Electronic Journal of Differential Equations, Vol. 2011 (2011), No. 26, pp. 1–7.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu

NONEXISTENCE OF RADIAL POSITIVE SOLUTIONS FOR A NONPOSITONE PROBLEM

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Abstract. In this article we study the nonexistence of radial positive solutions for a nonpositone problem when the nonliearity is superlinear and has more than one zero.

1. Introduction

We study the nonexistence of radial positive solutions for the boundary-value problem

$$\begin{gathered} -\Delta u(x) = \lambda f(u(x)) \quad x \in \Omega, \\ u(x) = 0 \quad x \in \partial\Omega, \end{gathered} \tag{1.1}$$

where $\lambda > 0$, $f : [0, +\infty) \to \mathbb{R}$ is a continuous nonlinear function that has more than one zero, and $\Omega \subset \mathbb{R}^N$; is the annulus: $\Omega = C(0, R, \widehat{R}) = \{x \in \mathbb{R}^N : R < |x| < \widehat{R}\}$ ($N > 2$, $0 < R < \widehat{R}$).

When f is a nondecreasing satisfying $f(0) < 0$ (the nonpositone case) and has only one zero, problem (1.1) has been studied by Brown, Castro and Shivaji [2] in the ball, and by Arcoya and Zertiti [1] in the annulus.

We observe that the nonexistence of radial positive solutions of (1.1) is equivalent to the nonexistence of positive solutions of the problem

$$\begin{gathered} -u''(r) - \frac{N-1}{r} u'(r) = \lambda f(u(r)) \quad R < r < \widehat{R} \\ u(R) = u(\widehat{R}) = 0, \end{gathered} \tag{1.2}$$

where $\lambda > 0$.

Our main objective in this article is to prove that the result of nonexistence of radial positive solutions of (1.1) remains valid when f has more than one zero and is not increasing entirely on $[0, +\infty)$; see [1, Theorem 3.1]. More precisely we assume that the map $f : [0, +\infty) \to \mathbb{R}$ satisfies the following hypotheses

- (H1) $f \in C^1([0, +\infty), \mathbb{R})$ such that f has three zeros $\beta_1 < \beta_2 < \beta_3$ with $f'(\beta_i) \neq 0$ for all $i \in \{1, 2, 3\}$. Moreover, $f' \geq 0$ on $[\beta_3, +\infty)$.
- (H2) $f(0) < 0$.
- (H3) $\lim_{u\to+\infty} f(u)/u = +\infty$,

2000 *Mathematics Subject Classification.* 35J25, 34B18.
Key words and phrases. Nonpositone problem; radial positive solutions.

Submitted June 14, 2010. Published February 10, 2011.

2. The main result

In this section, we give the main result in this work. More precisely we shall prove the following theorem.

Theorem 2.1. *Assume that the hypotheses* (H1)–(H3) *are satisfied. Then there exists a positive real number* λ_0 *such that if* $\lambda > \lambda_0$, *problem* (1.1) *has no radial positive solution.*

Remark. We do not know for what radius $r \in (R, \widehat{R})$ the solution u attains its maximum. In addition, f changes sign in $(\beta_1, +\infty)$. These two facts make our study more difficult, and change the proof of nonexistence in [1].

To prove Theorem 2.1, we need the next three technical lemmas. We note that the proofs of the first and the last lemma are analogous to those of [1, Lemma 3.2, Lemma 3.4]. On the other hand, the proof of the second lemma is different from that of [1, Lemma 3.3]. This is so because our f has no constant sign in $(\beta_1, +\infty)$.

Denote by $u_\lambda(r)$ a positive solution of (1.1) (if it exists) and let $R_0 = (R + \widehat{R})/2$. Following the work [1], we introduce the following notation: $\beta = \beta_1$, $\theta = \min\{\beta_2, \min \theta_i\}$ where θ_i are the zeros of F $(F(x) = \int_0^x f(t)dt)$.

Remark. In [3, Theorem B iii], F has at most one zero. On the opposite, in our case F may have more than one zero because f has a finite number of zeros. In this paper we assume, with out loss of generality, that f has three zeros. In fact, the number of zeros of F depends on f, but F has at most three zeros.

Lemma 2.2. *Let* $f \in C^1([0, +\infty))$ *satisfying* (H3) *and consider* $\lambda > 2$. *If* u_λ *is a positive solution of* (1.2), *then for every* $r \in (R_0, \widehat{R}]$ *there exists a positive number* $M = M(r) > 0$ *(independent of* λ*) such that* $u_\lambda(r) \le M$.

Proof. Let φ_1 be a positive eigenfunction associated to the first eigenvalue $\mu_1 > 0$ of the eigenvalue problem

$$-(r^{N-1}v')' = \mu r^{N-1} v, \quad R < r < \widehat{R}$$
$$v(R) = 0 = v(\widehat{R}),$$

Multiplying the equation in (1.2) by $r^{N-1}\varphi_1(r)$ and integrating from R to $\widehat{R}$, we obtain

$$\int_R^{\widehat{R}} r^{N-1} u_\lambda'(r)\varphi_1'(r)dr = -\int_R^{\widehat{R}} (r^{N-1}u_\lambda'(r))'\varphi_1(r)dr,$$

hence

$$\int_R^{\widehat{R}} r^{N-1} u_\lambda'(r)\varphi_1'(r)dr = \lambda \int_R^{\widehat{R}} r^{N-1} f(u_\lambda(r))\varphi_1(r)dr. \tag{2.1}$$

On the other hand, multiplying the equation $-(r^{N-1}\varphi_1'(r))' = \mu_1 r^{N-1}\varphi_1(r)$, $(R < r < \widehat{R})$ by u_λ and integrating from R to $\widehat{R}$, we obtain

$$\int_R^{\widehat{R}} r^{N-1} u_\lambda'(r)\varphi_1'(r)dr = -\int_R^{\widehat{R}} (r^{N-1}\varphi_1'(r))' u_\lambda(r)dr,$$

hence

$$\int_R^{\widehat{R}} r^{N-1} u_\lambda'(r)\varphi_1'(r)dr = \mu_1 \int_R^{\widehat{R}} r^{N-1}\varphi_1(r)u_\lambda(r)dr. \tag{2.2}$$

Combining (2.1), (2.2) and choosing $\mu > \mu_1/2$, $c > 0$ such that
$$f(\zeta) \geq \mu\zeta - c, \quad \forall \zeta \geq 0$$
(because f is superlinear), we deduce
$$\begin{aligned}\mu_1 \int_R^{\widehat{R}} r^{N-1}\varphi_1(r)u_\lambda(r)dr &= \lambda \int_R^{\widehat{R}} r^{N-1} f(u_\lambda(r))\varphi_1(r)dr \\ &\geq \lambda\mu \int_R^{\widehat{R}} r^{N-1}\varphi_1(r)u_\lambda(r)dr - \lambda c \int_R^{\widehat{R}} r^{N-1}\varphi_1(r)dr,\end{aligned}$$
from which
$$\int_R^{\widehat{R}} r^{N-1}\varphi_1(r)u_\lambda(r)dr \leq \frac{\lambda k}{\lambda\mu - \mu_1} \leq \frac{k}{\mu - \frac{\mu_1}{2}} := A, \quad \forall \lambda > 2,$$
with $k = c\int_R^{\widehat{R}} r^{N-1}\varphi_1(r)dr > 0$ and $A > 0$ is independent of λ. Now, let $r \in (R_0, \widehat{R}]$ and choosing $\delta > 0$ such that $R_0 < r - \delta$ and using the fact that u_λ is non-increasing in $(R_0, \widehat{R})$ (see [4]) implies
$$\begin{aligned}u_\lambda(r) &\leq \frac{\int_{r-\delta}^r t^{N-1}u_\lambda(t)\varphi_1(t)dt}{\int_{r-\delta}^r t^{N-1}\varphi_1(t)dt} \\ &\leq \frac{\int_R^{\widehat{R}} t^{N-1}u_\lambda(t)\varphi_1(t)dt}{\int_{r-\delta}^r t^{N-1}\varphi_1(t)dt} \\ &\leq \frac{A}{\int_{r-\delta}^r t^{N-1}\varphi_1(t)dt} = M, \quad \forall \lambda > 2\,.\end{aligned}$$
The proof is complete. □

Lemma 2.3. *Assume* (H1)–(H3) *and let $R_1 \in (R_0, \widehat{R})$, $c \in (\beta, \theta)$. Then there exists $\lambda_1 > 0$ such that for all positive solutions u_λ of* (1.2) *with $\lambda \geq \lambda_1$, there exists $t_1 = t_1(\lambda) \in (R_0, R_1)$ satisfying $u_\lambda(t_1) < c$.*

Proof. We argue by contradiction. Suppose that there exists a sequence $\{\lambda_n\} \subset (0, +\infty)$ converging to $+\infty$ such that
$$u_{\lambda_n}(r) \geq c, \quad \forall r \in (R_0, R_1],\ \forall n \in \mathbb{N}.$$
Consider $\bar{t}_n = \max\{r \in (R, \widehat{R}) : u'_{\lambda_n}(r) = 0\}$. Then $u'_{\lambda_n}(r) < 0$ for all $r \in (\bar{t}_n, \widehat{R})$, and we deduce
$$u_{\lambda_n}(r) \leq u_{\lambda_n}(\bar{t}_n), \quad \forall r \in (\bar{t}_n, \widehat{R}).$$
It follows that $u'_{\lambda_n}(\bar{t}_n) = 0$ and $u''_{\lambda_n}(\bar{t}_n) \leq 0$. So $f(u_{\lambda_n}(\bar{t}_n)) \geq 0$ by (1.2). Hence
$$u_{\lambda_n}(\bar{t}_n) \leq \beta_2 \quad \text{or} \quad u_{\lambda_n}(\bar{t}_n) \geq \beta_3.$$
Now, we study the following two cases:

Case 1: $u_{\lambda_n}(\bar{t}_n) \leq \beta_2$.

(i) If $\sup_n u_{\lambda_n}(\bar{t}_n) < \beta_2$, then
$$\begin{aligned}-r^{N-1}u'_{\lambda_n}(r) &= \lambda_n \int_{\bar{t}_n}^r s^{N-1} f(u_{\lambda_n}(s))ds, \quad \forall r \in (R_0, R_1) \\ &\geq \lambda_n \inf_{\xi \in (c, \sup_n u_{\lambda_n}(\bar{t}_n))} f(\xi) \int_{R_0}^r s^{N-1}ds.\end{aligned}$$

Since $\sup_n u_{\lambda_n}(\bar{t}_n) < \beta_2$, it follows that $\inf_{\xi\in(c,\sup_n u_{\lambda_n}(\bar{t}_n))} f(\xi) > 0$. Therefore,

$$\lim_{n\to+\infty} u'_{\lambda_n}(r) = -\infty, \text{ uniformly on compact subsets of } (R_0, R_1). \tag{2.3}$$

Now, let $r_1, r_2 \in (R_0, R_1)$ such that $R_0 < r_1 < r_2 < R_1$. By the mean value theorem, there exists $r_n \in (r_1, r_2)$ such that

$$u_{\lambda_n}(r_2) = u_{\lambda_n}(r_1) + (r_2 - r_1)u'_{\lambda_n}(r_n).$$

Also, for all $r \in (R_0, R_1)$ we have $c \le u_{\lambda_n}(r) < \beta_2$ for all n, and by (2.3) the second summand of the precedent equality tends to $-\infty$. Hence

$$\lim_{n\to+\infty} u_{\lambda_n}(r_2) = -\infty.$$

This contradicts $u_{\lambda_n} \ge 0$ for all $n \in \mathbb{N}$.

(ii) If $\sup_n u_{\lambda_n}(\bar{t}_n) = \beta_2$. Consider the following two sets:

$$\Phi_n = \{r \in [R_1, \widehat{R}] : \beta \le u_{\lambda_n}(r) \le \frac{3\beta + c}{4}\},$$
$$\Psi_n = \{r \in [R_1, \widehat{R}] : \frac{2(\beta + c)}{4} \le u_{\lambda_n}(r) \le \frac{\beta + 3c}{4}\}.$$

Since $(\beta, \frac{3\beta+c}{4}), (\frac{2(\beta+c)}{4}, \frac{\beta+3c}{4}) \subset u_{\lambda_n}((R_1, \widehat{R}))$, by the intermediate value theorem, Φ_n and Ψ_n are not empty. Consider $\underline{a}(n)$, $\overline{a}(n)$, $\underline{b}(n)$ and $\overline{b}(n)$ such that $\underline{a}(n) = \inf_r \Psi_n$, $\overline{a}(n) = \sup_r \Psi_n$, $\underline{b}(n) = \inf_r \Phi_n$ and $\overline{b}(n) = \sup_r \Phi_n$.
Let $r_0 \in \big[\underline{a}(n), \overline{b}(n)\big]$. Then

$$\begin{aligned}
-r_0^{N-1}u'_{\lambda_n}(r_0) &= \lambda_n \int_{\bar{t}_n}^{r_0} s^{N-1} f(u_{\lambda_n}(s))ds \\
&\ge \lambda_n R^{N-1} \int_{R_0}^{r_0} f(u_{\lambda_n}(s))ds \\
&\ge \lambda_n R^{N-1} \int_{u_{\lambda_n}(R_0)}^{u_{\lambda_n}(r_0)} \frac{f(t)}{u'_{\lambda_n}(u^{-1}_{\lambda_n}(t))} dt \\
&= \lambda_n R^{N-1} \int_{u_{\lambda_n}(r_0)}^{u_{\lambda_n}(R_0)} \frac{f(t)}{-u'_{\lambda_n}(u^{-1}_{\lambda_n}(t))} dt,
\end{aligned}$$

hence

$$\begin{aligned}
-r_0^{N-1}u'_{\lambda_n}(r_0)(-u'_{\lambda_n}(s_0)) &\ge \lambda_n R^{N-1} \int_{u_{\lambda_n}(r_0)}^{u_{\lambda_n}(R_0)} f(t)dt \\
&\ge \lambda_n R^{N-1} \int_{\frac{\beta+3c}{4}}^{c} f(t)dt,
\end{aligned}$$

where s_0 satisfies $u'_{\lambda_n}(s_0) = \inf_{[R_0, r_0]} u'_{\lambda_n}(s)$. Since the function $r \longmapsto -r^{N-1}u'_{\lambda_n}(r)$ is increasing on $\big(\underline{a}(n), \overline{b}(n)\big)$,

$$-r_0^{N-1}u'_{\lambda_n}(r_0)(-u'_{\lambda_n}(s_0)) \le (-r_0^{N-1}u'_{\lambda_n}(r_0))^2 \frac{1}{s_0^{N-1}}.$$

Then

$$(-r_0^{N-1}u'_{\lambda_n}(r_0))^2 \frac{1}{s_0^{N-1}} \ge \lambda_n R^{N-1} \int_{\frac{\beta+3c}{4}}^{c} f(t)dt.$$

Therefore,

$$\lim_{n\to+\infty} u'_{\lambda_n}(r_0) = -\infty. \tag{2.4}$$

Now, Let $r_1 \in [\underline{a}(n), \overline{a}(n)]$ and $r_2 \in \big[\underline{b}(n), \overline{b}(n)\big]$, then by the mean value theorem, there exists $r* \in (r_1, r_2)$ such that

$$\begin{aligned} u_{\lambda_n}(r_2) &= u_{\lambda_n}(r_1) + (r_2 - r_1)\, u'_{\lambda_n}(r*) \\ &< u_{\lambda_n}(R_1) + (\underline{b}(n) - \overline{a}(n))\, u'_{\lambda_n}(r*) \\ &\le u_{\lambda_n}(R_1) + \inf_n\, (\underline{b}(n) - \overline{a}(n))\, u'_{\lambda_n}(r*). \end{aligned}$$

Since $u_{\lambda_n}(R_1) \le M$, for all n and some $M = M(R_1) > 0$ (see Lemma 2.2) and $\inf_n\, (\underline{b}(n) - \overline{a}(n)) > 0$ and $\lim_{n\to+\infty} u'_{\lambda_n}(r*) = -\infty$ (by (2.4)), it follows that $\lim_{n\to+\infty} u_{\lambda_n}(r_2) = -\infty$, which contradicts $u_{\lambda_n} \ge 0$ for all $n \in \mathbb{N}$.

Case 2: $u_{\lambda_n}(\bar{t}_n) \ge \beta_3$. Let $r_0 \in \big[\underline{a}(n), \overline{b}(n)\big]$, then

$$-r_0^{N-1} u'_{\lambda_n}(r_0) = \lambda_n \int_{\bar{t}_n}^{r_0} s^{N-1} f(u_{\lambda_n}(s)) ds.$$

Consider t_{β_2} such that $t_{\beta_2} = \max\{r_n \in (R, \widehat{R}] : u_{\lambda_n}(r_n) = \beta_2\}$. Then

$$\begin{aligned} -r_0^{N-1} u'_{\lambda_n}(r_0) &= \lambda_n \int_{\bar{t}_n}^{r_0} s^{N-1} f(u_{\lambda_n}(s)) ds \\ &= \lambda_n \Big[\int_{\bar{t}_n}^{t_{\beta_2}} s^{N-1} f(u_{\lambda_n}(s)) ds + \int_{t_{\beta_2}}^{r_0} s^{N-1} f(u_{\lambda_n}(s)) ds \Big] \\ &\ge \lambda_n \int_{t_{\beta_2}}^{r_0} s^{N-1} f(u_{\lambda_n}(s)) ds, \end{aligned}$$

because $\int_{\bar{t}_n}^{t_{\beta_2}} s^{N-1} f(u_{\lambda_n}(s)) ds \ge 0$. Then as in Case 1, we obtain a contradiction with the positivity of u_{λ_n}. □

Lemma 2.4. *Assume* (H2). *Let* $R_2 \in (R_0, \widehat{R})$ *and* $\bar{c} > 1$. *Then there exists* $\lambda_2 > 0$ *such that every positive solution* u_λ *of* (1.2) *satisfies* $\frac{\beta}{\bar{c}} \in u_\lambda([R_2, \widehat{R}])$, *for all* $\lambda \ge \lambda_2$. *Where* $b_\lambda = \max\{r \in (R, \widehat{R}) : u_\lambda(r) = \frac{\beta}{\bar{c}}\}$.

Proof. This lemma will be proved if we show that

$$\lim_{\lambda\to+\infty} b_\lambda = \widehat{R} \tag{2.5}$$

To do this, we multiply the equation in (1.2) by r^{N-1}, integrate it from b_λ to $\widehat{R}$ and use that $u_\lambda(r) < \frac{\beta}{\bar{c}}$, for all $r \in (b_\lambda, \widehat{R}]$, to deduce that

$$\int_{b_\lambda}^{\widehat{R}} (r^{N-1} u'_\lambda(r))' dr \ge \int_{b_\lambda}^{\widehat{R}} \lambda r^{N-1} K dr$$

where $K = -\max\{f(\zeta) : \zeta \in [0, \frac{\beta}{\bar{c}}]\} > 0$. Hence

$$\widehat{R}^{N-1} u'_\lambda(\widehat{R}) - b_\lambda^{N-1} u'_\lambda(b_\lambda) \ge \frac{\lambda}{N} K(\widehat{R}^N - b_\lambda^N) > 0\,. \tag{2.6}$$

On the other hand, multiplying the same equation by $r^{2(N-1)} u'_\lambda(r)$ and integrating from b_λ to $\widehat{R}$, we have

$$-\int_{b_\lambda}^{\widehat{R}} [r^{N-1} u'_\lambda(r)]' u'_\lambda(r) r^{N-1} dr = \lambda \int_{b_\lambda}^{\widehat{R}} [F(u_\lambda(r))]' r^{2(N-1)} dr$$

Computing the two integrals by parts, we obtain

$$\begin{aligned}&\frac{1}{2}[b_\lambda^{2(N-1)}u_\lambda'(b_\lambda)^2-\widehat{R}^{2(N-1)}u_\lambda'(\widehat{R})^2]\\&=-\lambda b_\lambda^{2(N-1)}F(\frac{\beta}{\overline{c}})-2(N-1)\lambda\int_{b_\lambda}^{\widehat{R}}F(u_\lambda(r))r^{2N-3}dr\end{aligned}$$

Since $u_\lambda(r)<\frac{\beta}{\overline{c}}$, for all $r\in(b_\lambda,\widehat{R}]$ and F is decreasing in $(0,\beta)$ by (H2), we deduce that

$$\begin{aligned}&\frac{1}{2}[b_\lambda^{2(N-1)}u_\lambda'(b_\lambda)^2-\widehat{R}^{2(N-1)}u_\lambda'(\widehat{R})^2]\\&\le-\lambda b_\lambda^{2(N-1)}F(\frac{\beta}{\overline{c}})-2(N-1)F(\frac{\beta}{\overline{c}})\lambda\int_{b_\lambda}^{\widehat{R}}r^{2N-3}dr\\&=-\lambda\widehat{R}^{2(N-1)}F(\frac{\beta}{\overline{c}})\end{aligned}$$

By (2.6), the left hand of the precedent inequality is positive (because $u_\lambda'(b_\lambda)\le 0$ by definition of b_λ and $u_\lambda'(\widehat{R})\le 0$ by [4]). Consequently we can take square roots and using that $A-B\le\sqrt{A^2-B^2}$ for all $A\ge B\ge 0$, we obtain (by (2.6) again)

$$\frac{1}{N\sqrt{2}}K\frac{1}{\sqrt{-F(\frac{\beta}{\overline{c}})}}\sqrt{\lambda}(\widehat{R}^N-b_\lambda^N)\le\widehat{R}^{N-1}$$

and as a consequence (2.5) is satisfied. So the proof is complete. □

Proof of Theorem 2.1. Let $c\in(\beta,\theta)$, $\overline{c}>1$ and R_1, $R_2\in(R_0,\widehat{R})$ such that $R_1<R_2$. Consider λ_1,λ_2 given respectively by lemmas 2.3 and 2.4, and choose $\lambda^*\ge\max\{\lambda_1,\lambda_2\}$ such that

$$\lambda^*L+\frac{\mu^2}{2}<0,$$

where

$$L=\max\{F(\zeta):\frac{\beta}{\overline{c}}\le\zeta\le c\}.$$

Hence (1.2) has no positive solutions for $\lambda\ge\lambda^*$. Otherwise, there exists $\lambda\ge\lambda^*$ such that (1.2) has at least one positive solution u_λ.

Since $\lambda\ge\lambda_i$, $i=1,2$ we deduce from lemmas 2.3, 2.4 the existence of $t_1\in(R_0,R_1]$ and $t_2\in[R_2,\widehat{R}]$ satisfying $u_\lambda(t_1)<c$ and $u_\lambda(t_2)=\frac{\beta}{\overline{c}}$. Then by the mean value theorem there exists $t_3\in[t_1,t_2]$ such that

$$|u_\lambda'(t_3)|=\frac{|u_\lambda(t_2)-u_\lambda(t_1)|}{t_2-t_1}\le\mu,$$

where $\mu=(\frac{\beta}{\overline{c}}+c)/(R_2-R_1)$.

Consider the energy function $E(r)=\lambda F(u_\lambda(r))+\frac{u_\lambda'(r)^2}{2}$. Then for all $\lambda\ge\lambda^*$,

$$E(t_3)\le\lambda L+\frac{\mu^2}{2}\le\lambda^*L+\frac{\mu^2}{2}<0$$

(because $L<0$ and $u_\lambda(t_3)\in[\frac{\beta}{\overline{c}},c]$). This is a contradiction, since E is a non-increasing function (recall that $E'(r)=-\frac{N-1}{r}u'(r)^2\le 0$) and $E(\widehat{R})=\frac{u'(\widehat{R})^2}{2}\ge 0$. Hence the result follows. □

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