Electronic Journal of Differential Equations, Vol. 2010(2010), No. 151, pp. 1–8.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu

EXISTENCE OF SOLUTIONS TO N-TH ORDER NEUTRAL DYNAMIC EQUATIONS ON TIME SCALES

QIAOLUAN LI, ZHENGUO ZHANG

Abstract. In this article, we study n-th order neutral nonlinear dynamic equation on time scales. We obtain sufficient conditions for the existence of non-oscillatory solutions by using fixed point theory.

1. Introduction

This article concerns the n-th order neutral dynamic equation

$$(x(t)+p(t)x(\tau(t)))^{\Delta^n}+f_1(t,x(\tau_1(t)))-f_2(t,x(\tau_2(t)))=0, \tag{1.1}$$

for $t\geq t_0$, where $t\in\mathbb{T}$, $n\in\mathbb{N}$. We assume $p\in C_{rd}(\mathbb{T},\ \mathbb{R}),\tau,\tau_i\in C_{rd}(\mathbb{T},\mathbb{T})$, τ is strictly increasing, $\tau(t)<t$, $\tau(t)\to\infty$, $\tau_i(t)\to\infty$, $f_i\in C_{rd}(\mathbb{T}\times\mathbb{R},\mathbb{R})$, $f_1(t,u)f_2(t,u)>0$, and f_i is non-decreasing in u. In the sequel, without loss of generality, we assume that $f_i(t,u)>0$, $i=1,2$.

In 1988, Stephan Hilger [7] introduced the theory of time scales as a means of unifying discrete and continuous calculus. Several authors have expounded on various aspects of this new theory, see [1, 6, 12] and references therein. Recently, much attention is concerned with questions of existence of non-oscillatory solutions for dynamic equations on time scales. For significant works along this line, see [5, 8, 9, 10]. Many results have been obtained for first and second order dynamic equations, however, few results are available for higher order dynamic equations. Motivated by these works, we investigate the existence of non-oscillatory solutions of (1.1).

In Section 2, we present some preliminary material that we will need to show the existence of solutions of (1.1). We present our main results in Section 3.

2. Preliminaries

We assume the reader is familiar with the notation and basic results for dynamic equations on time scales. For a review of this topic we direct the reader to the monographs [2, 3].

We recall x is a solution of (1.1) provided that $x(t)+p(t)x(\tau(t))$ is n times differentiable, and x satisfies (1.1). A solution x of (1.1) is called non-oscillatory if x is of one sign when $t\geq T$.

2000 *Mathematics Subject Classification.* 34K40, 34N99, 39A10.
Key words and phrases. Time scales; dynamic equations; non-oscillatory solution.
©2010 Texas State University - San Marcos.
Submitted June 6, 2010. Published October 21, 2010.

We define a sequence of functions $g_k(s,t)$, $k = 1, 2, \dots$ as follows.

$$\begin{gathered} g_0(s,t) \equiv 1, \quad s, t \in \mathbb{T}^\kappa, \\ g_{k+1}(s,t) = \int_t^s g_k(\sigma(u), t)\Delta u, \quad s, t \in \mathbb{T}^\kappa. \end{gathered} \tag{2.1}$$

For $g_k(s,t)$, we have the following Lemma.

Lemma 2.1 ([11])**.** *Assume s is fixed, and let $g_k^\Delta(s,t)$ be the derivative of $g_k(s,t)$ with respect to t. Then*

$$g_k^\Delta(s,t) = -g_{k-1}(s,t), \quad k \in \mathbb{N},\ t \in \mathbb{T}^\kappa. \tag{2.2}$$

Lemma 2.2 ([4])**.** *Let X be a Banach space, Ω be a bounded closed convex subset of X and let A, B be maps from Ω to X such that $Ax + By \in \Omega$ for every pair $x, y \in \Omega$. If A is a contraction and B is completely continuous, then the equation $Ax + Bx = x$ has a solution in Ω.*

Lemma 2.3 ([4])**.** *Let X be a locally convex linear space, S be a compact convex subset of X, and $T : S \to S$ be a continuous mapping with $T(S)$ compact. Then T has a fixed point in S.*

3. Main Results

Theorem 3.1. *Assume that $0 < p(t) \le p < 1$, and there exists $b > 0$ such that*

$$\int_{t_0}^\infty g_{n-1}(\sigma(s), 0) f_i(s, b)\Delta s < \infty, \quad i = 1, 2. \tag{3.1}$$

Then (1.1) has a bounded non-oscillatory solution which is bounded away from zero.

Proof. Let BC be the set of bounded functions on $[t_0, \infty)$ with sup norm $\|x\| = \sup_{t \ge t_0} |x(t)|$, $t \in \mathbb{T}$. Let $\Omega \subset BC$, $\Omega = \{x \in BC, 0 < M_1 \le x(t) \le M_2 < b, t \ge t_0, t \in \mathbb{T}\}$, where $M_1 < (1-p)M_2$, then Ω is a closed bounded and convex subset of BC.

Choose α such that $pM_2 + M_1 < \alpha < M_2$, and $c = \min\{M_2 - \alpha, \alpha - pM_2 - M_1\}$. We choose $t_1 > t_0$, such that $\tau(t) \ge t_0$, $\tau_i(t) \ge t_0$, $i = 1, 2$ $t \ge t_1$ and $\int_{t_1}^\infty g_{n-1}(\sigma(s), 0) f_i(s, b)\Delta s \le c$, $i = 1, 2$. Define a mapping Γ on Ω as follows.

$$(\Gamma x)(t) = (\Gamma_1 x)(t) + (\Gamma_2 x)(t),$$

where

$$(\Gamma_1 x)(t) = \begin{cases} \alpha - p(t)x(\tau(t)), & t \ge t_1,\ t \in \mathbb{T}, \\ (\Gamma_1 x)(t_1), & t_0 \le t \le t_1,\ t \in \mathbb{T}. \end{cases}$$

$$(\Gamma_2 x)(t) = \begin{cases} (-1)^{n-1} \int_t^\infty g_{n-1}(\sigma(s), t)\big[f_1(s, x(\tau_1(s))) & \\ -f_2(s, x(\tau_2(s)))\big]\Delta s, & t \ge t_1, \\ (\Gamma_2 x)(t_1), & t_0 \le t \le t_1. \end{cases}$$

For any $x, y \in \Omega$, $t \ge t_0$, $t \in \mathbb{T}$, we have

$$\begin{gathered} (\Gamma_1 x)(t) + (\Gamma_2 y)(t) \le \alpha + c \le M_2, \\ (\Gamma_1 x)(t) + (\Gamma_2 y)(t) \ge \alpha - pM_2 - c \ge M_1. \end{gathered}$$

Hence for $t \geq t_0$, $t \in \mathbb{T}$, $\Gamma_1 x + \Gamma_2 y \in \Omega$. Clearly, Γ_1 is a contraction mapping on Ω and Γ_2 is continuous. We shall show that Γ_2 is completely continuous. In fact, for any $x \in \Omega$, for $t_0 \leq t \leq t_1$, $(\Gamma_2 x)(t) = (\Gamma_2 x)(t_1)$, and for $t \geq t_1$, we have

$$\begin{aligned}
|(\Gamma_2 x)(t)| &\leq \int_t^\infty g_{n-1}(\sigma(s), t)|f_1(s, x(\tau_1(s))) - f_2(s, x(\tau_2(s)))|\Delta s \\
&\leq \int_t^\infty g_{n-1}(\sigma(s), t) f_1(s, x(\tau_1(s)))\Delta s \\
&\leq \int_t^\infty g_{n-1}(\sigma(s), 0) f_1(s, b)\Delta s \leq c.
\end{aligned}$$

Hence $\Gamma_2\Omega$ is uniformly bounded. For $\varepsilon > 0$, there exists a T, such that for $t \geq T$,

$$\int_t^\infty g_{n-1}(\sigma(s), 0) f_i(s, b)\Delta s < \frac{\varepsilon}{2}.$$

For $t, t' > T$, we have

$$|(\Gamma_2 x)(t) - (\Gamma_2 x)(t')| \leq 2\int_T^\infty g_{n-1}(\sigma(s), 0) f_i(s, b)\Delta s < \varepsilon.$$

For $t, t' \in [t_1, T]$, we have

$$\begin{aligned}
&|(\Gamma_2 x)(t) - (\Gamma_2 x)(t')| \\
&= |\int_t^\infty g_{n-1}(\sigma(s), t)[f_1(s, x(\tau_1(s))) - f_2(s, x(\tau_2(s)))]\Delta s \\
&\quad - \int_{t'}^\infty g_{n-1}(\sigma(s), t')[f_1(s, x(\tau_1(s))) - f_2(s, x(\tau_2(s)))]\Delta s| \\
&\leq |\int_t^{t'} g_{n-1}(\sigma(s), t)[f_1(s, x(\tau_1(s))) - f_2(s, x(\tau_2(s)))]\Delta s| \\
&\quad + \int_{t'}^T |g_{n-1}(\sigma(s), t) - g_{n-1}(\sigma(s), t')||f_1(s, x(\tau_1(s))) - f_2(s, x(\tau_2(s)))|\Delta s \\
&\quad + \int_T^\infty |g_{n-1}(\sigma(s), t) - g_{n-1}(\sigma(s), t')||f_1(s, x(\tau_1(s))) - f_2(s, x(\tau_2(s)))|\Delta s.
\end{aligned}$$

There exists a δ, so that when $|t - t'| < \delta$, $|(\Gamma_2 x)(t) - (\Gamma_2 x)(t')| < \varepsilon$, which shows that the family $\Gamma_2\Omega$ is equicontinuous, Γ_2 is completely continuous.

By Lemma 2, there exists a fixed point $x \in \Omega$, such that $\Gamma x = x$. It is easily to see that x is a bounded non-oscillatory solution which is bounded away from zero. □

Theorem 3.2. *Assume that $1 < p_1 \leq p(t) \leq p_2$, and (3.1) holds. Then (1.1) has a bounded non-oscillatory solution which is bounded away from zero.*

Proof. We choose $t_1 > t_0$ such that

$$T_0 = \min\{\tau(t_1), \inf_{t \geq t_1}(\tau_1(t)), \inf_{t \geq t_1}(\tau_2(t))\} \geq t_0.$$

Let BC be the set of bounded functions on $[t_0, \infty)$ with supremum norm $\|x\| = \sup_{t \geq t_0}|x(t)|$, $t \in \mathbb{T}$. Define a set $\Omega \subset BC$ as follows:

$$\begin{aligned}
\Omega = \Big\{&x \in BC, x^\Delta(t) \leq 0, 0 < M_1 \leq x(t) \leq p_1 M_1 < b, t \geq t_1, \\
&x(t) = x(t_1), \quad T_0 \leq t \leq t_1.\Big\}
\end{aligned}$$

Then Ω is a closed bounded and convex subset of BC. Let $c=\min\{\alpha-M_1, p_1M_1-\alpha\}$, where $M_1<\alpha<p_1M_1$. We choose $t_2\ge t_1$, such that for $t\ge t_2$,

$$\int_t^\infty g_{n-1}(\sigma(s),0)f_i(s,b)\Delta s\le c.$$

For $x\in\Omega$, define

$$\psi(t)=\begin{cases}\sum_{i=1}^\infty\frac{(-1)^{i-1}x(\tau^{-i}(t))}{H_i(\tau^{-i}(t))}, & t\ge t_2,\\ \psi(t_2), & T_0\le t\le t_2,\end{cases}$$

where $\tau^0(t)=t$, $\tau^i(t)=\tau(\tau^{i-1}(t))$, $\tau^{-i}(t)=\tau^{-1}(\tau^{-(i-1)}(t))$, $H_0(t)=1$, $H_i(t)=\prod_{j=0}^{i-1}p(\tau^j(t))$, $i=1,2,\dots$. From $M_1\le x(t)\le p_1M_1$, we have

$$0<\psi(t)\le p_1M_1,\quad t\ge t_2,\ t\in\mathbb{T}.$$

Define a mapping Γ on Ω as follows

$$(\Gamma x)(t)=\begin{cases}\alpha+(-1)^{n-1}\int_t^\infty g_{n-1}(\sigma(s),t)\\ \times\sum_{i=1}^2(-1)^{i+1}f_i(s,\psi(\tau_i(s)))\Delta s, & t\ge t_2,\\ (\Gamma x)(t_2), & T_0\le t\le t_2.\end{cases}$$

Then Γ satisfies the following conditions:
(a) $\Gamma\Omega\subseteq\Omega$. In fact, for any $x\in\Omega$, $(\Gamma x)(t)\ge\alpha-c\ge M_1$, $(\Gamma x)(t)\le\alpha+c\le p_1M_1$.
(b) Γ is continuous which is easy to show.
(c) Similar to Theorem 1, Γ is equicontinuous.

By Lemma 3, there exists $x\in\Omega$, such that $x=\Gamma x$; i.e.,

$$x(t)=\alpha+(-1)^{n-1}\int_t^\infty g_{n-1}(\sigma(s),t)[f_1(s,\psi(\tau_1(s)))-f_2(s,\psi(\tau_2(s)))]\Delta s.$$

Since $\psi(t)+p(t)\psi(\tau(t))=x(t)$, we obtain

$$\begin{aligned}&\psi(t)+p(t)\psi(\tau(t))\\ &=\alpha+(-1)^{n-1}\int_t^\infty g_{n-1}(\sigma(s),t)[f_1(s,\psi(\tau_1(s)))-f_2(s,\psi(\tau_2(s)))]\Delta s.\end{aligned}$$

So $\psi(t)$ satisfies (1.1) for $t\ge t_0$, $t\in\mathbb{T}$, and $\frac{p_1-1}{p_1p_2}x(\tau^{-1}(t))\le\psi(t)\le x(t)$. The proof is complete. □

Theorem 3.3. *Assume that $-1<p_1\le p(t)\le 0$, and (3.1) holds. Then (1.1) has a bounded non-oscillatory solution which is bounded away from zero.*

Proof. Let BC be the set of bounded functions on $[t_0,\infty)$ with sup norm $\|x\|=\sup_{t\ge t_0}|x(t)|$. We choose $M_1,M_2<b$ such that $0<M_1<\alpha<(1+p_1)M_2$. Let $\Omega=\{x\in BC, M_1\le x(t)\le M_2<b, t\ge t_0\}$. Then Ω is a closed bounded and convex subset of BC. Let $c=\min\{\alpha-M_1,(1+p_1)M_2-\alpha\}$. We choose $t_1\ge t_0$, such that $\tau(t)\ge t_0,\tau_i(t)\ge t_0$, for $t\ge t_1$ and $\int_{t_1}^\infty g_{n-1}(\sigma(s),0)f_i(s,b)\Delta s\le c$, $i=1,2$. Define a mapping Γ on Ω as follows:

$$(\Gamma x)(t)=(\Gamma_1x)(t)+(\Gamma_2x)(t),$$

where

$$(\Gamma_1 x)(t) = \begin{cases} \alpha - p(t)x(\tau(t)), & t \geq t_1, \\ (\Gamma_1 x)(t_1), & t_0 \leq t \leq t_1, \end{cases}$$

$$(\Gamma_2 x)(t) = \begin{cases} (-1)^{n-1} \int_t^\infty g_{n-1}(\sigma(s),t)\big[f_1(s,x(\tau_1(s))) \\ -f_2(s,x(\tau_2(s)))\big]\Delta s, & t \geq t_1, \\ (\Gamma_2 x)(t_1), & t_0 \leq t \leq t_1. \end{cases}$$

For $x, y \in \Omega$, $t \geq t_0$, we have

$$(\Gamma_1 x)(t) + (\Gamma_2 y)(t) \leq \alpha - p_1 M_2 + c \leq M_2,$$
$$(\Gamma_1 x)(t) + (\Gamma_2 y)(t) \geq \alpha - c \geq M_1.$$

Hence for $t \geq t_0$, $\Gamma_1 x + \Gamma_2 y \in \Omega$. Clearly, Γ_1 is a contraction mapping on Ω and Γ_2 is continuous. Similar to Theorem 1, we can prove that Γ_2 is completely continuous. So that there exists $x \in \Omega$ such that $x = \Gamma x$. The proof is complete. □

Theorem 3.4. *Assume that $p_1 \leq p(t) \leq p_2 < -1$ and (3.1) holds. Then (1.1) has a bounded non-oscillatory solution which is bounded away from zero.*

Proof. Let BC be the bounded functions on $[t_0, \infty)$. We choose $0 < M_1 < M_2 < b$, such that $-p_1 M_1 < \alpha < (-p_2 - 1)M_2$. Let $\Omega = \{x \in BC, M_1 \leq x(t) \leq M_2, t \geq t_0\}$, $c = \min\{\frac{(\alpha + M_1 p_1)p_2}{p_1}, (-p_2 - 1)M_2 - \alpha\}$. Choose $t_1 \geq t_0$ such that for $t \geq t_1$,

$$\tau^{-1}(\tau_i(t)) \geq t_0, \quad \int_{\tau^{-1}(t)}^\infty g_{n-1}(\sigma(s),0) f_i(s,b)\Delta s \leq c, \quad i = 1,2.$$

Define two maps Γ_1, Γ_2 on Ω as follows:

$$(\Gamma_1 x)(t) = \begin{cases} -\frac{\alpha}{p(\tau^{-1}(t))} - \frac{x(\tau^{-1}(t))}{p(\tau^{-1}(t))}, & t \geq t_1, \\ (\Gamma_1 x)(t_1), & t_0 \leq t \leq t_1. \end{cases}$$

$$(\Gamma_2 x)(t) = \begin{cases} \frac{(-1)^{n-1}}{p(\tau^{-1}(t))} \int_{\tau^{-1}(t)}^\infty g_{n-1}(\sigma(s),t)\big[f_1(s,x(\tau_1(s))) \\ -f_2(s,x(\tau_2(s)))\big]\Delta s, & t \geq t_1, \\ (\Gamma_2 x)(t_1), & t_0 \leq t \leq t_1. \end{cases}$$

For $x, y \in \Omega$, $(\Gamma_1 x)(t) + (\Gamma_2 y)(t) \geq \frac{-\alpha}{p_1} + \frac{c}{p_2} \geq M_1$, $(\Gamma_1 x)(t) + (\Gamma_2 y)(t) \leq \frac{-\alpha}{p_2} - \frac{M_2}{p_2} - \frac{c}{p_2} \leq M_2$. So $(\Gamma_1 x)(t) + (\Gamma_2 y)(t) \in \Omega$. Since $p_1 \leq p(t) \leq p_2 \leq -1$, we get Γ_1 is contraction. We shall prove that Γ_2 is completely continuous. In fact, for all $x \in \Omega$, $t_0 \leq t \leq t_1$, $(\Gamma_2 x)(t) = (\Gamma_2 x)(t_1)$. For $t \geq t_1$,

$$|(\Gamma_2 x)(t)| \leq -\frac{1}{p_2} \int_{\tau^{-1}(t)}^\infty g_{n-1}(\sigma(s),0) f_i(s,x(\tau_i(s)))\Delta s \leq -\frac{c}{p_2},$$

so $\Gamma_2\Omega$ is uniformly bounded. By the conditions, for all $\varepsilon > 0$, there exists a $T > t_1$ such that

$$\int_{\tau^{-1}(T)}^\infty g_{n-1}(\sigma(s),0) f_i(s,b)\Delta s < \frac{-p_2\varepsilon}{2}.$$

For all $x \in \Omega$, $t, t' \geq T$, we have

$$|(\Gamma_2 x)(t) - (\Gamma_2 x)(t')| \leq -\frac{2}{p_2} \int_{\tau^{-1}(T)}^\infty g_{n-1}(\sigma(s),0) f_i(s,b)\Delta s < \varepsilon.$$

Since $\tau^{-1}(t)$, $\frac{1}{p(\tau^{-1}(t))}$ are continuous on $[t_1,T]$, they are uniformly continuous on $[t_1,T]$. Let $|g_{n-1}(\sigma(t),0)f_i(t,b)|\le M$, when $t\in[t_1,T]$. Hence for each $\varepsilon>0$, there exists a $\delta>0$ such that for $t,t'\in[t_1,T]$, $|t-t'|<\delta$, we have

$$
\begin{gathered}
|\frac{1}{p(\tau^{-1}(t))}-\frac{1}{p(\tau^{-1}(t'))}|<\frac{\varepsilon}{3c},\quad |\tau^{-1}(t)-\tau^{-1}(t')|<\frac{-p_2\varepsilon}{3M},\\
\int_{\tau^{-1}(t_1)}^{\infty}|g_{n-1}(\sigma(s),t)-g_{n-1}(\sigma(s),t')|f_i(s,b)\Delta s<\frac{|p_2|\varepsilon}{3}.
\end{gathered}
$$

For all $x\in\Omega$, when $t,t'\in[t_1,T]$ and $|t-t'|<\delta$, we have

$$
\begin{aligned}
&|(\Gamma_2x)(t)-(\Gamma_2x)(t')|\\
&=|\frac{1}{p(\tau^{-1}(t))}\int_{\tau^{-1}(t)}^{\infty}g_{n-1}(\sigma(s),t)[f_1(s,x(\tau_1(s)))-f_2(s,x(\tau_2(s)))]\Delta s\\
&\quad-\frac{1}{p(\tau^{-1}(t'))}\int_{\tau^{-1}(t')}^{\infty}g_{n-1}(\sigma(s),t')[f_1(s,x(\tau_1(s)))-f_2(s,x(\tau_2(s)))]\Delta s|\\
&\le|\frac{1}{p(\tau^{-1}(t))}-\frac{1}{p(\tau^{-1}(t'))}|\int_{\tau^{-1}(t)}^{\infty}g_{n-1}(\sigma(s),t)|f_1(s,x(\tau_1(s)))-f_2(s,x(\tau_2(s)))|\Delta s\\
&\quad+|\frac{1}{p(\tau^{-1}(t'))}||\int_{\tau^{-1}(t)}^{\tau^{-1}(t')}g_{n-1}(\sigma(s),t)[f_1(s,x(\tau_1(s)))-f_2(s,x(\tau_2(s)))]\Delta s|\\
&\quad+|\frac{1}{p(\tau^{-1}(t'))}|\int_{\tau^{-1}(t')}^{\infty}|g_{n-1}(\sigma(s),t)-g_{n-1}(\sigma(s),t')|\\
&\quad\times|\sum_{i=1}^{2}(-1)^{i+1}f_i(s,x(\tau_i(s)))|\Delta s\\
&<\frac{\varepsilon c}{3c}+\frac{M}{|p_2|}\cdot\frac{|p_2|\varepsilon}{3M}+\frac{|p_2|\varepsilon}{|p_2|3}=\varepsilon,
\end{aligned}
$$

which shows that the family $\Gamma_2\Omega$ is equicontinuous, so Γ_2 is completely continuous. By Lemma 2, there exists a fixed point $x\in\Omega$ such that $\Gamma x=x$. It is easily to see that x is a bounded non-oscillatory solution which is bounded away from zero. $\square$

Example. On the time scale $\mathbb{T}=\{q^n : n\in\mathbb{N}_0,\ q>1\}$, consider the dynamic equation

$$
\begin{aligned}
&(x(t)-\frac{1}{\sqrt{q}}x(\rho(t)))^{\Delta^4}+2\frac{(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}t^3(t+q^2)^2}x^2(\rho^2(t))\\
&-\frac{(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}t^3(t+q^2)^3}x^2(\rho^3(t))=0,
\end{aligned}
\tag{3.2}
$$

where ρ is the backward operator, $\rho^2(t)=\rho(\rho(t)),\rho^3(t)=\rho(\rho^2(t))$. In this equation, $n=4$, $p(t)=-\frac{1}{\sqrt{q}}$, $\tau(t)=\rho(t)=\frac{t}{q}$, $\tau_1(t)=\rho^2(t)$, $\tau_2(t)=\rho^3(t)$,

$$
\begin{aligned}
f_1(t,b)&=2\frac{(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}t^3(t+q^2)^2}b^2,\\
f_2(b)&=\frac{(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}t^3(t+q^2)^3}b^2.
\end{aligned}
$$

By the definition of $g_k(s,t)$,

$$\begin{aligned} g_{4-1}(\sigma(s),0)\cdot f_1(s,b) &\le s^3 2\frac{(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}s^3(s+q^2)^2}b^2\\ &\le 2\frac{(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}s^2}b^2, \end{aligned}$$

$$\begin{aligned} g_{4-1}(\sigma(s),0)\cdot f_2(s,b) &\le s^3\frac{(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}s^3(s+q^2)^3}b^2\\ &\le \frac{(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}s^3}b^2, \end{aligned}$$

and

$$\begin{gathered} \int_{t_0}^{\infty}\frac{(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}s^2}b^2\Delta s<\infty,\\ \int_{t_0}^{\infty}\frac{(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}s^3}b^2\Delta s<\infty. \end{gathered}$$

It is obviously that (3.2) satisfies all conditions of Theorem 3. Hence (3.2) has a bounded non-oscillatory solution which is bounded away from zero. In fact $x(t) = 1+\frac{1}{t}$ is a solution of (3.2).

Acknowledgements. This research was supported by grants L2009Z02 from the Main Foundation of Hebei Normal University, and L2006B01 from the Doctoral Foundation of Hebei Normal University. The authors would like to thank the anonymous referee for his or her careful reading and the comments on improving the presentation of this article.

References

[1] M. Bohner, G. Guseinov; *Line integrals and Green's formula on time scales,* J. Math. Anal. Appl., 326 (2007), 1124-1141.
[2] M. Bohner, A. Peterson; *Dynamic equations on time scales: An introduction with applications,* Birkhäuser, Boston, Massachusetts, 2001.
[3] M. Bohner, A. Peterson; *Advances in dynamic equations on time scales,* Birkhäuser, Boston, Massachusetts, 2003.
[4] L. Erbe, Q. Kong and B. Zhang; *Oscillation theory for functional differential equations,* New York: Marcel Dekker, 1995.
[5] L. Erbe, A. Peterson; *Positive solutions for a nonlinear differential equation on a measure chain,* Math. Comput. Modell., 32 (5/6) (2000), 571-585.
[6] L. Erbe, A. Peterson and S. Saker; *Oscillation criteria for second order nonlinear dynamic equations on time scales,* J. London. Math., 3 (2003), 701-714.
[7] S. Hilger; *Ein Masskettenkalkül mit Anwendung auf Zentrumsmanningfaltigkeiten,* PhD thesis, Universität Würzburg, 1988.
[8] M. Huang, W. Feng; *Oscillation of second-order nonlinear impulsive dynamic equations on time scales,* Electronic J. Differ. Equa. 2007 (2007), No. 72, 1-13.
[9] W. Li, H. Sun; *Multiple positive solutions for nonlinear dynamic systems on a measure chain,* J. Comput. Appl. Math., 162 (2004), 421-430.
[10] H. Sun; *Existence of positive solutions to second-order time scale systems,* Comput. Math. Appl., 49 (2005), 131-145.
[11] Z. Zhang, W. Dong, Q. Li and H. Liang; *Positive solutions for higher order nonlinear neutral dynamic equations on time scales,* Applied Mathematical Modelling, 33 (2009), 2455-2463.
[12] B. Zhang, Z. Liang; *Oscillation of second-order nonlinear delay dynamic equations on time scales,* Comput. Math. Appl., 49 (2005), 599-609.

Qiaoluan Li
College of Mathematics and Information Science, Hebei Normal University, Shijiazhuang, 050016, China
E-mail address: ql171125@163.com

Zhenguo Zhang
College of Mathematics and Information Science, Hebei Normal University, Shijiazhuang, 050016, China.
Information College, Zhejiang Ocean University, Zhoushan, 316000, China
E-mail address: zhangzhg@mail.hebtu.edu.cn