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GROWTH OF SOLUTIONS TO HIGHER ORDER LINEAR HOMOGENEOUS DIFFERENTIAL EQUATIONS IN ANGULAR DOMAINS

NAN WU

Abstract. In this article, we discuss the growth of meromorphic solutions to higher order homogeneous differential equations in some angular domains, instead of the whole complex plane.

1. Introduction and statement of main results

By a transcendental meromorphic function, we mean a function that is meromorphic on the whole complex plane, and is not a rational function; in other words, ∞ is an essential singular point. We assume the reader is familiar with the Nevanlinna theory of meromorphic functions and basic notation such as: Nevanlinna characteristic $T(r,f)$, integrated counting function $N(r,f)$, and proximity function $m(r,f)$, and the deficiency $\delta(a,f)$ of $f(z)$. For the details, see [4, 7]. The order λ and the lower order μ are defined as follows:

$$\lambda(f)=\limsup_{r\to\infty}\frac{\log T(r,f)}{\log r},\quad \mu(f)=\liminf_{r\to\infty}\frac{\log T(r,f)}{\log r}.$$

It is known the growth of meromorphic solutions of differential equations with meromorphic coefficients in the complex plane $\mathbb{C}$ attracted a lot research. In this article, we discuss the growth of meromorphic solutions of differential equations with transcendental meromorphic coefficients in a proper subset of $\mathbb{C}$. Let $f(z)$ be a meromorphic function in an angular region $\overline{\Omega}(\alpha,\beta)=\{z:\alpha\le\arg z\le\beta\}$. Recall the definition of Ahlfors-Shimizu characteristic in an angle (see [6]). Set

$$\Omega(r)=\Omega(\alpha,\beta)\cap\{z:1<|z|<r\}=\{z:\alpha<\arg z<\beta,1<|z|<r\}.$$

Define

$$\mathcal{S}(r,\Omega,f)=\frac{1}{\pi}\iint_{\Omega(r)}\Big(\frac{|f'(z)|}{1+|f(z)|^2}\Big)^2d\sigma,\quad \mathcal{T}(r,\Omega,f)=\int_1^r\frac{\mathcal{S}(t,\Omega,f)}{t}dt.$$

The order and lower order of f on Ω are defined as follows

$$\sigma_{\alpha,\beta}(f)=\limsup_{r\to\infty}\frac{\log\mathcal{T}(r,\Omega,f)}{\log r},\quad \mu_{\alpha,\beta}(f)=\liminf_{r\to\infty}\frac{\log\mathcal{T}(r,\Omega,f)}{\log r}.$$

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Remark 1.1. The order $\sigma_{\alpha,\beta}(f)$ of a meormorphic function f on an angular region here we give is reasonable, because $\mathcal{T}(r,\mathbb{C},f)=T(r,f)+O(1)$.

Nevanlinna theory on the angular domain plays an important role in value distribution of meromrorphic functions. Let us recall the following terms [3]:

$$A_{\alpha,\beta}(r,f)=\frac{\omega}{\pi}\int_1^r\Big(\frac{1}{t^\omega}-\frac{t^\omega}{r^{2\omega}}\Big)\{\log^+|f(te^{i\alpha})|+\log^+|f(te^{i\beta})|\}\frac{dt}{t},$$
$$B_{\alpha,\beta}(r,f)=\frac{2\omega}{\pi r^\omega}\int_\alpha^\beta\log^+|f(re^{i\theta})|\sin\omega(\theta-\alpha)d\theta,$$
$$C_{\alpha,\beta}(r,f)=2\sum_{1<|b_n|<r}\Big(\frac{1}{|b_n|^\omega}-\frac{|b_n|^\omega}{r^{2\omega}}\Big)\sin\omega(\theta_n-\alpha),$$

where $\omega=\pi/(\beta-\alpha)$, and $b_n=|b_n|e^{i\theta_n}$ is a pole of $f(z)$ in the angular domain $\Omega(\alpha,\beta)$, appears according to its multiplicity. The Nevanlinna's angular characteristic is defined as follows:

$$S_{\alpha,\beta}(r,f)=A_{\alpha,\beta}(r,f)+B_{\alpha,\beta}(r,f)+C_{\alpha,\beta}(r,f).$$

Some articles define the order and lower order of f on Ω as:

$$\overline{\sigma}_{\alpha,\beta}(f)=\limsup_{r\to\infty}\frac{\log S_{\alpha,\beta}(r,f)}{\log r},\quad \overline{\mu}_{\alpha,\beta}(f)=\liminf_{r\to\infty}\frac{\log S_{\alpha,\beta}(r,f)}{\log r}.$$

According to the inequality

$$S_{\alpha,\beta}(r,f)\le 2\omega^2\frac{\mathcal{T}(r,\Omega,f)}{r^\omega}+\omega^3\int_1^r\frac{\mathcal{T}(t,\Omega,f)}{t^{\omega+1}}dt+O(1),$$

showed by Zheng [13, Theorem 2.4.7], if $\sigma_{\alpha,\beta}(r,f)<\infty$, then $\overline{\sigma}_{\alpha,\beta}(r,f)<\infty$.

We consider q pairs of real numbers $\{\alpha_j,\beta_j\}$ such that

$$-\pi\le\alpha_1<\beta_1\le\alpha_2<\beta_2\le\dots\alpha_q<\beta_q\le\pi \tag{1.1}$$

and the angular domains $X=\cup_{j=1}^q\{z:\alpha_j\le\arg z\le\beta_j\}$. For a function f meromorphic in the complex plane $\mathbb{C}$, we define the order of f on X as

$$\sigma_X(f)=\limsup_{r\to\infty}\frac{\log\mathcal{T}(r,X,f)}{\log r}.$$

It is obvious that $\sigma_{\alpha_j,\beta_j}(f)\le\sigma_X(f)\le\sum_{j=1}^q\sigma_{\alpha_j,\beta_j}(f)$. $j=1,2,\dots,q$. And $\sigma_X(f)=+\infty$ if and only if there exists at least one $1\le j_0\le q$ such that $\sigma_{\alpha_{j_0},\beta_{j_0}}(f)=+\infty$. We will establish the following results.

Theorem 1.2. *Let $A_0(z)$ be a meromorphic function in $\mathbb{C}$ with finite lower order $\mu<\infty$ and nonzero order $0<\lambda\le\infty$ and $\delta=\delta(\infty,A_0)>0$. For q pair of real numbers $\{\alpha_j,\beta_j\}$ satisfying* (1.1) *and*

$$\sum_{j=1}^q(\alpha_{j+1}-\beta_j)<\frac{4}{\sigma}\arcsin\sqrt{\delta/2} \tag{1.2}$$

where $\sigma>0$ with $\mu\le\sigma\le\lambda$. If $A_j(z)(j=1,2,\dots,n)$ are meromorphic functions in $\mathbb{C}$ with $T(r,A_j)=o(T(r,A_0))$, then every solution $f\not\equiv 0$ to the equation

$$A_nf^{(n)}+A_{n-1}f^{(n-1)}+\dots+A_0f=0$$

has the order $\sigma_X(f)=+\infty$ in $X=\cup_{j=1}^q\{z:\alpha_j\le\arg z\le\beta_j\}$.

If we remove the condition $\mu(A_0) < \infty$ in Theorem 1.2, we can establish the following result.

Theorem 1.3. *Let $A_0(z)$ be a meromorphic function in $\mathbb{C}$ with nonzero order $0 < \lambda \leq \infty$ and $\delta(\infty, A_0) > 0$. Suppose that for q directions $\arg z = \alpha_j (1 \leq j \leq q)$, satisfying*

$$-\pi \leq \alpha_1 < \alpha_2 < \cdots < \alpha_q < \pi, \alpha_{q+1} = \alpha_1 + 2\pi,$$

$A_j(z)$, $j = 1, 2, \ldots, n$, are meromorphic functions in $\mathbb{C}$ with finite lower order and $T(r, A_j) = o(T(r, A_0))$. Then every solution $f \not\equiv 0$ to the equation

$$A_n f^{(n)} + A_{n-1} f^{(n-1)} + \cdots + A_0 f = 0$$

has order $\sigma_X(f) = +\infty$ in $X = \mathbb{C} \setminus \cup_{j=1}^{q} \{z : \arg z = \alpha_j\}$.

The method in this paper was firstly used by Zheng [14] to investigate the growth of transcendental meromorphic functions with radially distributed values.

2. Some auxiliary results

To prove the theorems, we give some lemmas. The following result is from [12, 13, 14].

Lemma 2.1. *Let $f(z)$ be a transcendental meromorphic function with lower order $\mu < \infty$ and order $0 < \lambda \leq \infty$, then for any positive number $\mu \leq \sigma \leq \lambda$ and any set E with finite measure, there exist a sequence $\{r_n\}$, such that*

(1) $r_n \notin E$, $\lim_{n\to\infty} \frac{r_n}{n} = \infty$;
(2) $\liminf_{n\to\infty} \frac{\log T(r_n, f)}{\log r_n} \geq \sigma$;
(3) $T(t, f) < (1 + o(1))(\frac{2t}{r_n})^{\sigma} T(r_n/2, f)$, $t \in [r_n/n, nr_n]$;
(4) $T(t, f)/t^{\sigma-\varepsilon_n} \leq 2^{\sigma+1} T(r_n, f)/r_n^{\sigma-\varepsilon_n}$, $1 \leq t \leq nr_n$, $\varepsilon_n = [\log n]^{-2}$.

We recall that $\{r_n\}$ is called the Pólya peaks of order σ outside E. Given a positive function $\Lambda(r)$ satisfying $\lim_{r\to\infty} \Lambda(r) = 0$. For $r > 0$ and $a \in \mathbb{C}$, define

$$D_\Lambda(r, a) = \{\theta \in [-\pi, \pi) : \log^+ \frac{1}{|f(re^{i\theta}) - a|} > \Lambda(r) T(r, f)\},$$

and

$$D_\Lambda(r, \infty) = \{\theta \in [-\pi, \pi) : \log^+ |f(re^{i\theta})| > \Lambda(r) T(r, f)\}.$$

The following result is called the spread relation, which was conjectured by Edrei [2] and proved by Baernstein [1].

Lemma 2.2. *Let $f(z)$ be transcendental and meromorphic in $\mathbb{C}$ with the finite lower order $\mu < \infty$ and the positive order $0 < \lambda \leq \infty$ and has one deficient values $a \in \widehat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$. Then for any sequence of Pólya peaks $\{r_n\}$ of order $\sigma > 0, \mu \leq \sigma \leq \lambda$ and any positive function $\Lambda(r) \to 0$ as $r \to +\infty$, we have*

$$\liminf_{n\to\infty} \operatorname{meas} D_\Lambda(r_n, a) \geq \min\{2\pi, \frac{4}{\sigma} \arcsin \sqrt{\delta(a, f)/2}\}.$$

To make it clearly, we give the definition of $\mathbb{R}$-set on the complex plane $\mathbb{C}$.

Definition 2.3. Let $B(z_n, r_n) = \{z : |z - z_n| < r_n\}$ be an open disk on the complex plane. If $\sum_{n=1}^{\infty} r_n < \infty$, $\cup_{n=1}^{\infty} B(z_n, r_n)$ is called an $\mathbb{R}$-set.

Lemma 2.4 ([8]). *Let f be a meromorphic function on the angular region $\overline{\Omega}(\alpha,\beta)$ with finite order ρ, let $\Gamma = \{(n_1,m_1),(n_2,m_2),\dots,(n_j,m_j)\}$ denote a finite set of distinct pair of integers which satisfying $n_i > m_i \geq 0$ for $i = 1,2,\dots,j$, and let $\varepsilon > 0$ and $\delta > 0$ be given constants. Then there exists $K > 0$ depending only on f,ε,δ such that*

$$\Big|\frac{f^{(n)}(z)}{f^{(m)}(z)}\Big| < K|z|^{(n-m)(k_\delta+2\rho+1+\varepsilon)}(\sin k_\delta(\varphi-\alpha-\delta))^{-2^{n-m}}, \tag{2.1}$$

for all $(n,m) \in \Gamma$ and all $z = re^{i\varphi} \in \Omega(\alpha+\delta,\beta-\delta)$ except for a $\mathbb{R}$-set, that is, a countable union of discs whose radii have finite sum, where $k_\delta = \frac{\pi}{\beta-\alpha-2\delta}$.

To prove Theorem 1.3, we need a result from Edrei [2].

Lemma 2.5. *Let $f(z)$ be a meromorphic function with $\delta = \delta(\infty,f) > 0$. Then given $\varepsilon > 0$, we have*

$$\operatorname{meas} E(r,f) > \frac{1}{T^\varepsilon(r,f)[\log r]^{1+\varepsilon}}, r \notin F,$$

where

$$E(r,f) = \{\theta \in [-\pi,\pi) : \log^+|f(re^{i\theta})| > \frac{\delta}{4}T(r,f)\}$$

and F is a set of positive real numbers with finite logarithmic measure depending on ε.

3. Proof of the Theorems

Proof of Theorem 1.2. We suppose that there exists a nontrival meromorphic solution f such that $\sigma_{\alpha_j,\beta_j}(f) < +\infty$, $j = 1,2,\dots,q$. In view of Lemma 2.4, there exists a constant $M > 0$ not depending on z such that

$$\Big|\frac{f^{(j)}(z)}{f(z)}\Big| < |z|^M, \quad j = 1,2,\dots,n\,.$$

for all $z \in \Omega(\alpha_j+\varepsilon,\beta_j-\varepsilon)$, $j = 1,2,\dots,q$, except for a $\mathbb{R}$-set E. For E, we can define a set $F = \{r > 0 | \exists z \in E, s.t. |z| = r\}$ thus

$$\operatorname{meas} F < \infty.$$

(I) $\lambda(A_0) > \mu(A_0)$. Then $\lambda(A_0) > \sigma \geq \mu(A_0)$. By the inequality (1.2), we can take a real number $\varepsilon > 0$ such that

$$\sum_{j=1}^{q}(\alpha_{j+1}-\beta_j+2\varepsilon)+2\varepsilon < \frac{4}{\sigma+2\varepsilon}\arcsin\sqrt{\delta/2}, \tag{3.1}$$

where $\alpha_{q+1} = 2\pi+\alpha_1$, and

$$\lambda(A_0) > \sigma+2\varepsilon > \mu(A_0).$$

Applying Lemma 2.1 to $A(z)$ gives the existence of the Pólya peak $\{r_n\}$ of order $\sigma+2\varepsilon$ of $A(z)$ such that $r_n \notin F$, and then from Lemma 2.2 for sufficiently large n we have

$$\operatorname{meas} D(r_n,\infty) > \frac{4}{\sigma+2\varepsilon}\arcsin\sqrt{\delta/2}-\varepsilon. \tag{3.2}$$

We can assume for all the n, above holds. Set

$$K := \operatorname{meas}(D(r_n,\infty)\cap\cup_{j=1}^{q}(\alpha_j+\varepsilon,\beta_j-\varepsilon)).$$

Then from (3.1) and (3.2) it follows that

$$\begin{aligned} K &\geq \operatorname{meas}(D(r_n,\infty)) - \operatorname{meas}([0,2\pi)\setminus \cup_{j=1}^{q} (\alpha_j+\varepsilon, \beta_j-\varepsilon)) \\ &= \operatorname{meas}(D(r_n,\infty)) - \operatorname{meas}(\cup_{j=1}^{q}(\beta_j-\varepsilon, \alpha_{j+1}+\varepsilon)) \\ &= \operatorname{meas}(D(r_n,\infty)) - \sum_{j=1}^{q}(\alpha_{j+1}-\beta_j+2\varepsilon) > \varepsilon > 0. \end{aligned}$$

It is easy to see that there exists a j_0 such that for infinitely many n, we have

$$\operatorname{meas}(D(r_n,\infty)\cap(\alpha_{j_0}+\varepsilon, \beta_{j_0}-\varepsilon)) > \frac{K}{q}. \tag{3.3}$$

We can assume for all the n, (3.3) holds. We define a real function by

$$\Lambda(r)^2 = \max\Big\{\frac{T(r_n,A_j)}{T(r_n,A_0)}, \frac{\log r_n}{T(r_n,A_0)}; j=1,2,\dots,n\Big\},$$

for $r_n \leq r < r_{n+1}$. Obviously $\lim_{r\to\infty}\Lambda(r)=0$ and

$$\operatorname{meas} D'_\Lambda(r_n) = \operatorname{meas}\{\theta : r_n e^{i\theta}\in E\} = 0.$$

Set

$$D_n = (\alpha_{j_0}+\varepsilon, \beta_{j_0}-\varepsilon)\setminus D'_\Lambda(r_n), \quad E_n = D(r_n,\infty)\cap(\alpha_{j_0}+\varepsilon, \beta_{j_0}-\varepsilon).$$

Thus from the definition of $D(r,\infty)$ it follows that

$$\begin{aligned} \int_{\alpha_{j_0}+\varepsilon}^{\beta_{j_0}-\varepsilon} \log^+|A_0(r_ne^{i\theta})|d\theta &\geq \int_{E_n} \log^+|A_0(r_ne^{i\theta})|d\theta \\ &\geq \operatorname{meas}(E_n)\Lambda(r_n)T(r_n,A_0) \\ &> \frac{K}{q}\Lambda(r_n)T(r_n,A_0). \end{aligned} \tag{3.4}$$

Thus, we have

$$\begin{aligned} &\int_{\alpha_{j_0}+\varepsilon}^{\beta_{j_0}-\varepsilon} \log^+|A_0(r_ne^{i\theta})|d\theta \\ &\leq \int_{\alpha_{j_0}+\varepsilon}^{\beta_{j_0}-\varepsilon} \sum_{j=1}^{n}\Big(\log^+\Big|\frac{f^{(j)}(r_ne^{i\theta})}{f(r_ne^{i\theta})}\Big| + \log^+|A_j(r_ne^{i\theta})|\Big)d\theta \\ &= \Big(\int_{D'_\Lambda(r_n)} + \int_{D_n}\Big)\sum_{j=1}^{n}\Big(\log^+\Big|\frac{f^{(j)}(r_ne^{i\theta})}{f(r_ne^{i\theta})}\Big| + \log^+|A_j(r_ne^{i\theta})|\Big)d\theta \\ &\leq \int_{\alpha_{j_0}+\varepsilon}^{\beta_{j_0}-\varepsilon} \sum_{j=1}^{n}\log^+|A_j(r_ne^{i\theta})|d\theta + O(\log r_n) \\ &\leq \sum_{j=1}^{n} T(r_n,A_j) + O(\log r_n) \\ &\leq \Lambda^2(r_n)T(r_n,A_0). \end{aligned} \tag{3.5}$$

Therefore,

$$\frac{K}{q}\Lambda(r_n) < \Lambda^2(r_n).$$

This contradicts that $\Lambda(r)\to 0$.

(II) $\lambda(A_0) = \mu(A_0)$. Then $\lambda(A_0) = \sigma = \mu(A_0)$. By the same argument as in (I) with all the $\sigma + 2\varepsilon$ replaced by σ, we can derive a contradiction. The proof is complete. $\square$

Proof of Theorem 1.3. Applying Lemma 2.1 to $A_0(z)$ confirms the existence of a sequence $\{r_n\}$ of positive numbers such that $r_n \notin E$ and

$$\operatorname{meas} E(r_n, A_0) > \frac{1}{T^{\varepsilon}(r_n, A_0)[\log r_n]^{1+\varepsilon}}, \tag{3.6}$$

where $E(r_n, A_0)$ is defined as in Lemma 2.5. Set

$$\varepsilon_n = \frac{1}{2q+1} \frac{1}{T^{\varepsilon}(r_n, A_0)[\log r_n]^{1+\varepsilon}}.$$

Then for (3.6) it follows that

$$\begin{aligned}
&\operatorname{meas}(E(r_n, A_0) \cap \cup_{j=1}^{q}(\alpha_j + \varepsilon_n, \alpha_{j+1} - \varepsilon_n))\\
&\geq \operatorname{meas} E(r_n, A_0) - \operatorname{meas}(\cup_{j=1}^{q}(\alpha_j + \varepsilon_n, \alpha_{j+1} - \varepsilon_n))\\
&\geq (2q+1)\varepsilon_n - 2q\varepsilon_n = \varepsilon_n > 0.
\end{aligned}$$

so that there exists a j such that for infinitely many n, we have

$$\operatorname{meas} E_n > \frac{\varepsilon_n}{q}, \tag{3.7}$$

where $E_n = E(r_n, A_0) \cap (\alpha_j + \varepsilon_n, \alpha_{j+1} - \varepsilon_n)$. We can assume that (3.7) holds for all the n. Thus

$$\begin{aligned}
\int_{\alpha_j+\varepsilon_n}^{\alpha_{j+1}-\varepsilon_n} \log^+ |A_0(r_n e^{i\theta})| d\theta &\geq \int_{E_n} \log^+ |A_0(r_n e^{i\theta})| d\theta\\
&\geq \operatorname{meas}(E_n) \frac{\delta}{4} T(r_n, A_0)\\
&\geq \frac{\delta\varepsilon_n}{4q} T(r_n, A_0).
\end{aligned} \tag{3.8}$$

On the other hand,

$$\int_{\alpha_j+\varepsilon_n}^{\alpha_{j+1}-\varepsilon_n} \log^+ |A_0(r_n e^{i\theta})| d\theta < \sum_{j=1}^{n} T(r_n, A_j) + O(\log r_n) \tag{3.9}$$

Combining (3.8) and (3.9) gives

$$\varepsilon_n T(r_n, A_0) \leq \frac{4q}{\delta} \sum_{j=1}^{n} T(r_n, A_j) + O(\log r_n),$$

so that

$$T^{1-\varepsilon}(r_n, A_0) \leq \frac{4q(2q+1)}{\delta} [\log r_n]^{1+\varepsilon} \sum_{j=1}^{n} T(r_n, A_j) + O(\log^{2+\varepsilon} r_n),$$

we have $\mu(A_0) \leq \max_{1\leq j\leq q}(\mu(A_j))/(1-\varepsilon)$. By the same method as in Theorem 1.2, we obtain a contradiction, which completes the proof. $\square$

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Nan Wu
Department of Mathematical Sciences, Tsinghua University, Beijing, 100084, China
E-mail address: wunan07@gmail.com