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PERIODIC SOLUTIONS FOR A STAGE-STRUCTURE ECOLOGICAL MODEL ON TIME SCALES

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Abstract. In this paper, by using the Mawhin's continuation theorem, we prove the existence of periodic solutions for a stage-structure ecological model on time scales. This unifies the results for differential and difference equations.

1. Introduction

Recently, Zheng and Cui constructed the following stage-structure population model, with patches and diffusion,

$$\begin{aligned} I_1'(t) &= aM_1(t) - bI_1(t) - cI_1(t), \\ M_1'(t) &= cI_1(t) - \alpha M_1^2(t) + D_{12}(M_2(t) - M_1(t)), \\ M_2'(t) &= -\beta M_2(t) + D_{21}(M_1(t) - M_2(t)), \end{aligned} \tag{1.1}$$

where $a, b, c, \alpha, \beta, D_{12}$ and D_{21} are positive constants. $I_1(t)$ is the density of immature rana chensinensis in patch 1, $M_i(t)$ denotes the density of mature rana chensinensis in the i-patch ($i = 1, 2$). D_{ij} is the diffusion coefficient of the species from patch j to patch i. The permanence and stability of equilibrium of system (1.1) were investigated in [9].

Taking into account the periodicity of environment, Zhang and Zheng reconstructed the model as follows:

$$\begin{aligned} I_1'(t) &= a(t)M_1(t) - b(t)I_1(t) - c(t)I_1(t), \\ M_1'(t) &= c(t)I_1(t) - \alpha(t)M_1^2(t) + D_{12}(t)(M_2(t) - M_1(t)), \\ M_2'(t) &= -\beta(t)M_2(t) + D_{21}(t)(M_1(t) - M_2(t)), \end{aligned} \tag{1.2}$$

where all the coefficients are positive continuous ω-periodic functions. Based on the theory of coincidence degree, the existence of positive periodic solution was

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established in [8]. The corresponding discrete system

$$\begin{gathered}
I_1(k+1) = I_1(k)\exp\big\{-b(k)-c(k)+a(k)\frac{M_1(k)}{I_1(k)}\big\},\\
M_1(k+1) = M_1(k)\exp\big\{-D_{12}(k)-\alpha(k)M_1(k)+\frac{c(k)I_1(k)+D_{12}(k)M_2(k)}{M_1(k)}\big\},\\
M_2(k+1) = M_2(k)\exp\big\{-\beta(k)-D_{21}(k)+\frac{D_{21}(k)M_1(k)}{M_2(k)}\big\},
\end{gathered}\tag{1.3}$$

was considered in [7]. The coefficients are all strictly positive ω-periodic sequences. The existence of periodic solutions for (1.3) was done. However, the work of [7, 8] was repeated to some extent. It is natural to ask whether there is a unified way to explore such kind of problem. To unify the continuous and discrete analysis, Stefan Hilger in his Ph.D. Thesis initiated the theory of calculus on time scales in [5]. The theme has received much attention in recent years, such as [1, 2, 3]. It is true that unification and extension are the two main features of the calculus on time scales.

In this paper, we consider the following ecological model with stage structure and diffusion on time scales:

$$\begin{gathered}
u_1^{\Delta}(t) = -b(t)-c(t)+a(t)\frac{e^{u_2(t)}}{e^{u_1(t)}},\\
u_2^{\Delta}(t) = -D_{12}(t)-\alpha(t)e^{u_2(t)}+\frac{c(t)e^{u_1(t)}+D_{12}(t)e^{u_3(t)}}{e^{u_2(t)}},\\
u_3^{\Delta}(t) = -\beta(t)-D_{21}(t)+\frac{D_{21}(t)e^{u_2(t)}}{e^{u_3(t)}},
\end{gathered}\tag{1.4}$$

where $a(t), b(t), c(t), \alpha(t), D_{12}(t), \beta(t)$ and $D_{21}(t)$ are rd-continuous positive ω-periodic functions on time scales $\mathbb{T}$. Set $I_1(t) = e^{u_1(t)}$, $M_1(t) = e^{u_2(t)}$ and $M_2(t) = e^{u_3(t)}$, if $\mathbb{T} = \mathbb{R}$ and $\mathbb{T} = \mathbb{Z}$, then (1.4) can be reduced to (1.2) and (1.3), respectively. As a result, it is unnecessary to investigate the periodic solutions of (1.2) and (1.3) separately.

Thus, we shall prove the periodicity of system (1.4) by Mawhin's continuation theorem in coincidence degree theory to unify the results in [7, 8]. This approach has been widely applied to deal with the existence of periodic solutions of differential equations and difference equations but rarely applied to the dynamic equations on time scales [2, 3].

2. Preliminary results

For the convenience of the reader, we first present some basic definitions and lemmas about time scales and the continuation theorem of the coincidence degree theory; more details can be found in [1, 4]. A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of real numbers $\mathbb{R}$. Throughout this paper, we assume that the time scale $\mathbb{T}$ is unbounded above and below, such as $\mathbb{R}$, $\mathbb{Z}$ and $\bigcup_{k\in\mathbb{Z}}[2k, 2k+1]$. The following definitions and lemmas about time scales are from [1].

Definition 2.1. The forward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$, the backward jump operator $\rho : \mathbb{T} \to \mathbb{T}$, and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+ = [0, +\infty)$ are defined, respectively, by

$$\sigma(t) := \inf\{s \in \mathbb{T} : s > t\}, \quad \rho(t) := \sup\{s \in \mathbb{T} : s < t\}, \quad \mu(t) = \sigma(t) - t.$$

If $\sigma(t) = t$, then t is called right-dense (otherwise: right-scattered), and if $\rho(t) = t$, then t is called left-dense(otherwise: left-scattered).

Definition 2.2. Assume $f : \mathbb{T} \to \mathbb{R}$ is a function and let $t \in \mathbb{T}$. Then we define $f^{\Delta}(t)$ to be the number (provided it exists) with the property that given any $\varepsilon > 0$, there is a neighborhood U of t such that

$$|f(\sigma(t)) - f(s) - f^{\Delta}(t)(\sigma(t) - s)| \leq \varepsilon|\sigma(t) - s| \quad \text{for all } s \in U.$$

In this case, $f^{\Delta}(t)$ is called the delta (or Hilger) derivative of f at t. Moreover, f is said to be delta or Hilger differentiable on $\mathbb{T}$ if $f^{\Delta}(t)$ exists for all $t \in \mathbb{T}$. A function $F : \mathbb{T} \to \mathbb{R}$ is called an antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided $F^{\Delta}(t) = f(t)$ for all $t \in \mathbb{T}$. Then we define

$$\int_r^s f(t)\Delta t = F(s) - F(r) \quad \text{for } r, s \in \mathbb{T}.$$

Definition 2.3. A function $f : \mathbb{T} \to \mathbb{R}$ is said to be rd-continuous if it is continuous at right-dense points in $\mathbb{T}$ and its left-sided limits exist(finite) at left-dense points in $\mathbb{T}$. The set of rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $C_{rd}(\mathbb{T})$.

Lemma 2.4. *Every rd-continuous function has an antiderivative.*

Lemma 2.5. *If $a, b \in \mathbb{T}$, α, $\beta \in \mathbb{R}$ and $f, g \in C_{rd}(\mathbb{T})$,then*

(a) $\int_a^b [\alpha f(t) + \beta g(t)]\Delta t = \alpha \int_a^b f(t)\Delta t + \beta \int_a^b g(t)\Delta t$;
(b) *if $f(t) \geq 0$ for all $a \leq t < b$, then $\int_a^b f(t)\Delta t \geq 0$;*
(c) *if $|f(t)| \leq g(t)$ on $[a, b) := \{t \in \mathbb{T} : a \leq t < b\}$, then $|\int_a^b f(t)\Delta t| \leq \int_a^b g(t)\Delta t$.*

For simplicity, we use the following notations throughout this paper. Let $\mathbb{T}$ be ω-periodic, that is $t \in \mathbb{T}$ implies $t + \omega \in \mathbb{T}$,

$$k = \min\{\mathbb{R}^+ \cap \mathbb{T}\}, \quad I_\omega = [k, k + \omega] \cap \mathbb{T}, \quad g^L = \inf_{t\in\mathbb{T}} g(t),$$
$$g^M = \sup_{t\in\mathbb{T}} g(t), \quad \bar{g} = \frac{1}{\omega}\int_{I_\omega} g(s)\Delta s = \frac{1}{\omega}\int_k^{k+\omega} g(s)\Delta s,$$

where $g \in C_{rd}(\mathbb{T})$ is an ω-periodic real function, i.e., $g(t + \omega) = g(t)$ for all $t \in \mathbb{T}$.

Now, we introduce some concepts and a useful result from [4].

Let X, Z be normed vector spaces, $L : \operatorname{Dom} L \subset X \to Z$ be a linear mapping, $N : X \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim \ker L = \operatorname{codim} \operatorname{Im} L < +\infty$ and $\operatorname{Im} L$ is closed in Z. If L is a Fredholm mapping of index zero and there exist continuous projections $P : X \to X$ and $Q : Z \to Z$ such that $\operatorname{Im} P = \ker L$, $\operatorname{Im} L = \ker Q = \operatorname{Im}(I - Q)$, then it follows that $L|\operatorname{Dom} L \cap \ker P : (I - P)X \to \operatorname{Im} L$ is invertible. We denote the inverse of that map by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_P(I - Q)N : \bar{\Omega} \to X$ is compact. Since $\operatorname{Im} Q$ is isomorphic to $\ker L$, there exists an isomorphism $J : \operatorname{Im} Q \to \ker L$.

Next, we state the Mawhin's continuation theorem, which is a main tool in the proof of our theorem.

Lemma 2.6 (Continuation Theorem). *Let L be a Fredholm mapping of index zero and N be L-compact on $\bar{\Omega}$. Suppose*

(a) *for each* $\lambda \in (0,1)$*, every solution* u *of* $Lu = \lambda Nu$ *is such that* $u \notin \partial\Omega$*;*
(b) $QNu \neq 0$ *for each* $u \in \partial\Omega \cap \ker L$ *and the Brouwer degree* $\deg\{JQN, \Omega \cap \ker L, 0\} \neq 0$.

Then the operator equation $Lu = Nu$ *has at least one solution lying in* $\operatorname{Dom} L \cap \bar{\Omega}$.

3. Existence of periodic solutions

In this section, we shall derive the sufficient conditions for the global existence of periodic solutions to system (1.4).

Theorem 3.1. *If*
$$\frac{c^L a^L}{(b+c)^M} + \frac{D_{12}^L D_{21}^L}{(\beta + D_{21})^M} - D_{12}^M > 0,$$
then (1.4) *has at least one* ω*-periodic solution.*

Proof. Let
$$X = Z = \big\{(u_1,u_2,u_3)^T \in C(\mathbb{T},\mathbb{R}^3) : u_i(t+\omega) = u_i(t),\ i=1,2,3, \forall t \in \mathbb{T}\big\},$$
$$\|(u_1,u_2,u_3)^T\| = \sum_{i=1}^{3} \max_{t\in I_\omega} |u_i(t)|, \quad (u_1,u_2,u_3)^T \in X \quad (\text{or in} Z).$$
Then X and Z are both Banach spaces when they are endowed with the above norm $\|\cdot\|$. Let
$$N\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = \begin{bmatrix}N_1\\N_2\\N_3\end{bmatrix} = \begin{bmatrix} -b(t) - c(t) + a(t)\frac{e^{u_2(t)}}{e^{u_1(t)}} \\ -D_{12}(t) - \alpha(t)e^{u_2(t)} + \frac{c(t)e^{u_1(t)} + D_{12}(t)e^{u_3(t)}}{e^{u_2(t)}} \\ -\beta(t) - D_{21}(t) + \frac{D_{21}(t)e^{u_2(t)}}{e^{u_3(t)}} \end{bmatrix},$$
$$L\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = \begin{bmatrix}u_1^\Delta\\u_2^\Delta\\u_3^\Delta\end{bmatrix}, \quad P\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = Q\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = \begin{bmatrix} \frac{1}{\omega}\int_k^{k+\omega} u_1(t)\Delta t \\ \frac{1}{\omega}\int_k^{k+\omega} u_2(t)\Delta t \\ \frac{1}{\omega}\int_k^{k+\omega} u_3(t)\Delta t \end{bmatrix}.$$
Then
$$\ker L = \big\{(u_1,u_2,u_3)^T \in X : (u_1(t),u_2(t),u_3(t))^T = (h_1,h_2,h_3)^T \in \mathbb{R}^3, t \in \mathbb{T}\big\},$$
$$\operatorname{Im} L = \big\{(u_1,u_2,u_3)^T \in Z : \bar{u}_1 = \bar{u}_2 = \bar{u}_3 = 0, t \in \mathbb{T}\big\},$$
$$\dim\ker L = 3 = \operatorname{codim}\operatorname{Im} L.$$
Since $\operatorname{Im} L$ is closed in Z, then L is a Fredholm mapping of index zero. It is not difficult to prove that P and Q are continuous projections such that $\operatorname{Im} P = \ker L$ and $\operatorname{Im} L = \ker Q = \operatorname{Im}(I-Q)$. Furthermore, the generalized inverse (of L) $K_P : \operatorname{Im} L \to \ker P \cap \operatorname{Dom} L$ exists and is given by
$$K_P\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = \begin{bmatrix} \int_k^t u_1(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t u_1(s)\Delta s\Delta t \\ \int_k^t u_2(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t u_2(s)\Delta s\Delta t \\ \int_k^t u_3(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t u_3(s)\Delta s\Delta t \end{bmatrix}.$$
Thus
$$QN\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = \begin{bmatrix} \frac{1}{\omega}\int_k^{k+\omega}\big(-b(t) - c(t) + a(t)\frac{e^{u_2(t)}}{e^{u_1(t)}}\big)\Delta t \\ \frac{1}{\omega}\int_k^{k+\omega}\big(-D_{12}(t) - \alpha(t)e^{u_2(t)} + \frac{c(t)e^{u_1(t)} + D_{12}(t)e^{u_3(t)}}{e^{u_2(t)}}\big)\Delta t \\ \frac{1}{\omega}\int_k^{k+\omega}\big(-\beta(t) - D_{21}(t) + \frac{D_{21}(t)e^{u_2(t)}}{e^{u_3(t)}}\big)\Delta t \end{bmatrix},$$

$$
\begin{aligned}
&K_P(I-Q)N\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix}\\
&=\begin{bmatrix}
\int_k^t u_1(s)\Delta s-\frac{1}{\omega}\int_k^{k+\omega}\int_k^t u_1(s)\Delta s\Delta t-\left(t-k-\frac{1}{\omega}\int_k^{k+\omega}(t-k)\Delta t\right)\bar{u}_1\\
\int_k^t u_2(s)\Delta s-\frac{1}{\omega}\int_k^{k+\omega}\int_k^t u_2(s)\Delta s\Delta t-\left(t-k-\frac{1}{\omega}\int_k^{k+\omega}(t-k)\Delta t\right)\bar{u}_2\\
\int_k^t u_3(s)\Delta s-\frac{1}{\omega}\int_k^{k+\omega}\int_k^t u_3(s)\Delta s\Delta t-\left(t-k-\frac{1}{\omega}\int_k^{k+\omega}(t-k)\Delta t\right)\bar{u}_3
\end{bmatrix}.
\end{aligned}
$$

Obviously, QN and $K_P(I-Q)N$ are continuous. According to Arzela-Ascoli theorem, it is easy to show that $K_P(I-Q)N(\bar{\Omega})$ is compact for any open bounded set $\Omega\subset X$ and $QN(\bar{\Omega})$ is bounded. Thus, N is L-compact on $\bar{\Omega}$.

Now, we shall search an appropriate open bounded subset Ω for the application of the continuation theorem, Lemma 2.6. For the operator equation $Lu=\lambda Nu$, where $\lambda\in(0,1)$, we have

$$
\begin{gathered}
u_1^{\Delta}(t)=\lambda\left(-b(t)-c(t)+a(t)\frac{e^{u_2(t)}}{e^{u_1(t)}}\right),\\
u_2^{\Delta}(t)=\lambda\left(-D_{12}(t)-\alpha(t)e^{u_2(t)}+\frac{c(t)e^{u_1(t)}+D_{12}(t)e^{u_3(t)}}{e^{u_2(t)}}\right),\\
u_3^{\Delta}(t)=\lambda\left(-\beta(t)-D_{21}(t)+\frac{D_{21}(t)e^{u_2(t)}}{e^{u_3(t)}}\right).
\end{gathered}\tag{3.1}
$$

Assume that $(u_1,u_2,u_3)^T\in X$ is a solution of system (3.1) for a certain $\lambda\in(0,1)$. Integrating (3.1) on both sides from k to $k+\omega$, we obtain

$$
\begin{gathered}
\bar{b}\omega+\bar{c}\omega=\int_k^{k+\omega}a(t)\frac{e^{u_2(t)}}{e^{u_1(t)}}\Delta t,\\
\bar{D}_{12}\omega+\int_k^{k+\omega}\alpha(t)e^{u_2(t)}\Delta t=\int_k^{k+\omega}\frac{c(t)e^{u_1(t)}+D_{12}(t)e^{u_3(t)}}{e^{u_2(t)}}\Delta t,\\
\bar{\beta}\omega+\bar{D}_{21}\omega=\int_k^{k+\omega}\frac{D_{21}(t)e^{u_2(t)}}{e^{u_3(t)}}\Delta t.
\end{gathered}\tag{3.2}
$$

Since $(u_1,u_2,u_3)^T\in X$, there exist $\xi_i,\eta_i\in[k,k+\omega]$, $i=1,2,3$, such that

$$
u_i(\xi_i)=\min_{t\in[k,k+\omega]}\{u_i(t)\},\quad u_i(\eta_i)=\max_{t\in[k,k+\omega]}\{u_i(t)\},\quad i=1,2,3. \tag{3.3}
$$

From (3.1) and (3.3), we have

$$
a(\eta_1)e^{u_2(\eta_1)}=(b(\eta_1)+c(\eta_1))e^{u_1(\eta_1)}, \tag{3.4}
$$

$$
c(\eta_2)e^{u_1(\eta_2)}+D_{12}(\eta_2)e^{u_3(\eta_2)}=D_{12}(\eta_2)e^{u_2(\eta_2)}+\alpha(\eta_2)e^{2u_2(\eta_2)}, \tag{3.5}
$$

$$
D_{21}(\eta_3)e^{u_2(\eta_3)}=\beta(\eta_3)e^{u_3(\eta_3)}+D_{21}(\eta_3)e^{u_3(\eta_3)}. \tag{3.6}
$$

Thus,

$$
(b+c)^Le^{u_1(\eta_1)}\le a^Me^{u_2(\eta_2)}, \tag{3.7}
$$

$$
(\beta+D_{21})^Le^{u_3(\eta_3)}\le D_{21}^Me^{u_2(\eta_2)}. \tag{3.8}
$$

From (3.5), (3.7) and (3.8), we have

$$\begin{aligned}\alpha^L e^{2u_2(\eta_2)} + D_{12}^L e^{u_2(\eta_2)} &\le c^M e^{u_1(\eta_1)} + D_{12}^M e^{u_3(\eta_3)}\\ &\le \frac{c^M a^M}{(b+c)^L} e^{u_2(\eta_2)} + \frac{D_{12}^M D_{21}^M}{(\beta+D_{21})^L} e^{u_2(\eta_2)}.\end{aligned}$$

From the above inequality, we get

$$e^{u_2(\eta_2)} \le \frac{1}{\alpha^L}\big[\frac{c^M a^M}{(b+c)^L} + \frac{D_{12}^M D_{21}^M}{(\beta+D_{21})^L} - D_{12}^L\big] := L_2. \tag{3.9}$$

Substituting (3.9) into (3.7) and (3.8), respectively, the following estimates hold:

$$e^{u_1(\eta_1)} \le \frac{a^M}{(b+c)^L} L_2 := L_1, \quad e^{u_3(\eta_3)} \le \frac{D_{21}^M}{(\beta+D_{21})^L} L_2 := L_3.$$

Similarly, we also get the results:

$$\begin{gathered} e^{u_2(\xi_2)} \ge \frac{1}{\alpha^M}\big[\frac{c^L a^L}{(b+c)^M} + \frac{D_{12}^L D_{21}^L}{(\beta+D_{21})^M} - D_{12}^M\big] := l_2,\\ e^{u_1(\xi_1)} \ge \frac{a^L}{(b+c)^M} l_2 := l_1\,, \quad e^{u_3(\xi_3)} \ge \frac{D_{21}^L}{(\beta+D_{21})^M} l_2 := l_3.\end{gathered}$$

So, we have

$$\begin{gathered} \max_{t\in[k,k+\omega]} |u_1(t)| \le \max\{|\ln L_1|, |\ln l_1|\} := R_1,\\ \max_{t\in[k,k+\omega]} |u_2(t)| \le \max\{|\ln L_2|, |\ln l_2|\} := R_2,\\ \max_{t\in[k,k+\omega]} |u_3(t)| \le \max\{|\ln L_3|, |\ln l_3|\} := R_3.\end{gathered}$$

Clearly, R_1, R_2 and R_3 are independent of λ. Let $R = R_1 + R_2 + R_3 + R_0$, where R_0 is taken sufficiently large such that $R_0 \ge \sum_{i=1}^3 |l_i| + \sum_{i=1}^3 |L_i|$.

Now, we consider the algebraic equations:

$$\begin{gathered} \bar{b} + \bar{c} - \bar{a}e^{y-x} = 0,\\ \bar{D}_{12} + \bar{\alpha}e^{y} - \bar{c}e^{x-y} - \bar{D}_{12}e^{z-y} = 0,\\ \bar{\beta} + \bar{D}_{21} - \bar{D}_{21}e^{y-z} = 0,\end{gathered} \tag{3.10}$$

every solution $(x^*, y^*, z^*)^T$ of (3.10) satisfies $\|(x^*, y^*, z^*)^T\| < R$. Now, we define $\Omega = \{(u_1(t), u_2(t), u_3(t))^T \in X, \|(u_1(t), u_2(t), u_3(t))^T\| < R\}$. Then it is clear that Ω verifies the requirement (a) of Lemma 2.6. If $(u_1(t), u_2(t), u_3(t))^T \in \partial\Omega \cap \ker L = \partial\Omega \cap \mathbb{R}^3$, then $(u_1(t), u_2(t), u_3(t))^T$ is a constant vector in $\mathbb{R}^3$ with $\|(u_1(t), u_2(t), u_3(t))^T\| = |u_1| + |u_2| + |u_3| = R$, so we have

$$QN\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = \begin{bmatrix}\bar{a}e^{u_2-u_1} - \bar{b} - \bar{c}\\ \bar{c}e^{u_1-u_2} + \bar{D}_{12}e^{u_3-u_2} - \bar{D}_{12} - \bar{\alpha}e^{u_2}\\ \bar{D}_{21}e^{u_2-u_3} - \bar{\beta} - \bar{D}_{21}\end{bmatrix} \ne \begin{bmatrix}0\\0\\0\end{bmatrix}.$$

Moreover, define

$$\phi(u_1, u_2, u_3, \mu) = \begin{bmatrix}\bar{a}e^{u_2-u_1} - \bar{b}\\ \bar{c}e^{u_1-u_2} - \bar{\alpha}e^{u_2}\\ -\bar{D}_{21} + \bar{D}_{21}e^{u_2-u_3}\end{bmatrix} + \mu\begin{bmatrix}-\bar{c}\\ \bar{D}_{12}(e^{u_3-u_2} - 1)\\ -\bar{\beta}\end{bmatrix},$$

where $\mu \in [0,1]$ is a parameter. If $(u_1,u_2,u_3)^T \in \partial\Omega \cap \ker L$, then $\phi(u_1,u_2,u_3,\mu) \neq 0$. In addition, we can easily see that the algebraic equation $\phi(u_1,u_2,u_3,0) = 0$ has a unique solution in $\mathbb{R}^3$. Thus the invariance of homotopy produces

$$\begin{aligned}
\deg(JQN, \Omega \cap \ker L, 0) &= \deg(QN, \Omega \cap \ker L, 0) \\
&= \deg(\phi(u_1,u_2,u_3,1), \Omega \cap \ker L, 0) \\
&= \deg(\phi(u_1,u_2,u_3,0), \Omega \cap \ker L, 0) \\
&\neq 0.
\end{aligned}$$

By now, we have verified that Ω fulfills all requirements of Lemma 2.6; therefore, system (1.4) has at least one ω-periodic solution in $\operatorname{Dom} L \cap \bar{\Omega}$. The proof is complete. □

Remark 3.2. Note that systems (1.2) and (1.3) are special cases of system (1.4), both (1.2) and (1.3) have at least one ω-periodic solution when the conditions of Theorem 3.1 hold. In addition, the conditions are weaker than those in [7, 8].

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