Electronic Journal of Differential Equations, Vol. 2009(2009), No. 63, pp. 1–7.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu

POSITIVE SOLUTIONS FOR NONLINEAR DIFFERENCE EQUATIONS INVOLVING THE P-LAPLACIAN WITH SIGN CHANGING NONLINEARITY

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Abstract. By means of fixed point index, we establish sufficient conditions for the existence of positive solutions to p-Laplacian difference equations. In particular, the nonlinear term is allowed to change sign.

1. Introduction

The aim of the paper is to prove the existence of positive solutions to the problem

$$\begin{gathered} \Delta[\phi_p(\Delta u(t-1))] + a(t)f(u(t)) = 0, \quad t \in [1, T+1], \\ \Delta u(0) = u(T+2) = 0, \end{gathered} \tag{1.1}$$

where ϕ_p is p-Laplacian operator, i.e. $\phi_p = |s|^{p-2}s$, $p > 1$, $(\phi_p)^{-1} = \phi_q$, $\frac{1}{p} + \frac{1}{q} = 1$, $T \geq 1$ is a fixed positive integer, Δ denotes the forward difference operator with step size 1, and $[a, b] = \{a, a+1, \dots, b-1, b\} \subset \mathbb{Z}$ the set of integers.

Our work focuses on the case when the nonlinear term $f(u)$ can change sign. By means of fixed point index, some new results are obtained for the existence of at least two positive solutions to the BVP (1.1), the method of this paper is motivated by [10, 16, 20]. Due to the wide application in many fields such as science, economics, neural network, ecology, cybernetics,etc., the theory of nonlinear difference equations has been widely studied since the 1970s: see, for example [1, 2, 11, 12]. At the same time, boundary value problem (BVP) of difference equations have received much attention from many author: see [1, 2, 3, 4, 5, 6, 8, 9, 14, 15, 17, 18, 19] and the reference therein.

The approach is mainly based on fixed point theorem. For example, using the Guo-Krasnosel'skii fixed point theorem in cone and a fixed point index theorem, He [9] considered the existence of one or two positive solutions of (1.1). Li and Lu [14] studied (1.1) and obtained at least two positive solutions by an application of a fixed point theorem due to Avery and Henderson. Motivated by [9, 14] Wang and Guan [17], showed that (1.1) has at least three positive solutions by applying the Avery Five Functionals Fixed Point Theorem.

2000 *Mathematics Subject Classification.* 39A10.
Key words and phrases. Difference equations; p-Laplacian; fixed point index; sign changing nonlinearity.

Submitted December 16, 2008. Published May 8, 2009.
Project supported by the Youth Science Fund of Shanxi Province (2009021001-2).

On the other hand, the application of critical point theory in difference equations has also been studied by Pasquale Candito [6] who considered the problem

$$\begin{gathered} -\Delta[\phi_p(\Delta u(k-1))] = \lambda f(k,u(k)), \quad k\in[1,T], \\ u(0)=u(T+1)=0, \end{gathered} \tag{1.2}$$

he established the existence of at least three solutions and two positive solutions to (1.2) using critical point theory. However, almost all of these works only considered the p-Laplacian equations with nonlinearity f being nonnegative. Therefore, it is a natural problem to consider the existence of positive solution of p-Laplacian equations with sign changing nonlinearity.

Throughout this paper, we assume that the following two conditions are satisfied:

(H1) $a:[1,T+1]\to(0,+\infty)$;
(H2) $f:[0,+\infty)\to\mathbb{R}$ is continuous.

2. Preliminaries

Let $E=\{u:[0,T+2]\to\mathbb{R}:\Delta u(0)=u(T+2)=0\}$, with norm $\|u\|=\max_{t\in[0,T+2]}|u(t)|$, then $(E,\|\cdot\|)$ is a Banach space. We define two cones by

$$P=\{u\in E: u(t)\ge 0,\ t\in[0,T+2]\},$$

$$P'=\{u\in E: u \text{ is concave, nonnegative and decreasing on } [0,T+2]\}.$$

Lemma 2.1 ([9, 14, 17])**.** *If $u\in P'$, then $u(t)\ge\frac{T+2-t}{T+2}\|u\|$ for $t\in[0,T+2]$.*

Let

$$\begin{aligned} K=\big\{&u\in E: u \text{ is nonnegative and decreasing on } [0,T+2],\\ &u(t)\ge\gamma\|u\|,\ t\in[0,l]\}, \end{aligned}$$

where $\gamma=\frac{T+2-l}{T+2}\gamma_1$, and for $l\in\mathbb{Z}$ with $l=T+1$,

$$\gamma_1=\frac{(T+2-l)\phi_q[\sum_{i=1}^{l}a(i)]}{\sum_{s=0}^{T+1}\phi_q[\sum_{i=1}^{s}a(i)]}\,.$$

Note that u is a solution of (1.1) if and only if

$$u(t)=\sum_{s=t}^{T+1}\phi_q\big[\sum_{i=1}^{s}a(i)f(u(i))\big],\quad t\in[0,T+2].$$

We define the operators $F:P\to E$ and $S:K\to E$ as follows

$$(Fu)(t)=\sum_{s=t}^{T+1}\phi_q\big[\sum_{i=1}^{s}a(i)f(u(i))\big],\quad t\in[0,T+2], \tag{2.1}$$

$$(Su)(t)=\sum_{s=t}^{T+1}\phi_q\big[\sum_{i=1}^{s}a(i)f^+(u(i))\big],\quad t\in[0,T+2], \tag{2.2}$$

where $f^+(u(t))=\max\{f(u(t)),0\}$, $t\in[0,T+2]$. It is obvious that $S:K\to K$ is completely continuous (see [9, Theorem 3.1]).

Lemma 2.2 ([7])**.** *Let K be a cone in a Banach space X. Let D be an open bounded subset of X with $D_K=D\cap K\ne\phi$ and $\overline{D_K}\ne K$. Assume that $A:\overline{D_K}\to K$ is a completely continuous map such that $x\ne Ax$ for $x\in\partial D_K$. Then the following results hold:*

(1) *If* $\|Ax\| \le \|x\|$, $x \in \partial D_K$, *then* $i_K(A, D_K) = 1$;
(2) *If there exists* $x_0 \in K\backslash\{\theta\}$ *such that* $x \neq Ax + \lambda x_0$, *for all* $x \in \partial D_K$ *and all* $\lambda > 0$, *then* $i_K(A, D_K) = 0$;
(3) *Let* U *be an open set in* X *such that* $\overline{U} \subset D_K$. *If* $i_K(A, D_K) = 1$ *and* $i_K(A, U_K) = 0$, *then* A *has a fixed point in* $D_K\backslash\overline{U}_K$. *The same results holds, if* $i_K(A, D_K) = 0$ *and* $i_K(A, U_K) = 1$.

Lemma 2.3 ([13]). *Let* $K_\rho = \{u(t) \in K : \|u\| < \rho\}$ *and* $\Omega_\rho = \{u(t) \in K : \min_{0\le t\le l} u(t) < \gamma\rho\}$. *Then the following properties are satsified:*
(a) $K_{\gamma\rho} \subset \Omega_\rho \subset K_\rho$;
(b) Ω_ρ *is open relative to K;*
(c) $u \in \partial\Omega_\rho$ *if and only if* $\min_{0\le t\le l} u(t) = \gamma\rho$;
(d) *If* $u \in \partial\Omega_\rho$, *then* $\gamma\rho \le u(t) \le \rho$ *for* $t \in [0, l]$.

Let
$$m = \Big\{ \sum_{s=0}^{T+1} \phi_q\big[\sum_{i=1}^{s} a(i)\big]\Big\}^{-1}, \tag{2.3}$$
$$M = \Big\{(T + 2 - l)\phi_q\big[\sum_{i=1}^{l} a(i)\big]\Big\}^{-1}. \tag{2.4}$$
We remark that by (H1), $0 < m$, $M < +\infty$ and
$$M\gamma = M\frac{T + 2 - l}{T + 2}\gamma_1 = m\frac{T + 2 - l}{T + 2} < m.$$

Lemma 2.4. *If* f *satisfies the condition*
$$f(u) \le \phi_p(m\rho), \quad \text{for } u \in [0, \rho],\ u \neq Su,\ u \in \partial K_\rho, \tag{2.5}$$
then $i_K(S, K_\rho) = 1$.

Proof. If $u \in \partial K_\rho$, then from (2.2), (2.3) and (2.5), we have
$$\begin{aligned}(Su)(t) &= \sum_{s=t}^{T+1} \phi_q\big[\sum_{i=1}^{s} a(i)f^+(u(i))\big]\\ &\le \sum_{s=0}^{T+1} \phi_q\big[\sum_{i=1}^{s} a(i)\phi_p(m\rho)\big]\\ &= \sum_{s=0}^{T+1} m\rho\phi_q\big[\sum_{i=1}^{s} a(i)\big] = \rho.\end{aligned}$$
This implies that $\|Su\| \le \|u\|$ for $u \in \partial K_\rho$. By Lemma 2.2(1), we have $i_K(S, K_\rho) = 1$. The proof is complete. □

Lemma 2.5. *If* f *satisfies the condition*
$$f(u) \ge \phi_p(M\gamma\rho), \quad \text{for } u \in [\gamma\rho, \rho],\ u \neq Su,\ u \in \partial\Omega_\rho, \tag{2.6}$$
then $i_K(S, \Omega_\rho) = 0$.

Proof. Let $e(t) \equiv 1$, $t \in [0, T + 2]$. Then $e \in \partial K_1$. Next we shall prove that
$$u \neq Su + \lambda e, \quad u \in \partial\Omega_\rho,\ \lambda > 0.$$

In fact, if it is not so, then there exist $u_0 \in \partial\Omega_\rho$, and $\lambda_0 > 0$ such that $u_0 = Su_0 + \lambda_0 e$. Then from (2.2), (2.4) and (2.6), we obtain

$$\begin{aligned}
u_0(t) &= (Su_0)(t) + \lambda_0 \\
&\geq (Su_0)(l) + \lambda_0 \\
&= \sum_{s=l}^{T+1} \phi_q\big[\sum_{i=1}^{s} a(i) f^+(u_0(i))\big] + \lambda_0 \\
&\geq (T+2-l)\phi_q\big[\sum_{i=1}^{l} a(i) f^+(u_0(i))\big] + \lambda_0 \\
&\geq (T+2-l)\phi_q\big[\sum_{i=1}^{l} a(i)\phi_p(M\gamma\rho)\big] + \lambda_0 \\
&= (T+2-l)M\gamma\rho\phi_q\big[\sum_{i=1}^{l} a(i)\big] + \lambda_0 \\
&= \gamma\rho + \lambda_0.
\end{aligned}$$

This together with Lemma 2.3(c) implies that $\gamma\rho \geq \gamma\rho + \lambda_0$, a contradiction. Hence, it follows from Lemma 2.2 (2) that $i_K(S, \Omega_\rho) = 0$. The proof is complete. $\square$

3. Existence of positive solutions

Theorem 3.1. *Assume* (H1), (H2) *and that one of the following two conditions holds:*

- (H3) *There exist $\rho_1, \rho_2 \in (0, +\infty)$ with $\rho_1 < \gamma\rho_2$ and $\rho_2 < \rho_3$ such that*
  - (i) *$f(u) \leq \phi_p(m\rho_1)$, for $u \in [0, \rho_1]$;*
  - (ii) *$f(u) \geq 0$, for $u \in [\gamma\rho_1, \rho_3]$, moreover, $f(u) \geq \phi_p(M\gamma\rho_2)$, for $u \in [\gamma\rho_2, \rho_2]$, $x \neq Sx$, $x \in \partial\Omega_{\rho_2}$;*
  - (iii) *$f(u) \leq \phi_p(m\rho_3)$, for $u \in [0, \rho_3]$.*
- (H4) *There exist ρ_1, ρ_2 and $\rho_3 \in (0, +\infty)$ with $\rho_1 < \rho_2 < \gamma\rho_3$ such that*
  - (i) *$f(u) \geq \phi_p(M\gamma\rho_1)$, for $u \in [\gamma^2\rho_1, \rho_2]$;*
  - (ii) *$f(u) \leq \phi_p(m\rho_2)$, for $u \in [0, \rho_2]$, $x \neq Sx$, $x \in \partial K_{\rho_2}$;*
  - (iii) *$f(u) \geq 0$, for $u \in [\gamma\rho_2, \rho_3]$, moreover, $f(u) \geq \phi_p(M\gamma\rho_3)$, for $u \in [\gamma\rho_3, \rho_3]$.*

Then (1.1) *has at least two positive solutions u_1 and u_2.*

Proof. Assuming (H3), we show that S has a fixed point u_1 either in ∂K_{ρ_1} or u_1 in $\Omega_{\rho_2} \setminus \overline{K_{\rho_1}}$. If $u \neq Su$, $u \in \partial K_{\rho_1} \cup \partial K_{\rho_3}$, by Lemmas 2.4 and 2.5, we have

$$i_K(S, K_{\rho_1}) = 1, \quad i_K(S, \Omega_{\rho_2}) = 0, \quad i_K(S, K_{\rho_3}) = 1.$$

By Lemma 2.3(b) and $\rho_1 < \gamma\rho_2$, we have $\overline{K}_{\rho_1} \subset K_{\gamma\rho_2} \subset \Omega_{\rho_2}$. By Lemma 2.2(3), we have S has a fixed point $u_1 \in \Omega_{\rho_2} \setminus \overline{K_{\rho_1}}$. Similarly, S has a fixed point $u_2 \in K_{\rho_3} \setminus \overline{\Omega_{\rho_2}}$. Clearly,

$$\|u_1\| > \rho_1, \quad \min_{t\in[0,l]} u_1(t) = u_1(l) \geq \gamma\|u_1\| > \gamma\rho_1.$$

This implies $\gamma\rho_1 \leq u_1(t) \leq \rho_2$, $t \in [0, l]$. By (H3)(ii), we have $f(u_1(t)) \geq 0$, $t \in [0, l]$; i.e., $f^+(u_1(t)) = f(u_1(t))$. Combining with the fact that $Su = Fu = 0$ if $t = T + 2$, we can get $Su_1 = Fu_1$. That means u_1 is a fixed point of F. From

$u_2 \in K_{\rho_3} \backslash \overline{\Omega_{\rho_2}}$, $\rho_2 < \rho_3$ and Lemma 2.3(b) we have $\overline{K}_{\gamma\rho_2} \subset \Omega_{\rho_2} \subset K_{\rho_3}$. Obviously, $\|u_2\| > \gamma\rho_2$. This implies that

$$\min_{t\in[0,l]} u_2(t) = u_2(l) \geq \gamma\|u_2\| > \gamma^2\rho_2.$$

Therefore,

$$\gamma^2\rho_2 \leq u_2(t) \leq \rho_3, \quad t \in [0,l].$$

By $\rho_1 < \gamma\rho_2$ and (H3)(ii), we have $f(u_2(t)) \geq 0$, $t \in [0,l]$; i.e., $f^+(u_2(t)) = f(u_2(t))$. So u_2 is another fixed point of F. Thus, we have proved that (1.1) has at least two positive solutions u_1 and u_2.

The proof under assumption (H4) is similar to the case above. This completes the proof. □

Theorem 3.2. *Assume* (H1), (H2) *that one of the following two conditions holds:*

- (H5) *There exist $\rho_1, \rho_2 \in (0,+\infty)$ with $\rho_1 < \gamma\rho_2$ such that*
  - (i) *$f(u) \leq \phi_p(m\rho_1)$, for $u \in [0,\rho_1]$;*
  - (ii) *$f(u) \geq 0$, for $u \in [\gamma\rho_1, \rho_2]$, moreover, $f(u) \geq \phi_p(M\gamma\rho_2)$, for $u \in [\gamma\rho_2, \rho_2]$.*
- (H6) *There exist ρ_1, $\rho_2 \in (0,+\infty)$ with $\rho_1 < \rho_2$ such that*
  - (i) *$f(u) \geq \phi_p(M\gamma\rho_1)$, for $u \in [\gamma^2\rho_1, \rho_2]$;*
  - (ii) *$f(u) \leq \phi_p(m\rho_2)$, for $u \in [0,\rho_2]$.*

Then (1.1) *has at least one positive solution.*

4. Example

In this section, we present a simple example to illustrate our results. Consider the following boundary-value problem

$$\begin{gathered} \Delta[\phi_p(\Delta u(t-1))] + a(t)f(u(t)) = 0, \quad t \in [1,4], \\ \Delta u(0) = u(5) = 0, \end{gathered} \tag{4.1}$$

where $p = 3/2$, $q = 3$, $a(t) \equiv 1$, $T = 3$ and

$$f(u) = \begin{cases} (u - \frac{8}{75})^{11}, & u \in [0, \frac{8}{75}]; \\ (\frac{1}{30})^{1/2} \sin(\frac{75}{67}\frac{\pi}{2}u - \frac{8}{67}\frac{\pi}{2}), & u \in [\frac{8}{75}, 1]; \\ (\frac{1}{30})^{1/2}(\frac{8}{3} - \frac{5}{3}u) + (\frac{1}{10})^{1/2}(\frac{5}{3}u - \frac{5}{3}) & u \in [1, \frac{8}{5}]; \\ (\frac{1}{10})^{1/2} + \frac{25}{52\times 67^2}(u - \frac{8}{5})^2, & u \in [\frac{8}{5}, 15]; \\ (\frac{1}{10})^{1/2} + \frac{25}{52\times 67^2}(15 - \frac{8}{5})^2 + [1 + (u-15)(25-u)], & u \in [15, +\infty). \end{cases}$$

It is easy to check that $f : [0,+\infty) \to \mathbb{R}$ is continuous, $l = 4$, it follows from a direct calculation that

$$\gamma_1 = \frac{\phi_q\big[\sum_{i=1}^4 a(i)\big]}{\sum_{s=0}^4 \big[\sum_{i=1}^s a(i)\big]} = \frac{(1+1+1+1)^2}{\sum_{s=0}^4 [a(1) + \cdots + a(s)]^2} = \frac{8}{15},$$

$$\begin{aligned}
m &= \Big\{\sum_{s=0}^{4}\phi_q\big[\sum_{i=1}^{s}a(i)\big]\Big\}^{-1} = \Big\{\sum_{s=0}^{4}[a(1)+a(2)+\dots+a(s)]^2\Big\}^{-1}\\
&= \{1^2+2^2+3^2+4^2\}^{-1} = \frac{1}{30},\\
M &= \Big\{\phi_q\big[\sum_{i=1}^{4}a(i)\big]\Big\}^{-1} = \{\phi_q[a(1)+a(2)+a(3)+a(4)]\}^{-1}\\
&= \{4^2\}^{-1} = \frac{1}{16},\\
\gamma &= \frac{T+2-l}{T+2}\gamma_1 = \frac{3+2-4}{3+2}\cdot\frac{8}{15} = \frac{1}{5}\cdot\frac{8}{15} = \frac{8}{75}.
\end{aligned}$$

Choose $\rho_1 = 1$, $\rho_2 = 15$, $\rho_3 = 25$. Then $\gamma\rho_1 < \rho_1 < \gamma\rho_2 < \rho_2 < \rho_3$.

In addition, by the definition of f, we have

(i) $f(u) \le \phi_p(m\rho_1) = (\frac{1}{30})^{1/2}$, for $u \in [0,1]$;

(ii) $f(u) \ge 0$, for $u \in [\frac{8}{75}\cdot 1, 25]$, moreover, $f(u) \ge \phi_p(M\gamma\rho_2) = (\frac{1}{16}\cdot\frac{8}{75}\cdot 15)^{1/2} = (\frac{1}{10})^{1/2}$, for $u \in [\frac{8}{75}\cdot 15, 15]$;

(iii) $f(u) \le \phi_p(m\rho_3) = (\frac{1}{30}\cdot 25)^{1/2} = (\frac{5}{6})^{1/2}$, for $u \in [0,25]$.

By (2.2), we have

$$(Su)(t) = \sum_{s=t}^{4}\phi_q\big[\sum_{i=1}^{s}a(i)f^+(u(i))\big] = \sum_{s=t}^{4}[a(1)f^+(u(1))+\dots+a(s)f^+(u(s))]^2.$$

Since $f^+(u) \le (\frac{1}{10})^{1/2} + \frac{25}{52\times 67^2}(15-\frac{8}{5})^2$, $u \in [0,15]$.

For $u \in \partial\Omega_{15}$, we have

$$\begin{aligned}
\|Su\| &= Su(0)\\
&= \sum_{s=0}^{4}[a(1)f^+(u(1))+\dots+a(s)f^+(u(s))]^2\\
&= [f^+(u(1))]^2 + [f^+(u(1))+f^+(u(2))]^2 + [f^+(u(1))+f^+(u(2))+f^+(u(3))]^2\\
&\quad + [f^+(u(1))+f^+(u(2))+f^+(u(3))+f^+(u(4))]^2\\
&\le 30[f^+(u)]^2\\
&< 15 = \|u\|.
\end{aligned}$$

This implies $Su \ne u$, for $u \in \partial\Omega_{15}$. Thus, (H3) of Theorem 3.1 is satisfied. Then (4.1) has two positive solutions u_1, u_2.

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