operators by

$$\sigma(t) = \inf\{s \in \mathbb{T}, s > t\}, \quad \rho(t) = \sup\{s \in \mathbb{T} : s < t\},$$

where $\inf \phi = \sup \mathbb{T}, \sup \phi = \inf \mathbb{T}$, and ϕ denotes the empty set. A nonmaximal element $t \in \mathbb{T}$ is called right-dense if $\sigma(t) = t$ and right-scattered if $\sigma(t) > t$. A nonminimal element $t \in \mathbb{T}$ is said to be left-dense if $\rho(t) = t$ and left-scattered if $\rho(t) < t$. The graininess μ of the time scale $\mathbb{T}$ is defined by $\mu(t) = \sigma(t) - t$.

A mapping $f : \mathbb{T} \to \mathbb{X}$ is said to be differentiable at $t \in \mathbb{T}$, if there exists $b \in \mathbb{X}$ such that for any $\varepsilon > 0$, there exists a neighborhood $\mathbf{U}$ of t satisfying $|[f(\sigma(t)) - f(s)] - b[\sigma(t) - s]| \le \varepsilon|\sigma(t) - s|$, for all $s \in \mathbf{U}$. We say that f is delta differentiable (or in short: differentiable) on $\mathbb{T}$ provided $f^{\Delta}(t)$ exist for all $t \in \mathbb{T}$.

A function $f : \mathbb{T} \to \mathbb{R}$ is called $rd-continuous$ provided it is continuous at right-dense points in $\mathbb{T}$ and its left-sided limits exist (finite) at left-dense points in $\mathbb{T}$. The set of rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $C_{rd}(\mathbb{T}, \mathbb{R})$.

The derivative and forward jump operator σ are related by the formula

$$f(\sigma(t)) = f(t) + \mu(t)f^{\Delta}(t). \tag{2.1}$$

Let f be a differentiable function on [a,b]. Then f is increasing, decreasing, nondecreasing and nonincreasing on $[a, b]$ if $f^{\Delta} > 0, f^{\Delta} < 0, f^{\Delta} \ge 0$ and $f^{\Delta} \le 0$ for all $t \in [a, b)$, respectively.

We will use the following product and quotient rules for derivative of two differentiable functions f and g:

$$(fg)^{\Delta} = f^{\Delta}g + f^{\sigma}g^{\Delta} = fg^{\Delta} + f^{\Delta}g^{\sigma}, \tag{2.2}$$

$$(\frac{f}{g})^{\Delta} = \frac{f^{\Delta}g - fg^{\Delta}}{gg^{\sigma}}, \tag{2.3}$$

where $f^{\sigma} = f \circ \sigma, gg^{\sigma} \neq 0$. The integration by parts formula reads

$$\int_a^b f^{\Delta}(t)g(t)\Delta t = f(t)g(t)|_a^b - \int_a^b f^{\sigma}(t)g^{\Delta}(t)\Delta t. \tag{2.4}$$

Chain Rule: Assume $g : \mathbb{T} \to \mathbb{R}$ is $\Delta-$differentiable on $\mathbb{T}$ and $f : \mathbb{R} \to \mathbb{R}$ is continuously differentiable. Then $f \circ g : \mathbb{T} \to \mathbb{R}$ is $\Delta-$differentiable and satisfies

$$(f \circ g)^{\Delta}(t) = \{\int_0^1 f'(g(t) + h\mu(t)g^{\Delta}(t))dh\}g^{\Delta}(t). \tag{2.5}$$

A function $p : \mathbb{T} \to \mathbb{R}$ is called regressive if for all $t \in \mathbb{T}$

$$1 + \mu(t)p(t) \neq 0.$$

The set of all $rd-continuous$ function f which satisfy $1+\mu(t)p(t) > 0$ for all $t \in \mathbb{T}$ will be denoted by $\mathcal{R}^+$. The generalized exponential function e_p is defined by

$$e_p(t, s) = \exp\big\{\int_s^t \xi_{\mu(\tau)}(p(\tau))\Delta\tau\big\},$$

with $\xi_h(z) = \log(1 + hz)/h$ if $h \neq 0$ and $\xi_h(z) = z$ if $h = 0$.

Lemma 2.1 (5, p. 255). *Let $y, f \in C_{rd}$ and $p \in \mathcal{R}^+$. Then*

$$y^{\Delta}(t) \le p(t)y(t) + f(t),$$

implies that for all $t \in \mathbb{T}$,

$$y(t) \le y(t_0)e_p(t,t_0) + \int_{t_0}^{t} e_p(t,\sigma(s))f(s)\Delta s\,.$$

3. Main results

Next, we prove some lemmas, which will be useful for establishing oscillation criteria for (1.1).

Lemma 3.1. *Assume that $m \in PC^1[\mathbb{T},\mathbb{R}]$ and*

$$\begin{gathered} m^{\Delta}(t) \le p(t)m(t) + q(t), \quad t \in \mathbb{J}_{\mathbb{T}} := [0,\infty) \cap \mathbb{T},\ t \ne t_k,\ k = 1,2,\dots, \\ m(t_k^+) \le d_k m(t_k^-) + b_k, \quad k = 1,2,\dots, \end{gathered} \tag{3.1}$$

then for $t \ge t_0$,

$$\begin{aligned} m(t) \le{}& m(t_0) \prod_{t_0<t_k<t} d_k e_p(t,t_0) + \sum_{t_0<t_k<t}\Big(\prod_{t_k<t_j<t} d_j e_p(t,t_k)\Big) b_k \\ &+ \int_{t_0}^{t} \prod_{s<t_k<t} d_k e_p(t,\sigma(s))q(s)\Delta s. \end{aligned} \tag{3.2}$$

Proof. Let $t \in [t_0,t_1]_{\mathbb{T}}$. then use Lemma 2.1 to obtain

$$m(t) \le m(t_0)e_p(t,t_0) + \int_{t_0}^{t} e_p(t,\sigma(s))q(s)\Delta s, \quad t \in [t_0,t_1]_{\mathbb{T}}.$$

Hence (3.2) is true for $t \in [t_0,t_1]_{\mathbb{T}}$. Now assume that (3.2) holds for $t \in [t_0,t_n]_{\mathbb{T}}$ for some integer $n > 1$. Then for $t \in (t_n,t_{n+1}]_{\mathbb{T}}$, it follows from (3.1) and Lemma 2.1, we get

$$m(t) \le m(t_n^+)e_p(t,t_n) + \int_{t_n}^{t} e_p(t,\sigma(s))q(s)\Delta s\,.$$

Using (3.1), we obtain, from (3.2),

$$\begin{aligned} m(t) &\le [d_n m(t_n^-) + b_n]e_p(t,t_n) + \int_{t_n}^{t} e_p(t,\sigma(s))q(s)\Delta s \\ &\le d_n e_p(t,t_n)\Big[m(t_0)\prod_{t_0<t_k<t_n} d_k e_p(t_n,t_0) + \sum_{t_0<t_k<t_n}\Big(\prod_{t_k<t_j<t_n} d_j e_p(t_n,t_k)\Big)b_k \\ &\quad + \int_{t_0}^{t_n}\prod_{s<t_k<t_n} d_k e_p(t_n,\sigma(s))q(s)\Delta s\Big] + b_n e_p(t,t_n) + \int_{t_n}^{t} e_p(t,\sigma(s))q(s)\Delta s \\ &\le m(t_0)\prod_{t_0<t_k<t} d_k e_p(t,t_0) + \sum_{t_0<t_k<t}\Big(\prod_{t_k<t_j<t} d_j e_p(t,t_k)\Big)b_k \\ &\quad + \int_{t_0}^{t}\prod_{s<t_k<t} d_k e_p(t,\sigma(s))q(s)\Delta s, \end{aligned}$$

which on simplification gives the estimate (3.2) for $t \in [t_0,t_{n+1}]_{\mathbb{T}}$, by induction, we get (3.2) holds for $t \ge t_0$. $\square$

Lemma 3.2. *Suppose that* (H1), (H2) *hold and* $y(t) > 0$, $t \geq t_0' \geq t_0$ *is a nonoscillatory solution of* (1.1). *If*

(H3) $\int_{t_j}^{\infty} \prod_{t_j<t_k<s} \frac{b_k^*}{a_k} \Delta s = \infty$ *for some* $t_j \geq t_0$.

Then $y^{\Delta}(t_k^+) \geq 0$ *and* $y^{\Delta}(t) \geq 0$ *for* $t \in (t_k, t_{k+1}]_{\mathbb{T}}$, *where* $t_k \geq t_0'$.

Proof. At first, we prove that $y^{\Delta}(t_k^-) \geq 0$ for $t_k \geq t_0'$, otherwise, there exists some j such that $t_j \geq t_0'$ and $y^{\Delta}(t_j^-) < 0$, hence

$$y^{\Delta}(t_j^+) = h_j\left(y^{\Delta}(t_j^-)\right) \leq b_j^* y^{\Delta}(t_j^-) < 0.$$

Let $y^{\Delta}(t_j^+) = -\alpha$ ($\alpha > 0$). From (1.1) and (H1), for $t \in (t_{j+i-1}, t_{j+i}]_{\mathbb{T}}$, $i = 1, 2, \dots$, we obtain

$$y^{\Delta\Delta}(t) = -f(t, y^{\sigma}(t)) \leq -p(t)\varphi(y^{\sigma}(t)) \leq 0;$$

i.e., $y^{\Delta}(t)$ is nonincreasing in $(t_{j+i-1}, t_{j+i}]_{\mathbb{T}}$, $i = 1, 2, \dots$, then

$$\begin{gathered} y^{\Delta}(t_{j+1}^-) \leq y^{\Delta}(t_j^+) = -\alpha < 0, \\ y^{\Delta}(t_{j+2}^-) \leq y^{\Delta}(t_{j+1}^+) = h_{j+1}\left(y^{\Delta}(t_{j+1}^-)\right) \leq b_{j+1}^* y^{\Delta}(t_{j+1}^-) \leq -b_{j+1}^*\alpha < 0. \end{gathered} \tag{3.3}$$

By induction, we obtain

$$y^{\Delta}(t) \leq -\alpha \prod_{t_j<t_k<t} b_k^* < 0 \quad t \in (t_{j+n}, t_{j+n+1}]_{\mathbb{T}}. \tag{3.4}$$

In view of (H2), we have $y(t_k^+) \leq a_k y(t_k^-)$. Applying Lemma 3.1, we obtain for $t > t_j$

$$\begin{aligned} y(t) &\leq y(t_j^+) \prod_{t_j<t_k<t} a_k - \alpha \int_{t_j}^{t} \prod_{s<t_k<t} a_k \prod_{t_j<t_k<s} b_k^* \Delta s \\ &= \prod_{t_j<t_k<t} a_k \Big[y(t_j^+) - \alpha \int_{t_j}^{t} \prod_{t_j<t_k<s} \frac{b_k^*}{a_k}\Big] \Delta s. \end{aligned} \tag{3.5}$$

Since $y(t_j^+) > 0$, one can find that (3.5) contradicts (H3) as $t \to \infty$. Therefore, $y^{\Delta}(t_k^-) \geq 0$ ($t_k \geq t_0'$). By condition (H2), we obtain, for any $t_k \geq t_0'$,

$$y^{\Delta}(t_k^+) \geq b_k^* y^{\Delta}(t_k^-) \geq 0.$$

Since $y^{\Delta}(t)$ is decreasing in $(t_k, t_{k+1}]_{\mathbb{T}}$, $t_k \geq t_0'$, we have $y^{\Delta}(t) \geq y^{\Delta}(t_k^-) \geq 0$, $t \in (t_k, t_{k+1}]_{\mathbb{T}}$, $t_k \geq t_0'$. The proof of Lemma 3.2 is complete. □

We remark that when y is eventually negative, under the hypothesis (H1)-(H3), it can be proved similarly that $y^{\Delta}(t_k^+) \leq 0$ and for $t \in (t_k, t_{k+1}]_{\mathbb{T}}$, $y^{\Delta}(t) \leq 0$ for $t_k \geq t_0' \geq t_0$.

Theorem 3.3. *Suppose that* (H1)-(H3) *hold and there exists a positive integer* k_0 *such that* $a_k^* \geq 1$ *for* $k \geq k_0$. *If*

$$\int_{t_0}^{\infty} \prod_{t_0<t_k<t} \frac{1}{b_k} p(t) \Delta t = \infty, \tag{3.6}$$

then (1.1) *is oscillatory.*

Proof. Suppose to the contrary that Eq.(1.1) has a nonoscillatory solution y, without loss of generality, we may assume that y is eventually positive solution of (1.1); i.e., $y(t) > 0, t \geq t_0$ and $k_0 = 1$. From lemma 3.2, we have $y^{\Delta}(t) \geq 0$, $t \in (t_k, t_{k+1}]_{\mathbb{T}}$, $k = 1, 2, \dots$. Let

$$w(t) = \frac{y^{\Delta}(t)}{\varphi(y(t))}\,. \tag{3.7}$$

Then $w(t_k^+) \geq 0$, $k = 1, 2, \dots$, and $w(t) > 0, t \geq t_0$. Using (H1) and (1.1), when $t \neq t_k$,

$$\begin{aligned} w^{\Delta}(t) &= -\frac{f(t, y^{\sigma}(t))}{\varphi(y^{\sigma}(t))} - \frac{y^{\Delta}(t)}{\varphi(y(t))\varphi(y^{\sigma}(t))}\int_0^1 \varphi'\left(y(t) + h\mu(t)y^{\Delta}(t)\right) dh y^{\Delta}(t) \\ &\leq -p(t) - \frac{\varphi(y(t))}{\varphi(y^{\sigma}(t))}\Big(\frac{y^{\Delta}(t)}{\varphi(y(t))}\Big)^2 \int_0^1 \varphi'\left(y(t) + h\mu(t)y^{\Delta}(t)\right) dh \\ &\leq -p(t). \end{aligned} \tag{3.8}$$

Since $\varphi'(y(t)) \geq 0$ and $\varphi(y(t)) > 0$, from (H2) and $a_k^* \geq 1$, we obtain

$$w(t_k^+) = \frac{y^{\Delta}(t_k^+)}{\varphi(y(t_k^+))} \leq \frac{b_k y^{\Delta}(t_k^-)}{\varphi(a_k^* y(t_k^-))} \leq \frac{b_k y^{\Delta}(t_k^-)}{\varphi(y(t_k^-))} = b_k w(t_k^-), \quad k = 1, 2, \dots. \tag{3.9}$$

Applying Lemma 3.1, we obtain from (3.8) and (3.9),

$$\begin{aligned} w(t) &\leq w(t_0)\prod_{t_0 < t_k < t} b_k - \int_{t_0}^{t} \prod_{s < t_k < t} b_k p(s)\Delta s \\ &= \prod_{t_0 < t_k < t} b_k\Big[w(t_0) - \int_{t_0}^{t}\prod_{t_0 < t_k < s}\frac{1}{b_k}p(s)\Delta s\Big]. \end{aligned} \tag{3.10}$$

In view of (3.6) and (3.10), we get a contradiction as $t \to \infty$. Then every solution of (1.1) is oscillatory. □

Theorem 3.4. *Assume that* (H1)-(H3) *hold and* $\varphi(ab) \geq \varphi(a)\varphi(b)$ *for any* $ab > 0$. *If*

$$\int_{t_0}^{\infty}\prod_{t_0 < t_k < t}\frac{\varphi(a_k^*)}{b_k}p(t)\Delta t = \infty, \tag{3.11}$$

then (1.1) *is oscillatory.*

Proof. As before, we may suppose $y(t) > 0$, $t \geq t_0$ be a nonoscillatory solution of (1.1), Lemma 3.2 yields $y^{\Delta}(t) \geq 0, t \geq t_0$, define $w(t)$ as in (3.7), we get $w(t) \geq 0, t \geq t_0, w(t_k^+) \geq 0, k = 1, 2, \dots$, (3.8) holds for $t \neq t_k$ and

$$w(t_k^+) = \frac{y^{\Delta}(t_k^+)}{\varphi(y(t_k^+))} \leq \frac{b_k y^{\Delta}(t_k^-)}{\varphi(a_k^* y(t_k^-))} \leq \frac{b_k y^{\Delta}(t_k^-)}{\varphi(a_k^*)\varphi(y(t_k^-))} = \frac{b_k}{\varphi(a_k^*)}w(t_k^-). \tag{3.12}$$

Using Lemma 3.1, we get from (3.8) and (3.12)

$$\begin{aligned} w(t) &\leq w(t_0)\prod_{t_0 < t_k < t}\frac{b_k}{\varphi(a_k^*)} - \int_{t_0}^{t}\prod_{s < t_k < t}\frac{b_k}{\varphi(a_k^*)}p(s)\Delta s \\ &= \prod_{t_0 < t_k < t}\frac{b_k}{\varphi(a_k^*)}\Big[w(t_0) - \int_{t_0}^{t}\prod_{t_0 < t_k < s}\frac{\varphi(a_k^*)}{b_k}p(s)\Delta s\Big]. \end{aligned}$$

Letting $t \to \infty$, the above inequality contradicts to (3.11). Then every solution of (1.1) is oscillatory. □

From Theorems 3.3 and 3.4, we have the following corollaries.

Corollary 3.5. *Suppose that* (H1)-(H3) *hold and there exists a positive integer* k_0 *such that* $a_k^* \geq 1, b_k \leq 1$ *for* $k \geq k_0$. *If* $\int^\infty p(t)\Delta t = \infty$, *then* (1.1) *is oscillatory.*

Proof. Without loss of generality, let $k_0 = 1$. By $b_k \leq 1$, we get $\frac{1}{b_k} \geq 1$, therefore

$$\int_{t_0}^{t} \prod_{t_0<t_k<s} \frac{1}{b_k} p(s)\Delta s \geq \int_{t_0}^{t} p(s)\Delta s.$$

Let $t \to \infty$ and using $\int^\infty p(t)\Delta t = \infty$, we obtain from Theorem 3.3 that (1.1) is oscillatory. $\square$

Corollary 3.6. *Suppose that* (H1)-(H3) *hold and there exist a positive integer* k_0 *and a constant* $\alpha > 0$ *such that*

$$a_k^* \geq 1, \quad \frac{1}{b_k} \geq \Big(\frac{t_{k+1}}{t_k}\Big)^\alpha, \quad \textit{for } k \geq k_0. \tag{3.13}$$

If

$$\int^\infty t^\alpha p(t)\Delta t = \infty. \tag{3.14}$$

Then (1.1) *is oscillatory.*

Proof. Without loss of generality let $k_0 = 1$. Then (3.6) yields

$$\begin{aligned}
&\int_{t_0}^{t} \prod_{t_0<t_k<s} \frac{1}{b_k} p(s)\Delta s \\
&= \int_{t_0}^{t_1} p(t)\Delta t + \frac{1}{b_1}\int_{t_1}^{t_2} p(t)\Delta t + \dots + \frac{1}{b_1 b_2 \dots b_n}\int_{t_n}^{t} p(t)\Delta t \\
&\geq \frac{1}{t_1^\alpha}\Big[\int_{t_1}^{t_2} t_2^\alpha p(t)\Delta t + \int_{t_2}^{t_3} t_3^\alpha p(t)\Delta t + \dots + \int_{t_n}^{t} t_{n+1}^\alpha p(t)\Delta t\Big] \\
&\geq \frac{1}{t_1^\alpha}\Big[\int_{t_1}^{t_2} s^\alpha p(s)\Delta s + \int_{t_2}^{t_3} s^\alpha p(s)\Delta s + \dots + \int_{t_n}^{t} s^\alpha p(s)\Delta s\Big] \\
&= \frac{1}{t_1^\alpha}\int_{t_1}^{t} s^\alpha p(s)\Delta s,
\end{aligned} \tag{3.15}$$

for $t \in (t_n, t_{n+1}]_{\mathbb{T}}$. Let $t \to \infty$ and use (3.15), (3.14) yields (3.6) holds. According to Theorem 3.3, we obtain (1.1) is oscillatory. $\square$

Corollary 3.7. *Assume that* (H1)-(H3) *hold and* $\varphi(ab) \geq \varphi(a)\varphi(b)$ *for any* $ab > 0$. *Suppose there exist a positive integer* k_0 *and a constant* $\alpha > 0$ *such that*

$$\frac{\varphi(a_k^*)}{b_k} \geq \Big(\frac{t_{k+1}}{t_k}\Big)^\alpha, \quad \textit{for } k \geq k_0.$$

If $\int^\infty t^\alpha p(t)\Delta t = \infty$, *then* (1.1) *is oscillatory.*

The above corollary can be deduced from Theorem 3.4. Its proof is similar to that of Corollary 3.6; so we omit it.

4. Example

Consider the second-order impulsive dynamic equation

$$\begin{gathered} y^{\Delta\Delta}(t)+\frac{1}{t\sigma^2(t)}y^{\gamma}(\sigma(t))=0, \quad t\geq 1,\ t\neq k,\ k=1,2,\dots, \\ y(k^+)=\frac{k+1}{k}y(k^-), \quad y^{\Delta}(k^+)=y^{\Delta}(k^-), \quad k=1,2,\dots, \\ y(1)=y_0, \quad y^{\Delta}(1)=y_0^{\Delta}. \end{gathered} \tag{4.1}$$

where $\gamma\geq 3$ and $\mu(t)\leq ct$, where c is a positive constant.

Since $a_k=a_k^*=(k+1)/k$, $b_k=b_k^*=1$, $p(t)=1/(t\sigma^2(t))$, $t_k=k$ and $\varphi(y)=y^{\gamma}$. It is easy to see that (H1)-(H3) hold. Let $k_0=1$, $\alpha=3$, hence

$$\frac{\varphi(a_k^*)}{b_k}=(k+1)/k^{\gamma}=\big(\frac{t_{k+1}}{t_k}\big)^{\gamma}\geq\Big(\frac{t_{k+1}}{t_k}\Big)^3,$$

and

$$\int^{\infty}t^{\alpha}p(t)\Delta t=\int^{\infty}t^3\frac{1}{t\sigma^2(t)}\Delta t=\int^{\infty}\Big(\frac{t}{\sigma(t)}\Big)^2\Delta t.$$

Since $\mu(t)\leq ct$, we get

$$\frac{t}{\sigma(t)}=\frac{t}{t+\mu(t)}\geq\frac{1}{1+c},$$

hence

$$\int^{\infty}\Big(\frac{t}{\sigma(t)}\Big)^2\Delta t\geq\frac{1}{(1+c)^2}\int^{\infty}\Delta t=\infty.$$

By Corollary 3.7, we obtain that (4.1) is oscillatory. But by [4] we know that the dynamic equation $y^{\Delta\Delta}(t)+\frac{1}{t\sigma^2(t)}y^{\gamma}(\sigma(t))=0$ is nonoscillatory.

In the above example, it is interesting that the dynamic equation without impulses is nonoscillatory, but when some impulses are added to it, it becomes oscillatory. Therefore, this example shows that impulses play an important part in the oscillations of dynamic equations on time scales.

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