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OSCILLATION THEOREMS FOR SECOND-ORDER NEUTRAL FUNCTIONAL DYNAMIC EQUATIONS ON TIME SCALES

CUNCHEN GAO, TONGXING LI, SHUHONG TANG, ETHIRAJU THANDAPANI

Abstract. In this article, we obtain several comparison theorems for the second-order neutral dynamic equation

$$\Big(r(t)\big([x(t)+p(t)x(\tau(t))]^{\Delta}\big)^{\gamma}\Big)^{\Delta}+q_1(t)x^{\lambda}(\delta(t))+q_2(t)x^{\beta}(\eta(t))=0,$$

where γ,λ,β are ratios of positive odd integers. We compare such equation with the first-order dynamic inequalities in the sense that the absence of the eventually positive solutions of these first-order inequalities implies the oscillation of the studied equation.

1. Introduction

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers. The theory of time scales was introduced in 1988 by Hilger [1] in order to unify continuous and discrete analysis. Several authors have expounded on various aspect of this new theory; see [2, 3, 4].

This article concerns the oscillation of solutions to the second-order nonlinear neutral dynamic equation

$$\Big(r(t)\big(\,[x(t)+p(t)x(\tau(t))]^{\Delta}\,\big)^{\gamma}\Big)^{\Delta}+q_1(t)x^{\lambda}(\delta(t))+q_2(t)x^{\beta}(\eta(t))=0 \tag{1.1}$$

on a time scale $\mathbb{T}$.

Since we are interested in oscillatory behavior of solutions we will assume that the time scale $\mathbb{T}$ is not bounded above; i.e., it is a time scale interval of the form $[t_0,\infty)_{\mathbb{T}}:=[t_0,\infty)\cap\mathbb{T}$.

Below we assume that γ,λ,β are ratios of positive odd integers; r,p,q_1,q_2 are real-valued rd-continuous functions; $r(t)>0$, $q_1(t)>0$, $q_2(t)>0$ for $t\in[t_0,\infty)_{\mathbb{T}}$, $\int_{t_0}^{\infty}r^{-1/\gamma}(t)\Delta t=\infty$, $\tau\in C_{rd}(\mathbb{T},\mathbb{T})$, τ is strictly increasing and $\tau([t_0,\infty)_{\mathbb{T}})=[\tau(t_0),\infty)_{\mathbb{T}}$, $\delta\in C_{rd}(\mathbb{T},\mathbb{T})$, $\eta\in C_{rd}(\mathbb{T},\mathbb{T})$, $\lim_{t\to\infty}\delta(t)=\lim_{t\to\infty}\eta(t)=\infty$, $\tau\circ\delta=\delta\circ\tau$ and $\tau\circ\eta=\eta\circ\tau$. We know from [7] that $\tau\circ\sigma=\sigma\circ\tau$.

By a solution of (1.1), we mean a nontrivial real-valued function $x\in C^1_{rd}[T_x,\infty)_{\mathbb{T}}$, $T_x\ge t_0$ which has the properties $x(t)+p(t)x(\tau(t))$ and $r(t)\big([x(t)+p(t)x(\tau(t))]^{\Delta}\big)^{\gamma}$ are defined, and is Δ-differentiable for $\mathbb{T}$, and satisfies (1.1) on $t\in[T_x,\infty)_{\mathbb{T}}$. The

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solutions vanishing in some neighbourhood of infinity will be excluded from our consideration. A solution x of (1.1) is said to be oscillatory if it is neither eventually positive nor eventually negative, otherwise it is nonoscillatory. Equation (1.1) is called oscillatory if all its solutions are oscillatory.

During the last few years, Ladde et al. [5] summarized some known oscillation criteria for differential equations. Tang and Liu [6] investigated the oscillatory behavior of the first-order nonlinear delay difference equation of the form

$$x(n+1) - x(n) + p(n)x^{\gamma}(n-l) = 0.$$

With the development of dynamic equations on time scales, there has been much research activity concerning the oscillation and nonoscillation of solutions of non-neutral dynamic equations and neutral functional dynamic equations on time scales, we refer the reader to the articles [7–25], and the references cited therein. Agarwal and Bohner [8], Bohner et al. [9], Şahiner and Stavroulakis [10], Braverman and B. Karpuz [11], and Zhang and Deng [12] studied the oscillation of first-order delay dynamic equation on time scales

$$x^{\Delta}(t) + p(t)x(\tau(t)) = 0.$$

Agarwal et al. [13] considered the second-order delay dynamic equation on time scales

$$x^{\Delta\Delta}(t) + p(t)x(\tau(t)) = 0.$$

Braverman and Karpuz [14] investigated the non-oscillation of second-order delay dynamic equation

$$(A_0 x^{\Delta})^{\Delta}(t) + \sum_{i\in[1,n]_{\mathbb{N}}} A_i(t)x(\alpha_i(t)) = f(t).$$

We note that [7, 8, 9, 10, 11] obtained some sufficient conditions for the nonexistence of eventually positive solutions of the first-order dynamic inequality

$$x^{\Delta}(t) + p(t)x(\tau(t)) \le 0,$$

where $\tau(t) < t$. For the oscillation of neutral dynamic equations, Agarwal et al. [16], Erbe et al. [17], Şahiner [18], Saker [19], Saker et al. [20], Saker and O'Regan [21], Tripathy [22], Chen [23], Zhang and Wang [24] and Wu et al. [25] investigated the oscillatory nature of following neutral dynamic equation

$$\big(r(t)\big([x(t) + p(t)x(\tau(t))]^{\Delta}\big)^{\gamma}\big)^{\Delta} + q(t)x^{\gamma}(\delta(t)) = 0. \tag{1.2}$$

Clearly, (1.2) is a special case of (1.1). However, there are few results to study the oscillation of (1.1). The purpose of this paper is to obtain some comparison theorems for the oscillation of (1.1). This paper is organized as follows: In Section 2, we present the basic definitions and the theory of calculus on time scales. In Section 3, we shall establish some oscillation criteria for (1.1).

In what follows, all functional inequalities considered in this paper are assumed to hold eventually; that is, they are satisfied for all sufficiently large t.

2. Preliminaries

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$. Since we are interested in oscillatory behavior, we suppose that the time scale under consideration is not bounded above; i.e., it is a time scale interval of the form

$[t_0, \infty)_{\mathbb{T}}$. On any time scale we define the forward and backward jump operators by

$$\sigma(t) := \inf\{s \in \mathbb{T} | s > t\}, \quad \text{and} \quad \rho(t) := \sup\{s \in \mathbb{T} | s < t\}.$$

A point $t \in \mathbb{T}$ is said to be left-dense if $\rho(t) = t$, right-dense if $\sigma(t) = t$, left-scattered if $\rho(t) < t$, and right-scattered if $\sigma(t) > t$. The graininess μ of the time scale is defined by $\mu(t) := \sigma(t) - t$.

For a function $f : \mathbb{T} \to \mathbb{R}$ (the range $\mathbb{R}$ of f may actually be replaced by any Banach space), the (delta) derivative is defined by

$$f^{\Delta}(t) = \frac{f(\sigma(t)) - f(t)}{\sigma(t) - t},$$

if f is continuous at t and t is right-scattered. If t is not right-scattered then the derivative is defined by

$$f^{\Delta}(t) = \lim_{s \to t^+} \frac{f(\sigma(t)) - f(s)}{t - s} = \lim_{s \to t^+} \frac{f(t) - f(s)}{t - s},$$

provided this limit exists.

A function $f : \mathbb{T} \to \mathbb{R}$ is said to be rd-continuous if it is continuous at each right-dense point and if there exists a finite left limit in all left-dense points. The set of rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ is denoted by $C_{rd}(\mathbb{T}, \mathbb{R})$.

A function f is said to be differentiable if its derivative exists. The set of functions $f : \mathbb{T} \to \mathbb{R}$ that are differentiable and whose derivative is rd-continuous function is denoted by $C^1_{rd}(\mathbb{T}, \mathbb{R})$.

The derivative and the shift operator σ are related by the formula

$$f^{\sigma}(t) = f(\sigma(t)) = f(t) + \mu(t) f^{\Delta}(t).$$

Let f be a real-valued function defined on an interval $[a, b]$. We say that f is increasing, decreasing, nondecreasing, and non-increasing on $[a, b]$ if $t_1,\ t_2 \in [a, b]$ and $t_2 > t_1$ imply $f(t_2) > f(t_1),\ f(t_2) < f(t_1),\ f(t_2) \geq f(t_1)$ and $f(t_2) \leq f(t_1)$, respectively. Let f be a differentiable function on $[a, b]$. Then f is increasing, decreasing, nondecreasing, and non-increasing on $[a, b]$ if $f^{\Delta}(t) > 0,\ f^{\Delta}(t) < 0,$ $f^{\Delta}(t) \geq 0$, and $f^{\Delta}(t) \leq 0$ for all $t \in [a, b)$, respectively.

We will make use of the following product and quotient rules for the derivative of the product fg and the quotient f/g (where $g(t)g(\sigma(t)) \neq 0$) of two differentiable functions f and g

$$(fg)^{\Delta}(t) = f^{\Delta}(t)g(t) + f(\sigma(t))g^{\Delta}(t) = f(t)g^{\Delta}(t) + f^{\Delta}(t)g(\sigma(t)),$$
$$\Big(\frac{f}{g}\Big)^{\Delta}(t) = \frac{f^{\Delta}(t)g(t) - f(t)g^{\Delta}(t)}{g(t)g(\sigma(t))}.$$

For $a, b \in \mathbb{T}$ and a differentiable function f, the Cauchy integral of f^{Δ} is defined by

$$\int_a^b f^{\Delta}(t)\Delta t = f(b) - f(a).$$

The integration by parts formula reads

$$\int_a^b f^{\Delta}(t)g(t)\Delta t = f(b)g(b) - f(a)g(a) - \int_a^b f^{\sigma}(t)g^{\Delta}(t)\Delta t,$$

and infinite integrals are defined as

$$\int_a^\infty f(s)\Delta s = \lim_{t\to\infty}\int_a^t f(s)\Delta s.$$

3. Main Results

In this section, we shall establish some comparison theorems for the oscillation of (1.1). Firstly, we give the following chain rule on time scales which will play an important role in the proofs of our results.

Lemma 3.1 ([3]). *Assume that* $\sup\mathbb{T} = \infty$, *and* $v \in C^1_{rd}([t_0,\infty)_\mathbb{T})$ *is a strictly increasing function and unbounded such that* $v([t_0,\infty)_\mathbb{T}) = [v(t_0),\infty)_\mathbb{T}$. *Then for* $x \in C^1_{rd}([t_0,\infty)_\mathbb{T},\mathbb{R})$, *we have*

$$(x\circ v)^\Delta(t) = x^\Delta(v(t))v^\Delta(t) \tag{3.1}$$

for $t \in [t_0,\infty)_\mathbb{T}$.

Below, we will give our results. For the sake of convenience, we denote

$$\begin{gathered}
z(t) = x(t) + p(t)x(\tau(t)), \quad Q_1(t) = \min\{q_1(t), q_1(\tau(t))\},\\
Q_2(t) = \min\{q_2(t), q_2(\tau(t))\}, \quad R(t) = \int_{t_0}^t \frac{1}{r^{1/\gamma}(s)}\Delta s,\\
H(t) = R(t) - R(t_1), \quad Q_3(t) = Q_1(t)\int_{t_1}^{\sigma(t)} \frac{1}{r(s)}\Delta s,\\
Q_4(t) = Q_2(t)\int_{t_1}^{\sigma(t)} \frac{1}{r(s)}\Delta s,
\end{gathered}$$

for $t_1 \ge t_0$ sufficiently large.

Without loss of generality we can deal only with the eventually positive solutions of (1.1) in our proofs.

Theorem 3.2. *Assume that* $\tau^\Delta(t) \ge \tau_0 > 0$, $\lambda \le 1$ *and* $\beta \le 1$. *Further, assume that there exists a* $p_0 > 0$ *such that* $0 \le p(t) \le p_0^{\gamma/\lambda} < \infty$, *and* $0 \le p(t) \le p_0^{\gamma/\beta} < \infty$. *If the first-order neutral dynamic inequality*

$$\begin{aligned}
&\Big(y(t) + \frac{p_0^\gamma}{\tau_0} y(\tau(t))\Big)^\Delta + Q_1(t)H^\lambda(\delta(t))y^{\lambda/\gamma}(\delta(t))\\
&+ Q_2(t)H^\beta(\eta(t))y^{\beta/\gamma}(\eta(t)) \le 0
\end{aligned}\tag{3.2}$$

has no eventually positive solution for all sufficiently large t_1, *then every solution of* (1.1) *is oscillatory.*

Proof. Assume that x is an eventually positive solution of (1.1). Then we have $\big(r(t)(z^\Delta(t))^\gamma\big)^\Delta < 0$. It follows from (1.1) and (3.1) that

$$\big(r(t)(z^\Delta(t))^\gamma\big)^\Delta + q_1(t)x^\lambda(\delta(t)) + q_2(t)x^\beta(\eta(t)) = 0 \tag{3.3}$$

and

$$\begin{aligned}
&\frac{p_0^\gamma}{\tau^\Delta(t)}\big(r(\tau(t))(z^\Delta(\tau(t)))^\gamma\big)^\Delta + p_0^\gamma q_1(\tau(t))x^\lambda(\delta(\tau(t)))\\
&+ p_0^\gamma q_2(\tau(t))x^\beta(\eta(\tau(t))) = 0.
\end{aligned}\tag{3.4}$$

In view of $\tau^{\Delta}(t) \geq \tau_0 > 0$ and (3.4), we see that

$$\frac{p_0^{\gamma}}{\tau_0}\big(r(\tau(t))(z^{\Delta}(\tau(t)))^{\gamma}\big)^{\Delta} + p_0^{\gamma} q_1(\tau(t)) x^{\lambda}(\delta(\tau(t))) + p_0^{\gamma} q_2(\tau(t)) x^{\beta}(\eta(\tau(t))) \leq 0.$$

Combining this inequality with (3.3), we have

$$\begin{aligned} &\big(r(t)(z^{\Delta}(t))^{\gamma}\big)^{\Delta} + \frac{p_0^{\gamma}}{\tau_0}\big(r(\tau(t))(z^{\Delta}(\tau(t)))^{\gamma}\big)^{\Delta} \\ &+ q_1(t) x^{\lambda}(\delta(t)) + p_0^{\gamma} q_1(\tau(t)) x^{\lambda}(\delta(\tau(t))) \\ &+ q_2(t) x^{\beta}(\eta(t)) + p_0^{\gamma} q_2(\tau(t)) x^{\beta}(\eta(\tau(t))) \leq 0. \end{aligned} \tag{3.5}$$

If $\lambda \leq 1$, from [29, Lemma 2], we obtain

$$x^{\lambda}(\delta(t)) + p_0^{\gamma} x^{\lambda}(\delta(\tau(t))) \geq [x(\delta(t)) + p_0^{\gamma/\lambda} x(\delta(\tau(t)))]^{\lambda} \geq z^{\lambda}(\delta(t)).$$

Similarly,

$$x^{\beta}(\eta(t)) + p_0^{\gamma} x^{\beta}(\eta(\tau(t))) \geq \Big[x(\eta(t)) + p_0^{\gamma/\beta} x(\eta(\tau(t)))\Big]^{\beta} \geq z^{\beta}(\eta(t)).$$

Hence by (3.5), we have

$$\begin{aligned} &\big(r(t)(z^{\Delta}(t))^{\gamma}\big)^{\Delta} + \frac{p_0^{\gamma}}{\tau_0}\big(r(\tau(t))(z^{\Delta}(\tau(t)))^{\gamma}\big)^{\Delta} \\ &+ Q_1(t) z^{\lambda}(\delta(t)) + Q_2(t) z^{\beta}(\eta(t)) \leq 0. \end{aligned} \tag{3.6}$$

It follows from (1.1) and $\int_{t_0}^{\infty} \frac{1}{r^{1/\gamma}(t)} \Delta t = \infty$ that $y(t) = r(t)(z^{\Delta}(t))^{\gamma} > 0$ is decreasing. Thus, there exists a $t_1 \geq t_0$ such that

$$z(t) \geq \int_{t_1}^{t} \frac{\big(r(s)(z^{\Delta}(s))^{\gamma}\big)^{1/\gamma}}{r^{1/\gamma}(s)} \Delta s \geq y^{1/\gamma}(t)\big(R(t) - R(t_1)\big). \tag{3.7}$$

Then, setting $y(t) = r(t)(z^{\Delta}(t))^{\gamma}$ in (3.6) and using (3.7), one can see that y is a positive solution of inequality (3.2). This is a contradiction and the proof is complete. □

Theorem 3.3. *Assume that* $\tau^{\Delta}(t) \geq \tau_0 > 0$, $\tau(t) \geq t$, $\lambda \leq 1$ *and* $\beta \leq 1$. *Moreover, assume that there exists a* $p_0 > 0$ *such that* $0 \leq p(t) \leq p_0^{\gamma/\lambda} < \infty$, *and* $0 \leq p(t) \leq p_0^{\gamma/\beta} < \infty$. *If the first-order dynamic inequality*

$$\begin{aligned} &u^{\Delta}(t) + \Big(\frac{\tau_0}{\tau_0 + p_0^{\gamma}}\Big)^{\lambda/\gamma} Q_1(t) H^{\lambda}(\delta(t)) u^{\lambda/\gamma}(\delta(t)) \\ &+ \Big(\frac{\tau_0}{\tau_0 + p_0^{\gamma}}\Big)^{\beta/\gamma} Q_2(t) H^{\beta}(\eta(t)) u^{\beta/\gamma}(\eta(t)) \leq 0 \end{aligned} \tag{3.8}$$

has no eventually positive solution for all sufficiently large t_1, *then every solution of* (1.1) *is oscillatory.*

Proof. Assume that x is a positive solution of (1.1). By the proof of Theorem 3.2, we find $y(t) = r(t)(z^{\Delta}(t))^{\gamma} > 0$ is decreasing and satisfies (3.2). Let $u(t) = y(t) + p_0^{\gamma} y(\tau(t))/\tau_0$. From $\tau(t) \geq t$, we have

$$u(t) \leq \big(1 + \frac{p_0^{\gamma}}{\tau_0}\big) y(t).$$

Hence, we get that u is a positive solution of (3.8). This is a contradiction and the proof is complete. □

From Theorem 3.3, we have the following results.

Corollary 3.4. *Assume that $\delta(t) \le \eta(t)$, $\tau^\Delta(t) \ge \tau_0 > 0$, $\tau(t) \ge t$, $\lambda = \beta \le 1$. Furthermore, assume that there exists a $p_0 > 0$ such that $0 \le p(t) \le p_0^{\gamma/\lambda} < \infty$. If the first-order dynamic inequality*

$$u^\Delta(t) + \frac{\tau_0^{\lambda/\gamma}}{(\tau_0 + p_0^\gamma)^{\lambda/\gamma}}[Q_1(t)H^\gamma(\delta(t)) + Q_2(t)H^\gamma(\eta(t))]u^{\lambda/\gamma}(\eta(t)) \le 0$$

has no positive solution for all sufficiently large t_1, then every solution of (1.1) is oscillatory.

Proof. Proceeding as in the proof of Theorem 3.3, u is decreasing and if $\delta(t) \le \eta(t)$, then $u(\delta(t)) \ge u(\eta(t))$. Therefore, u is a positive solution of the dynamic inequality

$$u^\Delta(t) + \frac{\tau_0^{\lambda/\gamma}}{(\tau_0 + p_0^\gamma)^{\lambda/\gamma}}[Q_1(t)H^\gamma(\delta(t)) + Q_2(t)H^\gamma(\eta(t))]u^{\lambda/\gamma}(\eta(t)) \le 0.$$

This is a contradiction and the proof is complete. □

Similar to the proof of Corollary 3.4, we have the another comparison result.

Corollary 3.5. *Assume that $\delta(t) \ge \eta(t)$, $\tau^\Delta(t) \ge \tau_0 > 0$, $\tau(t) \ge t$, $\lambda = \beta \le 1$. Moreover, assume that there exists a $p_0 > 0$ such that $0 \le p(t) \le p_0^{\gamma/\lambda} < \infty$. If the first-order dynamic inequality*

$$u^\Delta(t) + \frac{\tau_0^{\lambda/\gamma}}{(\tau_0 + p_0^\gamma)^{\lambda/\gamma}}[Q_1(t)H^\gamma(\delta(t)) + Q_2(t)H^\gamma(\eta(t))]u^{\lambda/\gamma}(\delta(t)) \le 0$$

has no positive solution for all sufficiently large t_1, then every solution of (1.1) is oscillatory.

Theorem 3.6. *Assume that $\tau(t) \ge t$, $\tau^\Delta(t) \ge \tau_0 > 0$, $\gamma = 1$, $\lambda \le 1$ and $\beta \le 1$. Further, assume that there exists a $p_0 > 0$ such that $0 \le p(t) \le p_0^{1/\lambda} < \infty$, and $0 \le p(t) \le p_0^{1/\beta} < \infty$. If the first-order dynamic inequality*

$$\phi^\Delta(t) - \frac{\tau_0}{\tau_0 + p_0}Q_3(t)\phi^\lambda(\delta(t)) - \frac{\tau_0}{\tau_0 + p_0}Q_4(t)\phi^\beta(\eta(t)) \ge 0 \tag{3.9}$$

has no eventually positive solution for all sufficiently large t_1, then every solution of (1.1) is oscillatory.

Proof. Assume that x is an eventually positive solution of (1.1). Then we have $\big(r(t)z^\Delta(t)\big)^\Delta < 0$ and $z^\Delta(t) > 0$. Proceeding as in the proof of Theorem 3.2, we have

$$\big(r(t)z^\Delta(t)\big)^\Delta + \frac{p_0}{\tau_0}\big(r(\tau(t))z^\Delta(\tau(t))\big)^\Delta + Q_1(t)z^\lambda(\delta(t)) + Q_2(t)z^\beta(\eta(t)) \le 0. \tag{3.10}$$

Integrating (3.10) from t to ∞, we obtain

$$r(t)z^\Delta(t) + \frac{p_0}{\tau_0}r(\tau(t))z^\Delta(\tau(t)) \ge \int_t^\infty \big(Q_1(s)z^\lambda(\delta(s)) + Q_2(s)z^\beta(\eta(s))\big)\Delta s. \tag{3.11}$$

Since $r(t)z^\Delta(t)$ is decreasing and $\tau(t) \ge t$, it follows that

$$(1 + \frac{p_0}{\tau_0})r(t)z^\Delta(t) \ge \int_t^\infty \big(Q_1(s)z^\lambda(\delta(s)) + Q_2(s)z^\beta(\eta(s))\big)\Delta s.$$

Integrating the last inequality from t_1 to t, from [26, Lemma 1], we obtain

$$\begin{aligned} z(t) &\geq \frac{\tau_0}{\tau_0+p_0}\int_{t_1}^{t}\frac{1}{r(u)}\int_{u}^{\infty}\big(Q_1(s)z^{\lambda}(\delta(s))+Q_2(s)z^{\beta}(\eta(s))\big)\Delta s\Delta u \\ &= \frac{\tau_0}{\tau_0+p_0}\int_{t_1}^{t}\big(Q_1(s)z^{\lambda}(\delta(s))+Q_2(s)z^{\beta}(\eta(s))\big)\int_{t_1}^{\sigma(s)}\frac{1}{r(u)}\Delta u\Delta s. \end{aligned}$$

Thus, we see that

$$z(t) \geq \frac{\tau_0}{\tau_0+p_0}\int_{t_1}^{t}\big(Q_3(s)z^{\lambda}(\delta(s))+Q_4(s)z^{\beta}(\eta(s))\big)\Delta s.$$

Denote the right hand side of the above inequality by $\phi(t)$. Since $z(t) \geq \phi(t)$, we find that ϕ is a positive solution of (3.9). This is a contradiction and the proof is complete. □

From Theorem 3.6, we get the following result.

Corollary 3.7. *Assume that* $\delta(t) \leq \eta(t)$, $\tau(t) \geq t$, $\tau^{\Delta}(t) \geq \tau_0 > 0$, $\gamma = 1$, $\lambda = \beta \leq 1$. *Furthermore, assume that there exists a* $p_0 > 0$ *such that* $0 \leq p(t) \leq p_0^{1/\lambda} < \infty$. *If the first-order dynamic inequality*

$$\phi^{\Delta}(t) - \frac{\tau_0}{\tau_0+p_0}\big(Q_3(t)+Q_4(t)\big)\phi^{\lambda}(\delta(t)) \geq 0$$

has no eventually positive solution for all sufficiently large t_1, *then every solution of* (1.1) *is oscillatory.*

Proof. Proceeding as in the proof of Theorem 3.6, ϕ is increasing and if $\delta(t) \leq \eta(t)$, then $\phi(\delta(t)) \leq \phi(\eta(t))$. Therefore, ϕ is a positive solution of the dynamic inequality

$$\phi^{\Delta}(t) - \frac{\tau_0}{\tau_0+p_0}\big(Q_3(t)+Q_4(t)\big)\phi^{\lambda}(\delta(t)) \geq 0.$$

This is a contradiction and the proof is complete. □

Similar to the proof of Corollary 3.7, we have another comparison result.

Corollary 3.8. *Assume that* $\delta(t) \geq \eta(t)$, $\tau(t) \geq t$, $\tau^{\Delta}(t) \geq \tau_0 > 0$, $\gamma = 1$, $\lambda = \beta \leq 1$. *Moreover, assume that there exists a* $p_0 > 0$ *such that* $0 \leq p(t) \leq p_0^{1/\lambda} < \infty$. *If the first-order dynamic inequality*

$$\phi^{\Delta}(t) - \frac{\tau_0}{\tau_0+p_0}\big(Q_3(t)+Q_4(t)\big)\phi^{\lambda}(\eta(t)) \geq 0$$

has no eventually positive solution for all sufficiently large t_1, *then every solution of* (1.1) *is oscillatory.*

Remark 3.9. Assume that $\tau^{\Delta}(t) \geq \tau_0 > 0$ and $\tau^{-1} \in C_{rd}(\mathbb{T},\mathbb{T})$, where τ^{-1} is the inverse function of τ. Similar to the methods of the above, we can derive some comparison theorems for (1.1) when $\tau(t) \leq t$, the details are left to the interested reader.

Our results can be extended to the equation of the general form

$$\Big(r(t)\big([x(t)+p(t)x(\tau(t))]^{\Delta}\big)^{\gamma}\Big)^{\Delta} + \sum_{i=1}^{n} q_i(t)x^{\lambda_i}(\delta_i(t)) = 0.$$

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Cunchen Gao
College of Information Science and Engineering, Ocean University of China, Qingdao, Shandong 266100, China
E-mail address: ccgao@ouc.edu.cn

Tongxing Li
School of Control Science and Engineering, Shandong University, Jinan, Shandong 250061, China.
School of Mathematical Science, University of Jinan, Jinan, Shandong 250022, China
E-mail address: litongx2007@hotmail.com

Shuhong Tang
School of Information and Control Engineering, Weifang University, Weifang, Shandong 261061, China.
College of Information Science and Engineering, Ocean University of China, Qingdao, Shandong 266100, China
E-mail address: wfxytang@163.com

Ethiraju Thandapani
Ramanujan Institute for Advanced Study in Mathematics, University of Madras, Chennai, India
E-mail address: ethandapani@yahoo.co.in