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CONTROLLABILITY OF NEUTRAL IMPULSIVE STOCHASTIC QUASILINEAR INTEGRODIFFERENTIAL SYSTEMS WITH NONLOCAL CONDITIONS

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Abstract. We establish sufficient conditions for controllability of neutral impulsive stochastic quasilinear integrodifferential systems with nonlocal conditions in Hilbert spaces. The results are obtained by using semigroup theory, evolution operator and a fixed point technique. An example is provided to illustrate the obtained results.

1. Introduction

Abstract differential systems in infinite-dimensional spaces appear in many branches of science and engineering, such as heat flow in materials with memory, viscoelasticity and other physical phenomena. In these fields many stochastic differential equations are obtained by including random fluctuations in ordinary differential equations which have been deduced from phenomological or physical laws. Quasilinear evolution equations forms a very important class of evolution equations as many time dependent phenomena in physics, chemistry and biology can be represented by such evolution equations. Some examples of quasi-stochastic systems are the system of price fluctuations in financial markets, earth climate or the seismic activity of the earth crust and a dice game. Of particular interest the following integrodifferential equation arises in the theory of one-dimensional viscoelasticity [18, 30] and also a special model for one-dimensional heat flow in materials with memory.

$$\begin{gathered} u_t(t,x) = \int_0^t k(t-s)(\sigma(u_x))_x(s,x)ds + f(t,x), \quad t \geq 0,\ x \in (0,1), \\ u(0,x) = u_0(x), \quad x \in [0,1], \quad u(t,0) = u(t,1) = 0,\ t > 0. \end{gathered} \tag{1.1}$$

In many of the papers, the mathematical model for certain problems in nonlinear viscoelasticity is discussed in the form

$$\begin{gathered} u_{tt}(t,x) = \phi(u_x(t,x))_x + \int_0^t a(t-s)\psi(u_x(s,x))_x ds + g(t,x), \quad t \geq 0, \\ u(0,x) = u_0(x), \quad x \in \mathbb{R}. \end{gathered} \tag{1.2}$$

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which is the same as (1.1) if $\phi = \psi = \sigma$, $k(0) = 1$ and $a = k'$ (see [13]). In [14], the following equation occurred during the study of the nonlinear behavior of elastic strings [21].

$$\begin{gathered}
u_{tt}(t,x) + c(t)u_t(t,x) - M\Big(\int_{-\infty}^{\infty} |u_x(t,s)|^2 ds\Big) u_{xx}(t,x) + u(t,x) = h(t,x,u(t,x)), \\
0 \le t < \infty, \\
u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), \quad x \in \mathbb{R}.
\end{gathered} \tag{1.3}$$

The above equations take the abstract form as

$$\frac{du(t)}{dt} = A(u)u(t) + f(t,u(t)), \quad u(0) = u_0. \tag{1.4}$$

where A is a linear operator in a Hilbert space H and f is a real function. Hence the natural generalization of (1.4) is the following quasilinear integrodifferential equation

$$\begin{gathered}
u'(t) = A(t,u)u(t) + f(t,u(t)) + \int_0^t g(t,s,u(s))ds, \\
u(0) = u_0.
\end{gathered} \tag{1.5}$$

Systems with short-term perturbations are often naturally described by impulsive differential equations. The theory of impulsive differential equations is much richer than the corresponding theory of differential equations without impulse effects [19, 27]. For instance, impulsive interruptions are observed in mechanics, radio engineering, communication security, control theory, optimal control, biology, mechanics, medicine, bio-technologies, electronics, neural networks and economics. The introduction of non-local conditions can improve the qualitative and quantitative characteristics of the problem which lead to good results concerning existence, uniqueness [8] and regularity of the solution. Problems related to non local conditions have applications such as in the theory of heat conduction, thermoelasticity, plasma physics, control theory etc. Many real systems are quite sensitive to sudden changes. This fact may suggest that proper mathematical models of systems should consist of some neutral equations. Indeed, we may find that neutral term effects can be quite significant in real mathematical models. The neutral equations find numerous applications in applied mathematics, natural sciences, biological and physical systems. For this reason these type of equations have received much attention in recent years.

Several authors have studied the existence of solutions of abstract quasilinear evolution equations in Banach spaces [1, 2, 4, 9, 12, 16, 22, 23]. Park et al. [25], Balachandran and Paul Samuel [3] studied the regularity of solutions and the existence of solutions of quasilinear delay integrodifferential equations respectively. Controllability of quasilinear systems has gained renewed interests and few papers appeared [5, 6, 7]. The controllability of nonlinear stochastic systems in finite and infinite-dimensional spaces have been extensively studied by many authors [11, 17, 20]. Park et al. [24] discussed the controllability of neutral stochastic functional integrodifferential infinite delay systems in abstract spaces. Karthikeyan and Balachandran [15] studied the controllability of nonlinear stochastic neutral impulsive systems. Subalakshmi and Balachandran [28, 29] investigated the approximate controllability of neutral and impulsive stochastic integrodifferential systems in Hilbert spaces.

Moreover, the controllability of neutral impulsive stochastic quasilinear integrodifferential systems is an untreated topic in the literature so far. Motivated by this fact, in this paper we study the controllability of neutral impulsive stochastic quasilinear integrodifferential systems with nonlocal conditions. For that, we impose neutral, impulse and nonlocal condition with random perturbations in (1.5) which gives the form

$$\begin{aligned} &d\big[x(t)-q(t,x(t))\big]\\ &= \Big[A(t,x)x(t)+Bu(t)+f(t,x(t))+\int_0^t g(t,s,x(s))ds\Big]dt\\ &\quad +\sigma(t,x(t))dw(t),\quad t\in J:=[0,a],\quad t\neq t_k,\\ &\Delta x(t_k)=x(t_k^+)-x(t_k^-)=I_k(x(t_k^-)),\quad k=1,2,\dots,m,\\ &\qquad x(0)+h(x)=x_0. \end{aligned}\tag{1.6}$$

Here, the state variable $x(\cdot)$ takes values in a real separable Hilbert space H with inner product $(\cdot,\cdot)$ and norm $\|\cdot\|$ and the control function $u(\cdot)$ takes values in $L^2(J,U)$, a Banach space of admissible control functions for a separable Hilbert space U. Also, $A(t,x)$ is the infinitesimal generator of a C_0-semigroup in H and B is a bounded linear operator from U into H. Let K be another separable Hilbert space with inner product $(\cdot,\cdot)_K$ and the norm $\|\cdot\|_K$. We employ the same notation $\|\cdot\|$ for the norm $\mathcal{L}(K,H)$, where $\mathcal{L}(K,H)$ denotes the space of all bounded linear operators from K into H. Further, $q:J\times H\to H$, $f:J\times H\to H$, $g:\Lambda\times H\to H$, $\sigma:J\times H\to\mathcal{L}_Q(K,H)$ are measurable mappings in H-norm and $\mathcal{L}_Q(K,H)$ norm respectively, where $\mathcal{L}_Q(K,H)$ denotes the space of all Q-Hilbert-Schmidt operators from K into H which will be defined in Section 2 and $\Lambda=\{(t,s)\in J\times J: s\le t\}$. Here, the nonlocal function $h:\mathcal{PC}[J:H]\to H$ and impulsive function $I_k\in C(H,H)$ $(k=1,2,\dots,m)$ are bounded functions. Furthermore, the fixed times t_k satisfies $0=t_0<t_1<t_2<\dots<t_m<a$, $x(t_k^+)$ and $x(t_k^-)$ denote the right and left limits of $x(t)$ at $t=t_k$. And $\Delta x(t_k)=x(t_k^+)-x(t_k^-)$ represents the jump in the state x at time t_k, where I_k determines the size of the jump.

2. Preliminaries

Let $(\Omega,\mathcal{F},P;\mathbf{F})\{\mathbf{F}=\{\mathcal{F}_t\}_{t\ge0}\}$ be a complete filtered probability space satisfying that $\mathcal{F}_0$ contains all P-null sets of $\mathcal{F}$. An H-valued random variable is an $\mathcal{F}$-measurable function $x(t):\Omega\to H$ and the collection of random variables $S=\{x(t,\omega):\Omega\to H\setminus t\in J\}$ is called a stochastic process. Generally, we just write $x(t)$ instead of $x(t,\omega)$ and $x(t):J\to H$ in the space of S. Let $\{e_i\}_{i=1}^\infty$ be a complete orthonormal basis of K. Suppose that $\{w(t):t\ge0\}$ is a cylindrical K-valued wiener process with a finite trace nuclear covariance operator $Q\ge0$, denote $\mathrm{Tr}(Q)=\sum_{i=1}^\infty\lambda_i=\lambda<\infty$, which satisfies that $Qe_i=\lambda_ie_i$. So, actually, $\omega(t)=\sum_{i=1}^\infty\sqrt{\lambda_i}\omega_i(t)e_i$, where $\{\omega_i(t)\}_{i=1}^\infty$ are mutually independent one-dimensional standard Wiener processes. We assume that $\mathcal{F}_t=\sigma\{\omega(s):0\le s\le t\}$ is the σ-algebra generated by ω and $\mathcal{F}_a=\mathcal{F}$. Let $\Psi\in\mathcal{L}(K,H)$ and define

$$\|\Psi\|_Q^2=\mathrm{Tr}(\Psi Q\Psi^*)=\sum_{n=1}^\infty\|\sqrt{\lambda_n}\Psi e_n\|^2.$$

If $\|\Psi\|_Q<\infty$, then Ψ is called a Q-Hilbert-Schmidt operator. Let $\mathcal{L}_Q(K,H)$ denote the space of all Q-Hilbert-Schmidt operators $\Psi:K\to H$. The completion

$\mathcal{L}_Q(K,H)$ of $\mathcal{L}(K,H)$ with respect to the topology induced by the norm $\|\cdot\|_Q$ where $\|\Psi\|_Q^2 = \langle \Psi, \Psi\rangle$ is a Hilbert space with the above norm topology. For more details in this section refer [10]. $L_2^{\mathcal{F}}(J,H)$ is the space of all $\mathcal{F}_t$ - adapted, H-valued measurable square integrable processes on $J\times\Omega$. Denote $J_0=[0,t_1], J_k=(t_k,t_{k+1}], k=1,2,\dots,m$, and define the following class of functions:

$$
\begin{aligned}
&\mathcal{PC}(J, L_2(\Omega,\mathcal{F},P;H))\\
&= \Big\{x : J\to L_2 : x(t) \text{ is continuous everywhere except for some } t_k \text{ at which}\\
&\quad x(t_k^-) \text{ and } x(t_k^+) \text{ exists and } x(t_k^-)=x(t_k),\ k=1,2,3,\dots,m\Big\}
\end{aligned}
$$

is the Banach space of piecewise continuous maps from J into $L_2(\Omega,\mathcal{F},P;H)$ satisfying the condition $\sup_{t\in J} E\|x(t)\|^2 < \infty$. Let $\mathcal{Z}\equiv\mathcal{PC}(J,L_2)$ be the closed subspace of $\mathcal{PC}(J,L_2(\Omega,\mathcal{F},P;H))$ consisting of measurable, $\mathcal{F}_t$ - adapted and H-valued processes $x(t)$. Then $\mathcal{PC}(J,L_2)$ is a Banach space endowed with the norm

$$
\|x\|_{\mathcal{PC}}^2 = \sup_{t\in J} \quad \big\{E\|x(t)\|^2 : x\in\mathcal{PC}(J,L_2)\big\}.
$$

Let H and Y be two Hilbert spaces such that Y is densely and continuously embedded in H. For any Hilbert space $\mathcal{Z}$ the norm of $\mathcal{Z}$ is denoted by $\|\cdot\|_{\mathcal{PC}}$ or $\|\cdot\|$. The space of all bounded linear operators from H to Y is denoted by $B(H,Y)$ and $B(H,H)$ is written as $B(H)$. We recall some definitions and known facts from Pazy [26].

Definition 2.1. Let S be a linear operator in H and let Y be a subspace of H. The operator $\tilde S$ defined by $D(\tilde S)=\{x\in D(S)\cap Y : Sx\in Y\}$ and $\tilde Sx = Sx$ for $x\in D(\tilde S)$ is called the part of S in Y.

Definition 2.2. Let Q be a subset of H and for every $0\le t\le a$ and $b\in Q$, let $A(t,b)$ be the infinitesimal generator of a C_0 semigroup $S_{t,b}(s), s\ge 0$ on H. The family of operators $\{A(t,b)\}$, $(t,b)\in J\times Q$, is stable if there are constants $M\ge 1$ and ω such that

$$
\rho(A(t,b))\supset(\omega,\infty) \quad \text{for } (t,b)\in J\times Q,
$$

$$
\|\prod_{j=1}^{k} R(\lambda : A(t_j,b_j))\| \le M(\lambda-\omega)^{-k} \quad \text{for } \lambda>\omega
$$

and every finite sequence $0\le t_1\le t_2\le\dots\le t_k\le a$, $b_j\in Q, 1\le j\le k$. The stability of $\{A(t,b)\}$, $(t,b)\in J\times Q$, implies [26] that

$$
\|\prod_{j=1}^{k} S_{t_j,b_j}(s_j)\| \le M\exp\{\omega\sum_{j=1}^{k} s_j\} \quad \text{for } s_j\ge 0
$$

and any finite sequences $0\le t_1\le t_2\le\dots\le t_k\le a$, $b_j\in Q, 1\le j\le k$. $k=1,2,\dots$.

Definition 2.3. Let $S_{t,b}(s)$, $s\ge 0$ be the C_0 semigroup generated by $A(t,b), (t,b)\in J\times Q$. A subspace Y of H is called $A(t,b)$-admissible if Y is invariant subspace of $S_{t,b}(s)$ and the restriction of $S_{t,b}(s)$ to Y is a C_0-semigroup in Y.

Let $Q\subset H$ be a subset of H such that for every $(t,b)\in J\times Q$, $A(t,b)$ is the infinitesimal generator of a C_0-semigroup $S_{t,b}(s), s\ge 0$ on H. We make the following assumptions:

(E1) The family $\{A(t,b)\}, (t,b) \in J \times Q$ is stable.
(E2) Y is $A(t,b)$- admissible for $(t,b) \in J \times Q$ and the family $\{\tilde{A}(t,b)\}, (t,b) \in J \times Q$ of parts $\tilde{A}(t,b)$ of $A(t,b)$ in Y, is stable in Y.
(E3) For $(t,b) \in J \times Q$, $D(A(t,b)) \supset Y$, $A(t,b)$ is a bounded linear operator from Y to H and $t \to A(t,b)$ is continuous in the $B(Y,H)$ norm $\|\cdot\|$ for every $b \in Q$.
(E4) There is a constant $L > 0$ such that

$$\|A(t,b_1) - A(t,b_2)\|_{Y \to H} \le L\|b_1 - b_2\|_H$$

holds for every $b_1, b_2 \in Q$ and $0 \le t \le a$.

Let Q be a subset of H and let $\{A(t,b)\}$, $(t,b) \in J \times Q$ be a family of operators satisfying the conditions $(E1) - (E4)$. If $x \in \mathcal{PC}(J, L_2)$ has values in Q then there is a unique evolution system $U(t,s;x), 0 \le s \le t \le a$ in H satisfying (see [26])

(i) $\|U(t,s;x)\| \le Me^{\omega(t-s)}$ for $0 \le s \le t \le a$, where M and ω are stability constants.
(ii) $\frac{\partial^+}{\partial t} U(t,s;x)y = A(s,x(s))U(t,s;x)y$ for $y \in Y$, $0 \le s \le t \le a$.
(iii) $\frac{\partial}{\partial s} U(t,s;x)y = -U(t,s;x)A(s,x(s))y$ for $y \in Y$, $0 \le s \le t \le a$.

Further we assume that

(E5) For every $x \in \mathcal{PC}(J, L_2)$ satisfying $x(t) \in Q$ for $0 \le t \le a$, we have

$$U(t,s;x)Y \subset Y, \quad 0 \le s \le t \le a$$

and $U(t,s;x)$ is strongly continuous in Y for $0 \le s \le t \le a$.
(E6) Closed bounded convex subsets of Y are closed in H.
(E7) For every $(t,b) \in J \times Q$, $q(t,b) \in Y$ and $f(t,b) \in Y$, $((t,s),b) \in \Lambda \times Q, g(t,s,b) \in Y$ and $(t,b) \in J \times Q$, $\sigma(t,b) \in Y$.

Definition 2.4 ([11])**.** A stochastic process x is said to be a mild solution of (1.6) if the following conditions are satisfied:

(a) $x(t,\omega)$ is a measurable function from $J \times \Omega$ to H and $x(t)$ is $\mathcal{F}_t$-adapted,
(b) $E\|x(t)\|^2 < \infty$ for each $t \in J$,
(c) $\Delta x(t_k) = x(t_k^+) - x(t_k^-) = I_k(x(t_k^-))$, $k = 1, 2, \dots, m$,
(d) For each $u \in L_2^{\mathcal{F}}(J,U)$, the process x satisfies the following integral equation

$$\begin{aligned} x(t) &= U(t,0;x)\big[x_0 - h(x) - q(0,x(0))\big] + q(t,x(t)) \\ &\quad + \int_0^t U(t,s;x)A(s,x(s))q(s,x(s))ds \\ &\quad + \int_0^t U(t,s;x)\big[Bu(s) + f(s,x(s))\big]ds \\ &\quad + \int_0^t U(t,s;x)\Big[\int_0^s g\big(s,\tau,x(\tau)d\tau\big)\Big]ds + \int_0^t U(t,s;x)\sigma(s,x(s))dw(s) \\ &\quad + \sum_{0<t_k<t} U(t,t_k;x)I_k(x(t_k^-)), \quad \text{for a.e. } t \in J, \\ &\qquad x(0) + h(x) = x_0 \in H. \end{aligned} \tag{2.1}$$

Definition 2.5. System (1.6) is said to be controllable on the interval J, if for every initial condition x_0 and $x_1 \in H$, there exists a control $u \in L^2(J,U)$ such that the solution $x(\cdot)$ of (1.6) satisfies $x(a) = x_1$.

Further there exists a constant $\mathcal{N} > 0$ such that for every $x, y \in \mathcal{PC}(J, L_2)$ and every $\tilde{y} \in Y$ we have

$$\|U(t,s;x)\tilde{y} - U(t,s;y)\tilde{y}\|^2 \le \mathcal{N} a^2 \|\tilde{y}\|_Y^2 \|x-y\|_{\mathcal{PC}}^2.$$

To establish our controllability result we assume the following hypotheses:

(H1) $A(t,x)$ generates a family of evolution operators $U(t,s;x)$ in H and there exists a constant $\mathcal{C}_U > 0$ such that

$$\|U(t,s;x)\|^2 \le \mathcal{C}_U \quad \text{for } 0 \le s \le t \le a,\ x \in \mathcal{Z}.$$

(H2) The linear operator $W : L^2(J,U) \to H$ defined by

$$Wu = \int_0^a U(a,s;x)Bu(s)ds$$

is invertible with inverse operator W^{-1} taking values in $L^2(J,U) \setminus \ker W$ and there exists a positive constant $\mathcal{C}_W$ such that

$$\|BW^{-1}\|^2 \le \mathcal{C}_W.$$

(H3) (i) The function $q : J \times \mathcal{Z} \to \mathcal{Z}$ is continuous and there exist constants $\mathcal{C}_q > 0$, $\tilde{\mathcal{C}_q} > 0$ for $s, t \in J$ and $x, y \in \mathcal{Z}$ such that the function $A(t,x)q$ satisfies the Lipschitz condition:

$$E\|A(t,x(t))q(t,x) - A(t,y(t))q(t,y)\|^2 \le C_q\|x-y\|^2,$$

and $\tilde{\mathcal{C}_q} = \sup_{t\in J} \|A(t,0)q(t,0)\|^2$.

(ii) There exist constants $\mathcal{C}_k > 0, \mathcal{C}_1 > 0$ and $\mathcal{C}_2 > 0$ such that

$$E\|q(t,x) - q(t,y)\|^2 \le \mathcal{C}_k[|t-s|^2 + \|x-y\|^2],$$
$$E\|q(t,x)\|^2 \le \mathcal{C}_1\|x\|^2 + \mathcal{C}_2,$$

where $\mathcal{C}_2 = \sup_{t\in J} \|q(t,0)\|^2$.

(H4) The nonlinear function $f : J \times \mathcal{Z} \to \mathcal{Z}$ is continuous and there exist constants $\mathcal{C}_f > 0$, $\tilde{\mathcal{C}_f} > 0$ for $t \in J$ and $x, y \in \mathcal{Z}$ such that

$$E\|f(t,x) - f(t,y)\|^2 \le \mathcal{C}_f\|x-y\|^2$$

and $\tilde{\mathcal{C}_f} = \sup_{t\in J} \|f(t,0)\|^2$.

(H5) The nonlinear function $g : \Lambda \times \mathcal{Z} \to \mathcal{Z}$ is continuous and there exist positive constants $\mathcal{C}_g$, $\tilde{\mathcal{C}_g}$, for $x, y \in \mathcal{Z}$ and $(t,s) \in \Lambda$ such that

$$E\Big\|g(t,s,x) - g(t,s,y)\Big\|^2 \le \mathcal{C}_g\|x-y\|^2$$

and $\tilde{\mathcal{C}_g} = \sup_{(t,s)\in\Lambda} \|g(t,s,0)\|^2$.

(H6) The function $\sigma : J \times \mathcal{Z} \to \mathcal{L}_Q(K,H)$ is continuous and there exist constants $\mathcal{C}_\sigma > 0$, $\tilde{\mathcal{C}_\sigma} > 0$ for $t \in J$ and $x, y \in \mathcal{Z}$ such that

$$E\|\sigma(t,x) - \sigma(t,y)\|_Q^2 \le \mathcal{C}_\sigma\|x-y\|^2$$

and $\tilde{\mathcal{C}_\sigma} = \sup_{t\in J} \|\sigma(t,0)\|^2$.

(H7) The nonlocal function $h : \mathcal{PC}(J : \mathcal{Z}) \to \mathcal{Z}$ is continuous and there exist constants $\mathcal{C}_h > 0$, $\tilde{\mathcal{C}_h} > 0$ for $x, y \in \mathcal{Z}$ such that

$$E\|h(x) - h(y)\|^2 \le \mathcal{C}_h\|x-y\|^2, \quad E\|h(x)\|^2 \le \tilde{\mathcal{C}_h}.$$

(H8) $I_k : \mathcal{Z} \to \mathcal{Z}$ is continuous and there exist constants $\beta_k > 0$, $\tilde{\beta_k} > 0$ for $x, y \in \mathcal{Z}$ such that
$$E\|I_k(x) - I_k(y)\|^2 \le \beta_k \|x-y\|^2, \quad k = 1, 2, \dots, m$$
and $\tilde{\beta_k} = \|I_k(0)\|^2$, $k = 1, 2, \dots, m$.

(H9) There exists a constant $r > 0$ such that
$$\begin{aligned}
&10\Big\{\mathcal{C}_U(\|x_0\|^2 + \tilde{\mathcal{C}_h}) + a^2\mathcal{C}_U\mathcal{G} + 2\mathcal{C}_U\big[\mathcal{C}_1(\|x_0\|^2 + \tilde{\mathcal{C}_h}) + \mathcal{C}_2\big] + \mathcal{C}_1 r + \mathcal{C}_2 \\
&+ 2a^2\mathcal{C}_U(\mathcal{C}_q r + \tilde{\mathcal{C}_q}) + 2a^2\mathcal{C}_U(\mathcal{C}_f r + \tilde{\mathcal{C}_f}) + 2a^3\mathcal{C}_U\big[\mathcal{C}_g r + \tilde{\mathcal{C}_g}\big] \\
&+ 2a\ \mathcal{C}_U\ \operatorname{Tr}(Q)\big(\mathcal{C}_\sigma r + \tilde{\mathcal{C}_\sigma}\big) + 2m\mathcal{C}_U\Big[\sum_{k=1}^m \beta_k r + \sum_{k=1}^m \tilde{\beta_k}\Big]\Big\} \\
&\le r
\end{aligned}$$
and
$$\nu = 10\Big\{(1 + 18a^2\mathcal{C}_U\mathcal{C}_W)(N_1 + N_2 + N_3 + N_4 + N_5 + N_6 + N_7) + 2a^3\mathcal{N}\mathcal{G}\Big\}$$
where
$$\begin{gathered}
N_1 = \mathcal{N}a^2\|x_0\|^2 + 2(\mathcal{N}a^2\tilde{\mathcal{C}_h} + \mathcal{C}_U\mathcal{C}_h), \\
N_2 = 2\Big[2\mathcal{N}a^2\big(\mathcal{C}_1(\|x_0\|^2 + \tilde{\mathcal{C}_h}) + \mathcal{C}_2\big) + \mathcal{C}_U\mathcal{C}_k\mathcal{C}_h\Big] + \mathcal{C}_q, \\
N_3 = 2a^2\Big[2\mathcal{N}a\big(\mathcal{C}_q r + \tilde{\mathcal{C}_q}\big) + \mathcal{C}_U\mathcal{C}_q\Big], \\
N_4 = 2a^2\Big[2\mathcal{N}a\big(\mathcal{C}_f r + \tilde{\mathcal{C}_f}\big) + \mathcal{C}_U\mathcal{C}_f\Big], \\
N_5 = 2a^3\Big[2\mathcal{N}a\big(\mathcal{C}_g r + \tilde{\mathcal{C}_g}\big) + \mathcal{C}_U\mathcal{C}_g\Big], \\
N_6 = 2a\Big[2\mathcal{N}a\operatorname{Tr}(Q)\big(\mathcal{C}_\sigma r + \tilde{\mathcal{C}_\sigma}\big) + \mathcal{C}_U\operatorname{Tr}(Q)\mathcal{C}_\sigma\Big], \\
N_7 = 2m\Big[2\mathcal{N}a^2\Big(\sum_{k=1}^m \beta_k r + \sum_{k=1}^m \tilde{\beta_k}\Big) + \mathcal{C}_U\sum_{k=1}^m \beta_k\Big].
\end{gathered}$$

3. Controllability Result

Theorem 3.1. *If the conditions* (H1)-(H9) *are satisfied and if* $0 \le \nu < 1$, *then system* (1.6) *is controllable on* J.

Proof. Using (H2) for an arbitrary function $x(\cdot)$, define the control
$$\begin{aligned}
u(t) = W^{-1}\Big[&x_1 - U(a,0;x)\big[x_0 - h(x) - q(0,x(0))\big] - q(a,x(a)) \\
&- \int_0^a U(a,s;x)A(s,x(s))q(s,x(s))ds - \int_0^a U(a,s;x)\sigma(s,x(s))dw(s) \\
&- \int_0^a U(a,s;x)\Big[f(s,x(s)) + \int_0^s g(s,\tau,x(\tau))d\tau\Big]ds \\
&- \sum_{0<t_k<a} U(a,t_k;x)I_k(x(t_k^-))\Big](t).
\end{aligned} \tag{3.1}$$
Let $\mathcal{Y}_r$ be a nonempty closed subset of $\mathcal{PC}(J, L_2)$ defined by
$$\mathcal{Y}_r = \{x : x \in \mathcal{PC}(J, L_2) | E\|x(t)\|^2 \le r\}.$$

Consider a mapping $\Phi : \mathcal{Y}_r \to \mathcal{Y}_r$ defined by

$$
\begin{aligned}
&(\Phi x)(t) \\
&= U(t,0;x)\big[x_0 - h(x) - q(0,x(0))\big] + q(t,x(t)) \\
&\quad + \int_0^t U(t,s;x)A(s,x(s))q(s,x(s))ds \\
&\quad + \int_0^t U(t,s;x)BW^{-1}\Big[x_1 - U(a,0;x)\big[x_0 - h(x) - q(0,x(0))\big] - q(a,x(a)) \\
&\quad - \int_0^a U(a,s;x)A(s,x(s))q(s,x(s))ds - \int_0^a U(a,s;x)\sigma(s,x(s))dw(s) \\
&\quad - \int_0^a U(a,s;x)\Big[f(s,x(s)) + \int_0^s g(s,\tau,x(\tau))d\tau\Big]ds \\
&\quad - \sum_{0<t_k<a} U(a,t_k;x)I_k(x(t_k^-))\Big](s)ds + \int_0^t U(t,s;x)f(s,x(s))ds \\
&\quad + \int_0^t U(t,s;x)\Big[\int_0^s g(s,\tau,x(\tau))d\tau\Big]ds + \int_0^t U(t,s;x)\sigma(s,x(s))dw(s) \\
&\quad + \sum_{0<t_k<t} U(t,t_k;x)I_k(x(t_k^-)).
\end{aligned}
$$

We have to show that by using the above control the operator Φ has a fixed point. Since all the functions involved in the operator are continuous therefore Φ is continuous. For convenience let us take

$$
\begin{aligned}
V(\mu,x) = BW^{-1}\Big[&x_1 - U(a,0;x)\big[x_0 - h(x) - q(0,x(0))\big] - q(a,x(a)) \\
&- \int_0^a U(a,s;x)A(s,x(s))q(s,x(s))ds - \int_0^a U(a,s;x)\sigma(s,x(s))dw(s) \\
&- \int_0^a U(a,s;x)\Big[f(s,x(s)) + \int_0^s g(s,\tau,x(\tau))d\tau\Big]ds \\
&- \sum_{0<t_k<a} U(a,t_k;x)I_k(x(t_k^-))\Big](\mu).
\end{aligned}
$$

From our assumptions we have

$$
\begin{aligned}
E\|V(\mu,x)\|^2 \le 10\mathcal{C}_W\Big\{&\|x_1\|^2 + \mathcal{C}_U(\|x_0\|^2 + \tilde{\mathcal{C}_h}) + 2\mathcal{C}_U\big[\mathcal{C}_1(\|x_0\|^2 + \tilde{\mathcal{C}_h}) + \mathcal{C}_2\big] + \mathcal{C}_1 r \\
&+ \mathcal{C}_2 + 2a^2\mathcal{C}_U(\mathcal{C}_q r + \tilde{\mathcal{C}_q}) + 2a^2\mathcal{C}_U(\mathcal{C}_f r + \tilde{\mathcal{C}_f}) + 2a^3\mathcal{C}_U\big[\mathcal{C}_g r + \tilde{\mathcal{C}_g}\big] \\
&+ 2a\ \mathcal{C}_U\ \ \mathrm{Tr}(Q)\big(\mathcal{C}_\sigma r + \tilde{\mathcal{C}_\sigma}\big) + 2m\mathcal{C}_U\Big[\sum_{k=1}^m \beta_k r + \sum_{k=1}^m \tilde{\beta_k}\Big]\Big\} := \mathcal{G}.
\end{aligned}
$$

and

$$
\begin{aligned}
&E\|V(\mu,x) - V(\mu,y)\|^2 \\
&\le 9\mathcal{C}_W\Big\{\mathcal{N}a^2\|x_0\|^2 + 2(\mathcal{N}a^2\tilde{\mathcal{C}_h} + \mathcal{C}_U\mathcal{C}_h) + 2\Big[2\mathcal{N}a^2\big(\mathcal{C}_1(\|x_0\|^2 + \tilde{\mathcal{C}_h}) \\
&\quad + \mathcal{C}_2\big) + \mathcal{C}_U\mathcal{C}_k\mathcal{C}_h\Big] + \mathcal{C}_q + 2a^2\Big[2\mathcal{N}a\big(\mathcal{C}_q r + \tilde{\mathcal{C}_q}\big) + \mathcal{C}_U\mathcal{C}_q\Big] \\
&\quad + 2a^2\Big[2\mathcal{N}a\big(\mathcal{C}_f r + \tilde{\mathcal{C}_f}\big) + \mathcal{C}_U\mathcal{C}_f\Big] + 2a^3\Big[2\mathcal{N}a\big(\mathcal{C}_g r + \tilde{\mathcal{C}_g}\big) + \mathcal{C}_U\mathcal{C}_g\Big]
\end{aligned}
$$

$$
\begin{aligned}
&+ 2a\Big[2\mathcal{N}a\ \operatorname{Tr}(Q)\big(\mathcal{C}_\sigma r+\tilde{\mathcal{C}_\sigma}\big)+\mathcal{C}_U\ \operatorname{Tr}(Q)\mathcal{C}_\sigma\Big]\\
&+ 2m\Big[2\mathcal{N}a^2\Big(\sum_{k=1}^m\beta_k r+\sum_{k=1}^m\tilde{\beta_k}\Big)+\mathcal{C}_U\sum_{k=1}^m\beta_k\Big]\Big\}.
\end{aligned}
$$

First we show that the operator Φ maps $\mathcal{Y}_r$ into itself. Now

$$
\begin{aligned}
&E\|(\Phi x)(t)\|^2\\
&\le 10\Big\{E\big\|U(t,0;x)\big[x_0-h(x)-q(0,x(0))\big]\big\|^2+E\|q(t,x(t))\|^2\\
&\quad+E\|\int_0^t U(t,s;x)A(s,x(s))q(s,x(s))ds\|^2+E\big\|\int_0^t U(t,\mu;x)V(\mu,x)d\mu\big\|^2\\
&\quad+E\big\|\int_0^t U(t,s;x)\Big[f(s,x(s))+\int_0^s g\big(s,\tau,x(\tau)\big)d\tau\Big]ds\big\|^2\\
&\quad+E\big\|\int_0^t U(t,s;x)\sigma(s,x(s))dw(s)\big\|^2+E\big\|\sum_{0<t_k<t}U(t,t_k;x)I_k(x(t_k^-))\big\|^2\Big\}\\
&\le 10\Big\{\mathcal{C}_U(\|x_0\|^2+\tilde{\mathcal{C}_h})+2\mathcal{C}_U\big[\mathcal{C}_1(\|x_0\|^2+\tilde{\mathcal{C}_h})+\mathcal{C}_2\big]+\mathcal{C}_1 r+\mathcal{C}_2\\
&\quad+2a^2\mathcal{C}_U(\mathcal{C}_q r+\tilde{\mathcal{C}_q})+a^2\mathcal{C}_U\mathcal{G}+2a^2\mathcal{C}_U(\mathcal{C}_f r+\tilde{\mathcal{C}_f})+2a^3\mathcal{C}_U\big[\mathcal{C}_g r+\tilde{\mathcal{C}_g}\big]\\
&\quad+2a\ \mathcal{C}_U\ \operatorname{Tr}(Q)\big(\mathcal{C}_\sigma r+\tilde{\mathcal{C}_\sigma}\big)+2m\mathcal{C}_U\Big[\sum_{k=1}^m\beta_k r+\sum_{k=1}^m\tilde{\beta_k}\Big]\Big\}\\
&\le r.
\end{aligned}
$$

From (H9) we obtain $E\|(\Phi x)(t)\|^2\le r$. Hence Φ maps $\mathcal{Y}_r$ into $\mathcal{Y}_r$. Let $x,y\in\mathcal{Y}_r$, then

$$
\begin{aligned}
&E\|(\Phi x)(t)-(\Phi y)(t)\|^2\\
&\le 10\Big\{E\big\|U(t,0;x)\big[x_0-h(x)-q(0,x(0))\big]-U(t,0;y)\big[x_0-h(y)-q(0,y(0))\big]\big\|^2\\
&\quad+E\big\|q(t,x(t))-q(t,y(t))\big\|^2+E\big\|\int_0^t\Big[U(t,s;x)A(s,x(s))q(s,x(s))\\
&\quad-U(t,s;y)A(s,y(s))q(s,y(s))\Big]ds\big\|^2\\
&\quad+E\big\|\int_0^t\Big[U(t,\mu;x)V(\mu,x)-U(t,\mu;y)V(\mu,y)\Big]d\mu\big\|^2\\
&\quad+E\big\|\int_0^t\Big[U(t,s;x)f(s,x(s))-U(t,s;y)f(s,y(s))\Big]ds\big\|^2\\
&\quad+E\big\|\int_0^t\Big[U(t,s;x)\Big[\int_0^s g(s,\tau,x(\tau))d\tau\Big]-U(t,s;y)\Big[\int_0^s g(s,\tau,y(\tau))d\tau\Big]\Big]ds\big\|^2\\
&\quad+E\big\|\int_0^t\Big[U(t,s;x)\sigma(s,x(s))-U(t,s;y)\sigma(s,y(s))\Big]dw(s)\big\|^2\\
&\quad+E\big\|\sum_{0<t_k<t}\Big[U(t,t_k;x)I_k(x(t_k^-))-U(t,t_k;y)I_k(y(t_k^-))\Big]\big\|^2\Big\}\\
&\le 10\Big\{(1+18a^2\mathcal{C}_U\mathcal{C}_W)(N_1+N_2+N_3+N_4+N_5+N_6+N_7)+2a^3\mathcal{N}\mathcal{G}\Big\}\|x-y\|^2\\
&\le \nu\|x-y\|^2.
\end{aligned}
$$

Since $\nu < 1$, the mapping Φ is a contraction and hence by Banach fixed point theorem there exists a unique fixed point $x \in \mathcal{Y}_r$ such that $(\Phi x)(t) = x(t)$. This fixed point is then the solution of the system (1.6) and clearly, $x(a) = (\Phi x)(a) = x_1$ which implies that the system (1.6) is controllable on J. □

Remark 3.2. Consider the neutral impulsive stochastic quasilinear system

$$\begin{gathered} d\Big[x(t) - q(t, x(t))\Big] = \Big[A(t, x)\big[x(t) - q(t, x(t))\big] + Bu(t) + f(t, x(t)) \\ + \int_0^t g(t, s, x(s))ds\Big]dt + \sigma(t, x(t))dw(t), \\ t \in J := [0, a], \quad t \neq t_k, \\ \Delta x(t_k) = x(t_k^+) - x(t_k^-) = I_k(x(t_k^-)), \quad k = 1, 2, \dots, m, \\ x(0) + h(x) = x_0. \end{gathered} \tag{3.2}$$

where A, B, q, f, g, σ are as before. The solution to the above equation is

$$\begin{aligned} x(t) = U(t, 0; x)\big[x_0 - h(x) - q(0, x(0))\big] + q(t, x(t)) + \int_0^t U(t, s; x)Bu(s)ds \\ + \int_0^t U(t, s; x)\Big[f(s, x(s)) + \int_0^s g\big(s, \tau, x(\tau)d\tau\big)\Big]ds \\ + \int_0^t U(t, s; x)\sigma(s, x(s))dw(s) + \sum_{0<t_k<t} U(t, t_k; x)I_k(x(t_k^-)), \end{aligned}$$

for a.e. $t \in J$. If the functions involved in (3.2) satisfy the lipschitz condition then the suitable control function will steer the system (3.2) from x_0 to x_1 provided the above equation is satisfied.

4. Neutral Stochastic Quasilinear Integrodifferential Systems

Consider the neutral stochastic quasilinear integrodifferential system

$$\begin{gathered} d\Big[x(t) - Q\Big(t, x(t), \int_0^t q(t, s, x(s))ds\Big)\Big] \\ = \Big[A(t, x)x(t) + Bu(t) + F\Big(t, x(t), \int_0^t f\big(t, s, x(s)\big)ds\Big)\Big]dt \\ + G\Big(t, x(t), \int_0^t \sigma\big(t, s, x(s)\big)ds\Big)dw(t), \quad t \in J,\ t \neq t_k, \\ \Delta x(t_k) = x(t_k^+) - x(t_k^-) = I_k(x(t_k^-)), \quad k = 1, 2, \dots, m, \\ x(0) + h(x) = x_0. \end{gathered} \tag{4.1}$$

where A, B, I_k, h are defined as before. Further,

$$\begin{gathered} Q : J \times H \times H \to H, \quad F : J \times H \times H \to H, \quad G : J \times H \times H \to \mathcal{L}_Q(K, H), \\ q : \Lambda \times H \to H, \quad f : \Lambda \times H \to H, \quad \sigma : \Lambda \times H \to H. \end{gathered}$$

are measurable mappings in H-norm and $\mathcal{L}_Q(K,H)$-norm , respectively. The solution of the above equation is

$$
\begin{aligned}
&x(t)\\
&= U(t,0;x)\Big[x_0 - h(x) - Q(0,x(0),0)\Big] + Q\Big(t, x(t), \int_0^t q(t,s,x(s))ds\Big)\\
&\quad + \int_0^t U(t,s;x)A(s,x(s))Q\Big(s,x(s),\int_0^s q(s,\tau,x(\tau))d\tau\Big)ds\\
&\quad + \int_0^t U(t,s;x)Bu(s)ds + \int_0^t U(t,s;x)F\Big(s,x(s),\int_0^s f\big(s,\tau,x(\tau)\big)d\tau\Big)ds\\
&\quad + \int_0^t U(t,s;x)G\Big(s,x(s),\int_0^s \sigma\big(s,\tau,x(\tau)\big)d\tau\Big)dw(s)\\
&\quad + \sum_{0<t_k<t} U(t,t_k;x)I_k(x(t_k^-)), \quad \text{for a.e. } t\in J.
\end{aligned}
\tag{4.2}
$$

Concerning the operators Q, q, F, f, G, σ we assume the following hypotheses:

(H10) (i) The function $Q : J\times\mathcal{Z}\times\mathcal{Z} \to \mathcal{Z}$ is continuous and there exist constants $\mathcal{C}_Q > 0$, $\tilde{\mathcal{C}_Q} > 0$ for $s,t\in J$ and $x,y,x_1,y_1\in\mathcal{Z}$ such that the function $A(t,x)Q$ satisfies the Lipschitz condition

$$E\|A(t,x(t))Q(t,x,x_1) - A(t,y(t))Q(t,y,y_1)\|^2 \le \mathcal{C}_Q\big(\|x-y\|^2 + \|x_1-y_1\|^2\big),$$

and $\tilde{\mathcal{C}_Q} = \sup_{t\in J}\|A(t,0)Q(t,0,0)\|^2$.

(ii) There exist constants $Q_k > 0, Q_1 > 0$ and $Q_2 > 0$ such that

$$E\|Q(t,x,x_1) - Q(t,y,y_1)\|^2 \le Q_k\big(|t-s|^2 + \|x-y\|^2 + \|x_1-y_1\|^2\big),$$
$$E\|Q(t,x,y)\|^2 \le Q_1\big(\|x\|^2 + \|y\|^2\big) + Q_2,$$

where $Q_2 = \sup_{t\in J}\|Q(t,0,0)\|^2$.

(H11) The nonlinear function $q : \Lambda\times\mathcal{Z} \to \mathcal{Z}$ is continuous and there exist positive constants $\mathcal{C}_q$, $\tilde{\mathcal{C}_q}$, for $x,y\in\mathcal{Z}$ and $(t,s)\in\Lambda$ such that

$$E\|\int_0^t \big(q(t,s,x) - q(t,s,y)\big)ds\|^2 \le \mathcal{C}_q\|x-y\|^2$$

and $\tilde{\mathcal{C}_q} = \sup_{(t,s)\in\Lambda}\|\int_0^t q(t,s,0)ds\|^2$.

(H12) The nonlinear function $F : J\times\mathcal{Z}\times\mathcal{Z} \to \mathcal{Z}$ is continuous and there exist constants $\mathcal{C}_F > 0$, $\tilde{\mathcal{C}_F} > 0$ for $t\in J$ and $x_1,x_2,y_1,y_2\in\mathcal{Z}$ such that

$$E\|F(t,x_1,y_1) - F(t,x_2,y_2)\|^2 \le \mathcal{C}_F\big(\|x_1-x_2\|^2 + \|y_1-y_2\|^2\big)$$

and $\tilde{\mathcal{C}_F} = \sup_{t\in J}\|F(t,0,0)\|^2$.

(H13) The nonlinear function $f : \Lambda\times\mathcal{Z} \to \mathcal{Z}$ is continuous and there exist positive constants $\mathcal{C}_f$, $\tilde{\mathcal{C}_f}$, for $x,y\in\mathcal{Z}$ and $(t,s)\in\Lambda$ such that

$$E\Big\|\int_0^t \big(f(t,s,x) - f(t,s,y)\big)ds\Big\|^2 \le \mathcal{C}_f\|x-y\|^2$$

and $\tilde{\mathcal{C}_f} = \sup_{(t,s)\in\Lambda}\|\int_0^t f(t,s,0)ds\|^2$.

(H14) The nonlinear function $G : J\times\mathcal{Z}\times\mathcal{Z} \to \mathcal{L}_Q(K,H)$ is continuous and there exist constants $\mathcal{C}_G > 0$, $\tilde{\mathcal{C}_G} > 0$ for $t\in J$ and $x_1,x_2,y_1,y_2\in\mathcal{Z}$ such that

$$E\|G(t,x_1,y_1) - G(t,x_2,y_2)\|^2 \le \mathcal{C}_G\big(\|x_1-x_2\|^2 + \|y_1-y_2\|^2\big)$$

and $\tilde{\mathcal{C}_G} = \sup_{t\in J} \|G(t,0,0)\|^2$.

(H15) The nonlinear function $\sigma : \Lambda\times\mathcal{Z} \to \mathcal{Z}$ is continuous and there exist positive constants $\mathcal{C}_\sigma$, $\tilde{\mathcal{C}_\sigma}$, for $x,y\in\mathcal{Z}$ and $(t,s)\in\Lambda$ such that

$$E\Big\|\int_0^t \big(\sigma(t,s,x)-\sigma(t,s,y)\big)ds\Big\|^2 \le \mathcal{C}_\sigma\|x-y\|^2$$

and $\tilde{\mathcal{C}_\sigma} = \sup_{(t,s)\in\Lambda} \|\int_0^t \sigma(t,s,0)ds\|^2$.

(H16) There exists a constant $r^* > 0$ such that

$$\begin{aligned}
&9\Big\{\mathcal{C}_U(\|x_0\|^2+\tilde{\mathcal{C}_h}) + a^2\mathcal{C}_U\mathcal{G} + 2\mathcal{C}_U\big[Q_1(\|x_0\|^2+\tilde{\mathcal{C}_h})+Q_2\big]\\
&+ Q_1\big[(1+2\mathcal{C}_q)r+2\tilde{\mathcal{C}_q}\big] + Q_2 + 2a^2\mathcal{C}_U\big[\mathcal{C}_Q\big((1+2\mathcal{C}_q)r+2\tilde{\mathcal{C}_q}\big)+\tilde{\mathcal{C}_Q}\big]\\
&+ 2a^2\mathcal{C}_U\big[\mathcal{C}_F\big((1+2\mathcal{C}_f)r+2\tilde{\mathcal{C}_f}\big)+\tilde{\mathcal{C}_F}\big]\\
&+ 2a\mathcal{C}_U\,\mathrm{Tr}(Q)\big[\mathcal{C}_G\big((1+2\mathcal{C}_\sigma)r+2\tilde{\mathcal{C}_\sigma}\big)+\tilde{\mathcal{C}_G}\big] + 2m\mathcal{C}_U\Big[\sum_{k=1}^m \beta_k r + \sum_{k=1}^m \tilde{\beta_k}\Big]\Big\}\\
&\le r^*
\end{aligned}$$

and

$$\nu^* = 9\Big\{(1+16a^2\mathcal{C}_U\mathcal{C}_W)(N_1+N_2+N_3+N_4+N_5+N_6) + 2a^3\mathcal{N}\mathcal{G}\Big\}$$

where

$$\begin{gathered}
N_1 = \mathcal{N}a^2\|x_0\|^2 + 2(\mathcal{N}a^2\tilde{\mathcal{C}_h}+\mathcal{C}_U\mathcal{C}_h)\\
N_2 = 2\Big[2\mathcal{N}a^2\big(Q_1(\|x_0\|^2+\tilde{\mathcal{C}_h})+Q_2\big)+\mathcal{C}_UQ_k\mathcal{C}_h\Big] + Q_k(1+\mathcal{C}_q)\\
N_3 = 2a^2\Big[2\mathcal{N}a\Big[\mathcal{C}_Q\big((1+2\mathcal{C}_q)r+2\tilde{\mathcal{C}_q}\big)+\tilde{\mathcal{C}_Q}\Big] + \mathcal{C}_U\mathcal{C}_Q(1+\mathcal{C}_q)\Big]\\
N_4 = 2a^2\Big[2\mathcal{N}a\Big[\mathcal{C}_F\big((1+2\mathcal{C}_f)r+2\tilde{\mathcal{C}_f}\big)+\tilde{\mathcal{C}_F}\Big] + \mathcal{C}_U\mathcal{C}_F(1+\mathcal{C}_f)\Big]\\
N_5 = 2a\Big[2\mathcal{N}a\,\mathrm{Tr}(Q)\Big[\mathcal{C}_G\big((1+2\mathcal{C}_\sigma)r+2\tilde{\mathcal{C}_\sigma}\big)+\tilde{\mathcal{C}_G}\Big] + \mathcal{C}_U\ \ \mathrm{Tr}(Q)C_G(1+\mathcal{C}_\sigma)\Big]\\
N_6 = 2m\Big[2\mathcal{N}a^2\Big(\sum_{k=1}^m\beta_k r + \sum_{k=1}^m\tilde{\beta_k}\Big) + \mathcal{C}_U\sum_{k=1}^m\beta_k\Big].
\end{gathered}$$

To apply the contraction mapping, we define the nonlinear operator $\Phi^* : \mathcal{Y}_r \to \mathcal{Y}_r$ as

$$\begin{aligned}
&(\Phi^*x)(t)\\
&= U(t,0;x)\Big[x_0 - h(x) - Q(0,x(0),0)\Big] + Q\Big(t,x(t),\int_0^t q(t,s,x(s))ds\Big)\\
&\quad + \int_0^t U(t,s;x)A(s,x(s))Q\Big(s,x(s),\int_0^s q(s,\tau,x(\tau))d\tau\Big)ds\\
&\quad + \int_0^t U(t,s;x)Bu(s)ds + \int_0^t U(t,s;x)F\Big(s,x(s),\int_0^s f\big(s,\tau,x(\tau)\big)d\tau\Big)ds\\
&\quad + \int_0^t U(t,s;x)G\Big(s,x(s),\int_0^s \sigma\big(s,\tau,x(\tau)\big)d\tau\Big)dw(s) + \sum_{0<t_k<t} U(t,t_k;x)I_k(x(t_k^-)).
\end{aligned}$$

where

$$\begin{aligned} u(t) = W^{-1}\Big[&x_1 - U(a,0;x)\big[x_0 - h(x) - Q(0,x(0),0)\big] \\ &- Q\Big(a, x(a), \int_0^a q(a,s,x(s))ds\Big) \\ &- \int_0^a U(a,s;x)A(s,x(s))Q\Big(s,x(s),\int_0^s q(s,\tau,x(\tau))d\tau\Big)ds \\ &- \int_0^a U(a,s;x)F\Big(s,x(s),\int_0^s f\big(s,\tau,x(\tau)\big)d\tau\Big)ds \\ &- \int_0^a U(a,s;x)G\Big(s,x(s),\int_0^s \sigma\big(s,\tau,x(\tau)\big)d\tau\Big)dw(s) \\ &- \sum_{0<t_k<a} U(a,t_k;x)I_k(x(t_k^-))\Big](t). \end{aligned}$$

Clearly the above control transfers the system (4.1) from the initial state x_0 to the final state x_1 provided that the operator $\Phi^* x$ has a fixed point. Hence, if the operator $\Phi^* x$ has a fixed point then the system (4.1) is controllable.

Theorem 4.1. *If* (H10)–(H16) *hold, then system* (4.1) *is controllable provided that*

$$9\Big\{(1+16a^2\mathcal{C}_U\mathcal{C}_W)(N_1+N_2+N_3+N_4+N_5+N_6)+2a^3\mathcal{N}\mathcal{G}\Big\} < 1.$$

The proof of the above theorem is similar to that of Theorem 3.1 and hence it is omitted.

5. Example

Consider the partial integrodifferential equation

$$\begin{aligned} \partial\Big(z(t,y) - \frac{1}{2}\cos z(t,y)\Big) &= \Big(\frac{\partial^3}{\partial y^3}z(t,y) + z(t,y)\frac{\partial}{\partial y}z(t,y) + \mu(t,y) \\ &\quad + \frac{1}{2}e^{-t}\sin z(t,y) + \frac{z(t,y)}{t(1+t^2)}\Big[\int_0^t e^{-z(s,y)}ds\Big]\Big)\partial t \\ &\quad + \frac{1}{2}\cos t\ z(t,y)dw(t),\ t\in J:=[0,1], \quad t\neq t_k, \\ z(0,y) + \int_0^1 & m(s)\log(1+|z(s,y)|)ds = z_0(y), \\ \Delta z|_{t=t_k} = I_k(z(y)) &= \int_\Omega d_k(y,s)\cos^2(z(s,y))ds, \quad k=1,2,\dots,m. \end{aligned} \tag{5.1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary, $m(\cdot)\in L^1([0,1];\mathbb{R})$ and $d_k\in C(\bar\Omega\times\bar\Omega,\mathbb{R})$ for $k=1,2,\dots,m$. For every real s we introduce a Hilbert space $H^s(R)$ as follows [26]. Let $z\in L^2(R)$ and set

$$\|z\|_s = \Big(\int_R (1+\xi^2)^s|\hat z(\xi)|^2d\xi\Big)^{1/2},$$

where $\hat z$ is the Fourier transform of z. The linear space of functions $z\in L^2(R)$ for which $\|z\|_s$ is finite is a pre-Hilbert space with the inner product

$$(z,y)_s = \Big(\int_R (1+\xi^2)^s\hat z(\xi)\overline{\hat y(\xi)}d\xi\Big)^{1/2}.$$

The completion of this space with respect to the norm $\|\cdot\|_s$ is a Hilbert space which we denote by $H^s(R)$. It is clear that $H^0(R) = L^2(R)$.

Take $H = U = K = L^2(R) = H^0(R)$ and $Y = H^s(R), s \geq 3$. Define an operator A_0 by $D(A_0) = H^3(R)$ and $A_0 z = D^3 z$ for $z \in D(A_0)$ where $D = d/dy$. Then A_0 is the inifinitesimal generator of a C_0-group of isometries on H. Next we define for every $v \in Y$ an operator $A_1(v)$ by $D(A_1(v)) = H^1(R)$ and $z \in D(A_1(v))$, $A_1(v)z = vDz$. Then for every $v \in Y$ the operator $A(v) = A_0 + A_1(v)$ is the infinitesimal generator of C_0 semigroup $U(t, 0; v)$ on H satisfying $\|U(t, 0; v)\| \leq e^{\beta t}$ for every $\beta \geq c_0\|v\|_s$, where c_0 is a constant independent of $v \in Y$. Let $\mathcal{Y}_r$ be the ball of radius $r > 0$ in Y and it is proved that the family of operators $A(v), v \in \mathcal{Y}_r$, satisfies the conditions (E1)–(E4) and (H1) (see [26]). Put $x(t) = z(t, \cdot)$ and $u(t) = \mu(t, \cdot)$ where $\mu : J \times \mathbb{R} \to \mathbb{R}$ is continuous,

$$f(t, x(t)) = \frac{1}{2}e^{-t}\sin z(t, y), \quad \sigma(t, x(t)) = \frac{1}{2}\cos t\; z(t, y),$$

$$q(t, x(t)) = \frac{1}{2}\cos z(t, y), \quad h(x) = \int_0^1 m(s)\log(1 + |z(s, y)|)ds$$

$$\int_0^t g(t, s, x(s))ds = \frac{z(t, y)}{t(1 + t^2)}\Big[\int_0^t e^{-z(s,y)}ds\Big].$$

With this choice of $A(v)$, I_k, q, f, g, h, σ, $B = I$, the identity operator and $w(t)$ denotes a one dimensional standard wiener process, we see that (5.1) is an abstract formulation of the system (1.6). Further we have

$$\Big\|\frac{z(t, y)}{t(1 + t^2)}\Big[\int_0^t e^{-z(s,y)}ds\Big]\Big\| \leq \frac{1}{1 + t^2}\|z\|.$$

Assume that the operator $W : L^2(J, U)/KerW \to H$ defined by

$$Wu = \int_0^1 U(1, s; x)\mu(s, \cdot)ds$$

has an inverse operator and satisfies (H2) for every $x \in \mathcal{Y}_r$. Further the other assumptions (H3)–(H9) are obviously satisfied and it is possible to choose a suitable control function $u(t)$ in such a way that the constant $\nu < 1$ which will steer the system from x_0 to x_1. Hence, by Theorem 4.1, system (5.1) is controllable on J.

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