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COMPARISON THEOREMS FOR SECOND-ORDER NEUTRAL DIFFERENTIAL EQUATIONS OF MIXED TYPE

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Abstract. Three comparison theorems are established for the oscillation of the second-order neutral differential equations of mixed type

$$\big(r(t)[x(t)+p_1(t)x(t-\sigma_1)+p_2(t)x(t+\sigma_2)]'\big)'+q_1(t)x(t-\sigma_3)+q_2(t)x(t+\sigma_4)=0.$$

Our results are new even when $p_2(t)=q_2(t)=0$. An example is provided to illustrate the main results.

1. Introduction

This article concerns the oscillatory behavior of the second-order linear neutral differential equation of mixed type

$$\big(r(t)[x(t)+p_1(t)x(t-\sigma_1)+p_2(t)x(t+\sigma_2)]'\big)'+q_1(t)x(t-\sigma_3)+q_2(t)x(t+\sigma_4)=0, \tag{1.1}$$

for $t \ge t_0$.

We will use the following conditions:

(H1) $r \in C^1([t_0,\infty),\mathbb{R})$, $r(t)>0$ for $t \ge t_0$;
(H2) $p_i \in C([t_0,\infty),[0,a_i])$, where a_i are constants for $i=1,\,2$;
(H3) $q_j \in C([t_0,\infty),[0,\infty))$, and q_j are not eventually zero on any half line $[t_*,\infty)$ for $t_* \ge t_0$, $j=1,\,2$;
(H4) $\sigma_i \ge 0$ are constants, for $i=1,2,3,4$.

We put $z(t)=x(t)+p_1(t)x(t-\sigma_1)+p_2(t)x(t+\sigma_2)$. By a solution of (1.1), we mean a function $x \in C([T_x,\infty),\mathbb{R})$ for some $T_x \ge t_0$ which has the properties that $z \in C^1([T_x,\infty),\mathbb{R})$ and $rz' \in C^1([T_x,\infty),\mathbb{R})$ and satisfying (1.1) on $[T_x,\infty)$. We consider only those solutions x of (1.1) which satisfy $\sup\{|x(t)| : t \ge T\}>0$ for all $T \ge T_x$. We assume that (1.1) possesses such a solution. As is customary, a solution of (1.1) is called oscillatory if it has arbitrarily large zeros on $[t_0,\infty)$; otherwise, it is called non-oscillatory. Equation (1.1) is said to be oscillatory if all its solutions are oscillatory.

Recently, there has been much research activity concerning the oscillation and non-oscillation of solutions of varietal types of differential equations. We refer the reader to [2, 3, 4, 6, 7, 11, 17, 18, 21, 25, 26] and the references cited therein.

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Džurina [7] presented sufficient conditions for the oscillation of the second-order differential equation with mixed argument

$$\Big(\frac{1}{r(t)}u'(t)\Big)' + p(t)u(\tau(t)) + q(t)u(\sigma(t)) = 0, \quad t \geq t_0.$$

Some oscillation results for the second-order neutral differential equation

$$(r(t)|z'(t)|^{\gamma-1}z'(t))' + q(t)|x(\sigma(t))|^{\gamma-1}x(\sigma(t)) = 0,$$

where $z(t) = x(t) + p(t)x(\tau(t))$ and $t \geq t_0$ were obtained by [8, 10, 16, 19].

Regarding the oscillatory behavior of neutral differential equations with mixed arguments; see e.g., the papers [1, 9, 12, 13, 14, 23, 24]. Agarwal and Grace [1] studied the oscillation of the even-order equation

$$(x(t) + ax(t-\tau) - bx(t+\tau))^{(n)} + q(t)x(t-g) + p(t)x(t+h) = 0.$$

Džurina et al. [9] established some oscillation criteria for the mixed neutral equation

$$(x(t) + p_1x(t-\tau_1) + p_2x(t+\tau_2))'' = q_1(t)x(t-\sigma_1) + q_2(t)x(t+\sigma_2).$$

Grace and Lalli [12] examined the oscillatory behavior for the second-order equation

$$(x(t) + \lambda x(t-\tau))'' = q(t)x(t-\sigma) + p(t)x(t+\beta).$$

Grace [13] obtained some oscillation theorems for the odd-order neutral differential equation

$$(x(t) + p_1x(t-\tau_1) + p_2x(t+\tau_2))^{(n)} = q_1x(t-\sigma_1) + q_2x(t+\sigma_2).$$

Grace [14] and Yan [23] established several sufficient conditions for the oscillation of solutions of odd-order neutral functional differential equation

$$(x(t) + cx(t-h) + Cx(t+H))^{(n)} + qx(t-g) + Qx(t+G) = 0.$$

Yan [24] considered the oscillation of even-order mixed neutral differential equation

$$(x(t) - c_1x(t-h_1) - c_2x(t+h_2))^{(n)} + qx(t-g_1) + px(t+g_2) = 0.$$

To the best of our knowledge, there are only few results on the oscillation of (1.1). It is interesting to study (1.1) since it has some applications in the study of vibrating masses attached to an elastic bar (see [15]). The aim of this paper is to establish some oscillation results for (1.1). The organization of this paper is as follows: In Section 2, we reduce the problem of the oscillation of (1.1) to the oscillation of the first-order inequalities under the case when

$$\int_{t_0}^{\infty} \frac{1}{r(t)}\,\mathrm{d}t = \infty. \tag{1.2}$$

In Section 3, we give an example and a remark to illustrate our results.

Below, when we write a functional inequality without specifying its domain of validity we assume that it holds for all sufficiently large t.

2. Main results

In the following, we will establish some oscillation criteria for (1.1).

Throughout this paper, we denote

$$\begin{gathered} Q(t)=Q_1(t)+Q_2(t),\\ Q_1(t)=\min\{q_1(t),q_1(t-\sigma_1),q_1(t+\sigma_2)\},\\ Q_2(t)=\min\{q_2(t),q_2(t-\sigma_1),q_2(t+\sigma_2)\}. \end{gathered}$$

Theorem 2.1. *Assume that* (1.2) *holds. Further, assume that*

$$[y(t)+a_1y(t-\sigma_1)+a_2y(t+\sigma_2)]'+Q(t)\Big(\int_{t_1}^{t-\sigma_3}\frac{1}{r(s)}\,\mathrm{d}s\Big)y(t-\sigma_3)\le 0 \tag{2.1}$$

has no eventually positive solution for all sufficiently large t_1, $t_1\ge t_0$. Then (1.1) *is oscillatory.*

Proof. Let x be a non-oscillatory solution of (1.1). Without loss of generality, we assume that there exists $t_1\ge t_0$ such that $x(t)>0$, $x(t-\sigma_1)>0$, $x(t+\sigma_2)>0$, $x(t-\sigma_3)>0$ and $x(t+\sigma_4)>0$ for all $t\ge t_1$. Then $z(t)>0$ for $t\ge t_1$. In view of (1.1), we obtain

$$(r(t)z'(t))'=-q_1(t)x(t-\sigma_3)-q_2(t)x(t+\sigma_4)\le 0,\quad t\ge t_1. \tag{2.2}$$

Thus, $r(t)z'(t)$ is non-increasing function. Consequently, it is easy to conclude that there exist two possible cases of the sign of $z'(t)$, that is, $z'(t)>0$ or $z'(t)<0$ eventually. If there exists $t_2\ge t_1$ such that $z'(t_2)<0$, then from (2.2), we see that

$$r(t)z'(t)\le r(t_2)z'(t_2)<0,\quad t\ge t_2.$$

Integrating the above inequality from t_2 to t, we obtain

$$z(t)\le z(t_2)+r(t_2)z'(t_2)\int_{t_2}^{t}\frac{1}{r(s)}\,\mathrm{d}s.$$

Letting $t\to\infty$, we obtain $\lim_{t\to\infty}z(t)=-\infty$ due to (1.2), which is a contradiction. Thus, there exists a $t_2\ge t_1$ such that

$$z'(t)>0 \tag{2.3}$$

for $t\ge t_2$. Using (1.1), for all sufficiently large t, we have

$$\begin{aligned} &(r(t)z'(t))'+q_1(t)x(t-\sigma_3)+q_2(t)x(t+\sigma_4)+a_1(r(t-\sigma_1)z'(t-\sigma_1))'\\ &+a_1q_1(t-\sigma_1)x(t-\sigma_1-\sigma_3)+a_1q_2(t-\sigma_1)x(t+\sigma_4-\sigma_1)\\ &+a_2(r(t+\sigma_2)z'(t+\sigma_2))'+a_2q_1(t+\sigma_2)x(t+\sigma_2-\sigma_3)\\ &+a_2q_2(t+\sigma_2)x(t+\sigma_2+\sigma_4)=0. \end{aligned}$$

Thus

$$\begin{aligned} &(r(t)z'(t))'+a_1(r(t-\sigma_1)z'(t-\sigma_1))'+a_2(r(t+\sigma_2)z'(t+\sigma_2))'\\ &+Q_1(t)z(t-\sigma_3)+Q_2(t)z(t+\sigma_4)\le 0. \end{aligned} \tag{2.4}$$

By (2.3), we have $z(t+\sigma_4)\ge z(t-\sigma_3)$. Then, from (2.4), we obtain

$$(r(t)z'(t))'+a_1(r(t-\sigma_1)z'(t-\sigma_1))'+a_2(r(t+\sigma_2)z'(t+\sigma_2))'+Q(t)z(t-\sigma_3)\le 0. \tag{2.5}$$

It follows from (2.2) that

$$z(t)=z(t_2)+\int_{t_2}^{t}\frac{r(s)z'(s)}{r(s)}\,\mathrm{d}s\ge r(t)z'(t)\int_{t_2}^{t}\frac{1}{r(s)}\,\mathrm{d}s. \tag{2.6}$$

Set $y(t) = r(t)z'(t) > 0$. From (2.5) and (2.6), we see that y is an eventually positive solution of

$$[y(t) + a_1 y(t-\sigma_1) + a_2 y(t+\sigma_2)]' + Q(t)y(t-\sigma_3)\int_{t_2}^{t-\sigma_3} \frac{1}{r(s)}\,\mathrm{d}s \le 0.$$

This completes the proof. □

Theorem 2.2. *Assume that* (1.2) *holds and*

$$u'(t) + Q(t)\frac{\int_{t_1}^{t-\sigma_3}\frac{1}{r(s)}\,\mathrm{d}s}{1+a_1+a_2}u(t+\sigma_1-\sigma_3) \le 0 \tag{2.7}$$

has no eventually positive solution for all sufficiently large t_1, $t_1 \ge t_0$. *Then* (1.1) *is oscillatory.*

Proof. Let x be a non-oscillatory solution of (1.1). Without loss of generality, we assume that there exists $t_1 \ge t_0$ such that $x(t) > 0$, $x(t-\sigma_1) > 0$, $x(t+\sigma_2) > 0$, $x(t-\sigma_3) > 0$ and $x(t+\sigma_4) > 0$ for all $t \ge t_1$. Then $z(t) > 0$ for $t \ge t_1$. Proceeding as in the proof of Theorem 2.1, we obtain that $y(t) = r(t)z'(t) > 0$ is non-increasing and satisfies inequality (2.1). Define

$$u(t) = y(t) + a_1 y(t-\sigma_1) + a_2 y(t+\sigma_2) > 0.$$

Then

$$u(t) \le (1+a_1+a_2)y(t-\sigma_1).$$

Substituting the above formulas into (2.1), we find u is an eventually positive solution of

$$u'(t) + Q(t)\frac{\int_{t_1}^{t-\sigma_3}\frac{1}{r(s)}\,\mathrm{d}s}{1+a_1+a_2}u(t+\sigma_1-\sigma_3) \le 0.$$

The proof is complete. □

From Theorem 2.2 and [18, Theorem 2.1.1], we establish the following corollary.

Corollary 2.3. *Assume that* (1.2) *holds,* $\sigma_1 - \sigma_3 < 0$ *and*

$$\liminf_{t\to\infty}\int_{t+\sigma_1-\sigma_3}^{t} Q(u)\Big(\int_{t_1}^{u-\sigma_3}\frac{1}{r(s)}\,\mathrm{d}s\Big)\,\mathrm{d}u > \frac{1+a_1+a_2}{\mathrm{e}} \tag{2.8}$$

for all sufficiently large t_1, $t_1 \ge t_0$. *Then* (1.1) *is oscillatory.*

Theorem 2.4. *Assume that* (1.2) *holds and*

$$w'(t) - \frac{Q(t+\sigma_1)}{1+a_1+a_2}\Big(\int_{t_1}^{t+\sigma_1}\frac{\mathrm{d}u}{r(u-\sigma_1)}\Big)w(t+\sigma_1-\sigma_3) \ge 0 \tag{2.9}$$

has no eventually positive solution for all sufficiently large t_1, $t_1 \ge t_0$. *Then* (1.1) *is oscillatory.*

Proof. Let x be a non-oscillatory solution of (1.1). Without loss of generality, we assume that there exists $t_1 \ge t_0$ such that $x(t) > 0$, $x(t-\sigma_1) > 0$, $x(t+\sigma_2) > 0$, $x(t-\sigma_3) > 0$ and $x(t+\sigma_4) > 0$ for all $t \ge t_1$. Then $z(t) > 0$ for $t \ge t_1$. Proceeding as in the proof of Theorem 2.1, we obtain (2.2)–(2.5) for $t \ge t_2 \ge t_1$. Integrating (2.5) from t to ∞ yields

$$r(t)z'(t) + a_1 r(t-\sigma_1)z'(t-\sigma_1) + a_2 r(t+\sigma_2)z'(t+\sigma_2) \ge \int_t^\infty Q(s)z(s-\sigma_3)\,\mathrm{d}s. \tag{2.10}$$

Since $r(t)z'(t)$ is non-increasing, we get

$$r(t)z'(t)+a_1r(t-\sigma_1)z'(t-\sigma_1)+a_2r(t+\sigma_2)z'(t+\sigma_2) \le (1+a_1+a_2)r(t-\sigma_1)z'(t-\sigma_1). \tag{2.11}$$

In view of (2.10) and (2.11), we have

$$z'(t-\sigma_1) \ge \frac{1}{(1+a_1+a_2)r(t-\sigma_1)} \int_t^\infty Q(s)z(s-\sigma_3)\,\mathrm{d}s. \tag{2.12}$$

Integrating (2.12) from t_2 to t, we see that

$$\begin{aligned} z(t-\sigma_1) &\ge \int_{t_2}^t \frac{1}{(1+a_1+a_2)r(u-\sigma_1)} \int_u^\infty Q(s)z(s-\sigma_3)\,\mathrm{d}s\,\mathrm{d}u \\ &\ge \int_{t_2}^t \frac{1}{1+a_1+a_2} Q(s)z(s-\sigma_3) \int_{t_2}^s \frac{1}{r(u-\sigma_1)}\,\mathrm{d}u\,\mathrm{d}s. \end{aligned}$$

Thus

$$z(t) \ge \frac{1}{1+a_1+a_2} \int_{t_2}^{t+\sigma_1} Q(s)z(s-\sigma_3) \int_{t_2}^s \frac{1}{r(u-\sigma_1)}\,\mathrm{d}u\,\mathrm{d}s.$$

Let

$$w(t) = \frac{1}{1+a_1+a_2} \int_{t_2}^{t+\sigma_1} Q(s)z(s-\sigma_3) \int_{t_2}^s \frac{1}{r(u-\sigma_1)}\,\mathrm{d}u\,\mathrm{d}s > 0.$$

Then $z(t) \ge w(t)$ and

$$\begin{aligned} w'(t) &= \frac{1}{1+a_1+a_2} Q(t+\sigma_1)z(t+\sigma_1-\sigma_3) \int_{t_2}^{t+\sigma_1} \frac{1}{r(u-\sigma_1)}\,\mathrm{d}u \\ &\ge \frac{1}{1+a_1+a_2} Q(t+\sigma_1)w(t+\sigma_1-\sigma_3) \int_{t_2}^{t+\sigma_1} \frac{1}{r(u-\sigma_1)}\,\mathrm{d}u. \end{aligned}$$

Hence, we find w is an eventually positive solution of

$$w'(t) - \frac{Q(t+\sigma_1)}{1+a_1+a_2} \Big(\int_{t_2}^{t+\sigma_1} \frac{\mathrm{d}u}{r(u-\sigma_1)} \Big) w(t+\sigma_1-\sigma_3) \ge 0.$$

This completes the proof. □

Due to Theorem 2.4 and [18, Theorem 2.4.1], we obtain the following corollary.

Corollary 2.5. *Assume that* (1.2) *holds,* $\sigma_1 - \sigma_3 > 0$ *and*

$$\liminf_{t\to\infty} \int_t^{t+\sigma_1-\sigma_3} Q(u+\sigma_1) \Big(\int_{t_1}^{u+\sigma_1} \frac{1}{r(s-\sigma_1)}\,\mathrm{d}s \Big) \mathrm{d}u > \frac{1+a_1+a_2}{\mathrm{e}} \tag{2.13}$$

for all sufficiently large t_1*,* $t_1 \ge t_0$*. Then* (1.1) *is oscillatory.*

3. Example and remark

For an application of our results, we will give the following example. Consider the equation

$$[x(t)+a_1x(t-\sigma_1)+a_2x(t+\sigma_2)]'' + \frac{\alpha}{t}x(t-\sigma_3) + \frac{\beta}{t}x(t+\sigma_4) = 0, \quad t \ge t_0, \tag{3.1}$$

where a_1, a_2, α and β are positive constants.

Let $r(t)=1$, $p_1(t)=a_1$, $q_1(t)=\alpha/t$ and $q_2(t)=\beta/t$. Then $Q_1(t)=\alpha/(t+\sigma_2)$, $Q_1(t)=\beta/(t+\sigma_2)$ and $Q(t)=(\alpha+\beta)/(t+\sigma_2)$. Assume that $\sigma_3>\sigma_1$. Since

$$\liminf_{t\to\infty}\int_{t+\sigma_1-\sigma_3}^{t} Q(u)\Big(\int_{t_1}^{u-\sigma_3}\frac{1}{r(s)}\,\mathrm{d}s\Big)\,\mathrm{d}u=(\alpha+\beta)(\sigma_3-\sigma_1),$$

we conclude that (3.1) is oscillatory if

$$(\alpha+\beta)(\sigma_3-\sigma_1)>\frac{1+a_1+a_2}{\mathrm{e}}$$

due to Corollary 2.3.

Suppose that $\sigma_3<\sigma_1$. Since

$$\liminf_{t\to\infty}\int_{t}^{t+\sigma_1-\sigma_3} Q(u+\sigma_1)\Big(\int_{t_1}^{u+\sigma_1}\frac{1}{r(s-\sigma_1)}\,\mathrm{d}s\Big)\,\mathrm{d}u=(\alpha+\beta)(\sigma_1-\sigma_3),$$

we conclude that (3.1) is oscillatory if

$$(\alpha+\beta)(\sigma_1-\sigma_3)>\frac{1+a_1+a_2}{\mathrm{e}}$$

due to Corollary 2.5.

Remark 3.1. The equation

$$[x(t)+a_1x(t-\sigma_1)]''+q_1(t)x(t-\sigma_3)=0,\quad \sigma_1<\sigma_3,\ t\ge t_0 \tag{3.2}$$

is a special case of (1.1). Applying results of [25, Theorem 2] and [26, Corollary 1], we obtain a sufficient condition for (3.2) to be oscillatory, that is, if $a_1<1$ and

$$\liminf_{t\to\infty}\int_{t-\sigma_3}^{t} q_1(s)(s-\sigma_3)\,\mathrm{d}s>\frac{1}{(1-a_1)\mathrm{e}}, \tag{3.3}$$

then (3.2) is oscillatory.

Note that Corollary 2.3 transforms (3.3) into

$$\liminf_{t\to\infty}\int_{t+\sigma_1-\sigma_3}^{t} Q_1(s)(s-\sigma_3-t_1)\,\mathrm{d}s>\frac{1+a_1}{\mathrm{e}}, \tag{3.4}$$

for all sufficiently large t_1, $t_1\ge t_0$, where $Q_1(t)=\min\{q_1(t),q_1(t-\sigma_1)\}$. Since

$$\frac{1}{(1-a_1)\mathrm{e}}>\frac{1+a_1}{\mathrm{e}}$$

for $a_1>0$, our results improve their results in some sense. Moreover, our results can be applied to (3.2) when $a_1\ge 1$.

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