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PROPERTIES OF AN EQUATION FOR NEURAL FIELDS IN A BOUNDED DOMAIN

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ABSTRACT. In this work we study the global dynamics of an evolution equation for neural fields, where the flow generated by this equation in the phase space $L^2(S^1)$, is C^1. Furthermore we exhibit a continuous Lyapunov functional and use it for proving that this flow has the gradient property.

1. INTRODUCTION

We consider the non local evolution equation

$$\frac{\partial u(x,t)}{\partial t} = -u(x,t) + J * (f \circ u)(x,t) + h, \quad h > 0, \tag{1.1}$$

where $u(x,t)$ is a real-valued function on , $J \in C^1(\mathbb{R})$ is a non negative even function supported in the interval $[-1,1]$, f is a non negative nondecreasing function and h is a positive constant. The symbol $*$ above denotes convolution product; that is, $(J * v)(x) = \int_{\mathbb{R}} J(x-y)v(y)dy$.

Equation (1.1) was derived by Wilson and Cowan [26] for modeling neuronal activity, and arise through a limiting argument from a discrete synaptically-coupled network of excitatory and inhibitory neurons, [8]. Here the function $u(x,t)$ denotes the mean membrane potential of a patch of tissue located at position $x \in (-\infty, \infty)$ at time $t \geq 0$. The connection function $J(x)$ determines the coupling between the elements at position x and position y. The function $f(u)$ gives the neural firing rate, or average rate at which spikes are generated, corresponding to an activity level u. The parameter h denotes a constant external stimulus applied uniformly to the entire neural field. Let $S(x,t) = f(u(x,t))$ be the firing rate of a neuron at position x at time t, we say that the neurons at a point x is active if $S(x,t) > 0$.

In the literature, there are several works dedicated to the analysis of this model; see [1, 5, 7, 9, 15, 16, 17, 21, 22, 23, 24]. Most of these works concern with the existence and stability of characteristic solutions, such as localized excitation [1, 15, 17, 21] or traveling front [5, 7, 9]. Although there are some works on the global dynamics of this model [16, 22, 23, 24], it has not been fully analyzed; for example, existence of one continuous Lyapunov functional defined in the whole phase space,

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property of smoothness of the flow and lower simicontinuity of global attractors are not known.

We consider additional conditions on f and J which will be used as hypotheses in our results.

(H1) $f \in C^1(\mathbb{R})$ and f' locally Lipschitz and for some positive constant k_1,
$$0 < f'(r) < k_1, \quad \forall\, r \in \mathbb{R}. \tag{1.2}$$

(H2) f is a nondecreassing function taking value between 0 and $S_{\max} > 0$ and satisfying, for $0 \leq s \leq S_{\max}$,
$$\Big| \int_0^s f^{-1}(r)dr \Big| < L < \infty.$$

(H3) $J \in C^1(\mathbb{R})$ and satisfies $k_1 \|J\|_{L^1} < 1$.

From (H1) it follows that
$$|f(x) - f(y)| \leq k_1 |x - y|, \quad \forall\, x, y \in \mathbb{R}, \tag{1.3}$$
and, in particular, there exists constant $k_2 \geq 0$ such that
$$|f(x)| \leq k_1 |x| + k_2. \tag{1.4}$$

This article is organized as follows. In Section 2, following the techniques in [3, 18, 19], we repeated the process in [24] to formulate the Cauchy problem for (1.1) in $L^2(S^1)$, and to check that, in this space under hypothesis (H1), the Cauchy problem for (1.1) is well posed with globally defined solutions. In Section 3, under hypothesis (H1) we prove that the flow generated by (1.1), in $L^2(S^1)$, is of class C^1. For this, we apply one classic result from [20]. In Section 4 motivated by energy functionals from [2, 10, 11, 14, 16, 27], under hypotheses (H1) and (H2), we exhibit a continuous Lyapunov functional for the flow of (1.1), and use it to prove that, under hypotheses (H1)–(H3), the flow is gradient in the sense of [12]. Finally, in Section 5, we illustrate our results with a concrete example, where $f(x) = (1 + e^{-x})^{-1}$ and $J(x) = e^{-1/(1-x^2)}$, if $|x| < 1$ and $J(x) = 0$ if $|x| \geq 1$.

2. Well posedness in $L^2(S^1)$

In this section we use the same the technique as in [3, 18, 19] to obtain the formulation given in [24]. We repeat this technique, only to facilitate the readers work.

The Cauchy problem for (1.1) is well posed in the space of continuous bounded functions, $C_b(\mathbb{R})$, with the supremum norm, since the function given by the right hand side of (1.1) is uniformly Lipschitz in this space. It is an easy consequence of the uniqueness theorem that the subspace $\mathbb{P}_{2\tau}$ of 2τ periodic functions is invariant.

We considerer here equation (1.1) restricted to $\mathbb{P}_{2\tau}$, $\tau > 1$. As we will see below, this leads naturally to the consideration of a flow in $L^2(S^1)$, where S^1 denotes the unit sphere.

Now, if $\tau > 1$ is a given positive number, we define J^τ as the 2τ periodic extension of the restriction of J to interval $[-\tau, \tau]$. It is then easy to show that, if $u \in \mathbb{P}_{2\tau}$, then
$$(J * u)(x) = \int_{-\tau}^{\tau} J^\tau(x - y) u(y) dy. \tag{2.1}$$

In view of (2.1), equation (1.1), restricted to $\mathbb{P}_{2\tau}$, with $\tau > 1$, can be written as

$$\frac{\partial m(x,t)}{\partial t} = -m(x,t) + \int_{-\tau}^{\tau} J^\tau(x-y) f(m(y,t))dy + h.$$

Define $\varphi : \mathbb{R} \to S^1$ by

$$\varphi(x) = e^{i\pi x/\tau}$$

and, for $u \in \mathbb{P}_{2\tau}$, $v : S^1 \to \mathbb{R}$ by

$$v(\varphi(x)) = u(x).$$

In particular, we write $\widetilde{J}(\varphi(x)) = J^\tau(x)$. Then we have the following result:

Proposition 2.1 ([24])**.** *The function $u(x,t)$ is a 2τ periodic solution of* (1.1) *if and only if $v(w,t) = u(\varphi^{-1}(w),t)$ is a solution of*

$$\frac{\partial m(w,t)}{\partial t} = -m(w,t) + \widetilde{J} * (f \circ m)(w,t) + h \tag{2.2}$$

where, $(*)$ *denotes convolution in S^1; that is,*

$$(\widetilde{J} * m)(w) = \int_{S^1} \widetilde{J}(w \cdot z^{-1}) m(z) dz$$

and $dz = \frac{\tau}{\pi} d\theta$, where $d\theta$ denote integration with respect to arc length.

From now on we will write J instead of $\widetilde{J}$ for simplicity.

Remark 2.2. Using the triangle inequality, Young's inequality and (1.3), it follows that the function F given by right hand side of (2.2),

$$F(u) = -u + J * (f \circ u) + h,$$

is uniformly Lipschitz in $L^2(S^1)$. Hence (see [4] and [6]) the Cauchy problem for (2.2) is well posed in this space. More precisely, we have that (2.2) has a unique solution for any initial condition in $L^2(S^1)$, which is globally defined.

3. Smoothness of the orbits

In this section, we prove that (2.2) generates one flow C^1 with respect to initial conditions.

Proposition 3.1. *Assume that* (H1) *holds. Then the function*

$$F(u) = -u + J * (f \circ u) + h$$

is continuously Fréchet differentiable in $L^2(S^1)$ with derivative given by

$$F'(u)v = -v + J * (f'(u))v.$$

Proof. By a simple computation, using (H1), it follows that the Gateaux's derivative of F is given by

$$DF(u)v = -v + J * (f'(u)v).$$

Now, note that for each $u \in L^2(S^1)$, due to linearity of the convolution, $DF(u)$ is a linear operator. Furthermore,

$$\|DF(u)v\|_{L^2} \le \|v\|_{L^2} + \|J * f'(u)v\|_{L^2} \le \|v\|_{L^2} + \|J\|_{L^1} \|f'(u)v\|_{L^2}.$$

But, using (1.2), we have

$$\|f'(u)v\|_{L^2} \le k_1 \|v\|_{L^2}.$$

Hence
$$\|DF(u)v\|_{L^2} \le (1 + k_1\|J\|_{L^1})\|v\|_{L^2}.$$
Furthermore, DF is a continuous operator. In fact, given $v \in L^2(S^1)$, we have
$$\|DF(u_1)v - DF(u_2)v\|_{L^2} = \|J * [(f' \circ u_1)v] - J * [(f' \circ u_2)]v\|_{L^2}.$$
Since
$$\begin{aligned}
&|(J * f'(u_1)v)(w) - (J * f'(u_2)v)(w)| \\
&= |J * [f'(u_1)v - f'(u_2)v](w)| \\
&\le \int_{S^1} |J(wz^{-1})[f'(u_1(z)) - f'(u_2(z))]v(z)|dz \\
&\le \|J\|_\infty \int_{S^1} |f'(u_1(z)) - f'(u_2(z))||v(z)|dz.
\end{aligned}$$
Using Hölder's inequality [4], we obtain
$$\begin{aligned}
&\|DF(u_1)v - DF(u_2)v\|_{L^2} \\
&\le \|J\|_\infty \Big(\int_{S^1} |f'(u_1(z)) - f'(u_2(z))|^2 dz\Big)^{1/2} \Big(\int_{S^1} |v(z)|^2 dz\Big)^{1/2} \\
&= \|J\|_\infty \|f' \circ u_1 - f' \circ u_2\|_{L^2}\|v\|_{L^2}.
\end{aligned}$$
Thus
$$\|DF(u_1)v - DF(u_2)v\|_{L^2}^2 \le \sqrt{2\tau}\|J\|_\infty^2 \|f' \circ u_1 - f' \circ u_2\|_{L^2}^2 \|v\|_{L^2}^2.$$
Keeping $u_1 \in L^2(S^1)$ fixed and letting $u_2 \to u_1$ in $L^2(S^1)$ it follows that $u_2(w) \to u_1(w)$ almost everywhere in S^1. From (H1) follows that, there exists $M > 0$ such that
$$|f'(u_2(w)) - f'(u_1(w))| \le M|u_2(w) - u_1(w)|, \quad \text{almost everywhere.}$$
Then
$$\begin{aligned}
\|f' \circ u_1 - f' \circ u_2\|_{L^2}^2 &= \int_{S^1} |f'(u_1(w)) - f'(u_2(w))|^2 dw \\
&\le \int_{S^1} M^2 |u_1(w) - u_2(w)|^2 dw \\
&= M^2 \|u_2 - u_1\|_{L^2}^2.
\end{aligned}$$
Hence
$$\|DF(u_1)v - DF(u_2)v\|_{L^2}^2 \le \sqrt{2\tau}\|J\|_\infty^2 M^2 \|u_1 - u_2\|_{L^2}^2 \|v\|_{L^2}^2.$$
Therefore, from Proposition 3.2 below it follows that F is Fréchet differentiable with continuous derivative in $L^2(S^1)$. □

Proposition 3.2 ([20])**.** *Let X and Y be normed linear spaces, $F : X \to Y$ a map and suppose that the Gateaux derivative of F, $DF : X \to \mathcal{L}(X, Y)$ exists and is continuous at $x \in X$. Then the Fréchet derivative F' of F exists and is continuous at x.*

Remark 3.3. If $u(w, t)$ is a solution of (2.2) with initial condition u_0 then by the variation of constants formula
$$u(w, t) = e^{-t}u_0 + \int_0^t e^{-(t-s)}[J * (f \circ u)(w, s) + h]ds.$$

Since the right-hand side of (2.2) is a C^1 function, the flow generated by (2.2), which is given by $T(t)u_0 = u(w,t)$ is C^1 with respect to initial conditions (see [13]).

4. Gradient property

In this section, we exhibit a continuous Lyapunov functional for the flow of (2.2), which is well defined in the whole space $L^2(S^1)$, and as used it to prove that this flow has the gradient property, in the sense of [12].

We recall that a C^r-semigroup, $T(t)$, is *gradient* if each bounded positive orbit is precompact and there exists a continuous Lyapunov Functional for $T(t)$ (see [12]).

Remark 4.1. As shown in [24], under hypotheses (H1) and (H3), there exists a global attractor, $\mathcal{A}$, for the flow $T(t)$ generated by (2.2), in $L^2(S^1)$, which is given by ω-limit set of the ball of radius $\frac{2\sqrt{2\tau}(k_2\|J\|_{L^1}+h)}{1-k_1\|J\|_{L^1}}$. This implies that, for any $u_0 \in L^2(S^1)$, the positive orbit by u_0

$$\gamma^+(u_0) = \{T(t)u_0,\, t \geq 0\}$$

is precompact.

Motivated by energy functionals from [2, 11], [14, 16, 27] (see also [10] for similar functional), we define the functional $F : L^2(S^1) \to \mathbb{R}$ by

$$F(u) = \int_{S^1} \Big[-\frac{1}{2}S(w)\int_{S^1} J(wz^{-1})S(z)dz + \int_0^{S(w)} f^{-1}(r)dr - hS(w)\Big]dw, \tag{4.1}$$

where $S(w) = f(u(w))$.

Remark 4.2. From hypotheses (H1) and (H2), follows that the functional given in (4.1) is defined in the whole space $L^2(S^1)$ and it is lower bounded.

Theorem 4.3. *Assume* (H1) *holds. Then the functional given in* (4.1) *is continuous in the topology of* $L^2(S^1)$.

Proof. Let (u_n) be a sequence converging to u in the norm of $L^2(S^1)$. We can extract a subsequence u_{n_k}, such that, $u_{n_k}(w) \to u(w)$ *a.e.* in S^1. Now, from (H1), it follows that f is continuous, then $S_{n_k}(w) = f(u_{n_k}(w)) \to f(u(w)) = S(u(w))$ a.e. Thus

$$\lim_{k\to\infty} \int_0^{S_{n_k}(w)} f^{-1}(r)dr = \int_0^{S(w)} f^{-1}(r)dr.$$

And from Lebesgue's Dominated Convergence Theorem follows that

$$\begin{gathered}
\lim_{k\to\infty} \int_{S^1} J(wz^{-1})S_{n_k}(z)dz = \int_{S^1} J(wz^{-1})S(z)dz, \\
\lim_{k\to\infty} \int_{S^1} hS_{n_k}(w)dw = \int_{S^1} hS(w)dw, \\
\lim_{k\to\infty} \int_{S^1} \Big[-\frac{1}{2}S_{n_k}(w)\int_{S^1} J(wz^{-1})S_{n_k}(z)dz\Big] \\
= \int_{S^1} \Big[-\frac{1}{2}S(w)\int_{S^1} J(wz^{-1})S(z)dz\Big],
\end{gathered}$$

Thus $F(u_{n_k})$ converges to $F(u)$, as $k \to \infty$. Therefore $F(u_n)$ is a sequence such that every subsequence has a subsequence that converges to $F(u)$. Hence $F(u_n) \to F(u)$, as $n \to \infty$. $\square$

Theorem 4.4. *Suppose that* (H1)-(H2) *hold. Let* $u(\cdot,t)$ *be a solutions of* (2.2)*. Then* $F(u(\cdot,t))$ *is differentiable with respect to* t *and*

$$\frac{dF}{dt} = -\int_{S^1}[-u(w,t) + J*(f\circ u)(w,t) + h]^2 f'(u(w,t))dw \leq 0.$$

Proof. Let

$$\varphi(w,s) = -\frac{1}{2}S(w,s)\int_{S^1} J(wz^{-1})S(z,s)dz + \int_0^{S(w,s)} f^{-1}(r)dr - hS(w,s)\,.$$

From (H1) and (H2) it follows that $\|\frac{\partial\varphi(\cdot,s)}{\partial s}\|_{L^1} < \infty$, for all $s\in\mathbb{R}_+$. Hence, deriving under the integration sign, we obtain

$$\begin{aligned}
&\frac{d}{dt}F(u(\cdot,t))\\
&= \int_{S^1}[-\frac{1}{2}\frac{\partial S(w,t)}{\partial t}\int_{S^1} J(wz^{-1})S(z,t)dz - \frac{1}{2}S(w,t)\int_{S^1} J(wz^{-1})\frac{\partial S(z,t)}{\partial t}dz\\
&\quad + f^{-1}(S(w,t)))\frac{\partial S(w,t)}{\partial t} - h\frac{\partial S(w,t)}{\partial t}]dw\\
&= -\frac{1}{2}\int_{S^1}\int_{S^1} J(wz^{-1})S(z,t)\frac{\partial S(w,t)}{\partial t}dzdw\\
&\quad -\frac{1}{2}\int_{S^1}\int_{S^1} J(wz^{-1})S(w,t)\frac{\partial S(z,t)}{\partial t}dzdw + \int_{S^1}[u(w,t)-h]\frac{\partial S(w,t)}{\partial t}dw.
\end{aligned}$$

Since

$$\frac{1}{2}\int_{S^1}\int_{S^1} J(wz^{-1})S(z,t)\frac{\partial S(w,t)}{\partial t}dzdw = \frac{1}{2}\int_{S^1}\int_{S^1} J(wz^{-1})S(w,t)\frac{\partial S(z,t)}{\partial t}dzdw,$$

it follows that

$$\begin{aligned}
&\frac{d}{dt}F(u(\cdot,t))\\
&= -\int_{S^1}\int_{S^1} J(wz^{-1})S(z,t)\frac{\partial S(w,t)}{\partial t}dzdw + \int_{S^1}[u(w,t)-h]\frac{\partial S(w,t)}{\partial t}dw\\
&= -\int_{S^1}[-u(w,t) + \int_{S^1} J(wz^{-1})S(z,t)dz + h]\frac{\partial S(w,t)}{\partial t}dw\\
&= -\int_{S^1}[-u(w,t) + J*(f\circ u)(w,t) + h]\frac{\partial f(u(w,t))}{\partial t}dw\\
&= -\int_{S^1}[-u(w,t) + J*(f\circ u)(w,t) + h]f'(u(w,t))\frac{\partial u(w,t)}{\partial t}dw\\
&= -\int_{S^1}[-u(w,t) + J*(f\circ u)(w,t) + h]^2 f'(u(w,t))dw.
\end{aligned}$$

Using (H1) the result follows. □

Remark 4.5. From Theorem 4.4 it follows that, if $F(T(t)u) = F(u)$ for $t\in\mathbb{R}$, then u is an equilibrium point for $T(t)$.

Proposition 4.6. *Assume* (H1)-(H3)*. Then the flow generated by equation* (2.2) *is gradient.*

Proof. The precompacity of the orbits follows from Remark 4.1. From Remark 4.2, Theorem 4.3, Theorem 4.4 and Remark 4.5 follows that the functional given in (4.1) is a continuous Lyapunov functional. □

Remark 4.7. *As a consequence of the Proposition 4.6, we have that the global attractor given in* [24] *coincides with the unstable set of the equilibria* [12, Theorem 3.8.5]*; that is,*

$$\mathcal{A} = W^u(E),$$

where $E = \{u \in L^2(S^1) : u(w) = J * (f \circ u)(w) + h\}$.

5. A concrete example

In this section we illustrate the results of previous sections to the particular case of (1.1) where f and J are given by $f(x) = (1 + e^{-x})^{-1}$ and $J(x) = e^{-1/(1-x^2)}$, if $|x| < 1$ and $J(x) = 0$ if $|x| \geq 1$. Considering J^τ as the 2τ periodic extension of the restriction of J to interval $[-\tau, \tau]$, $\tau > 1$, we can rewrite (1.1), in the space $\mathbb{P}_{2\tau}$, as

$$\frac{\partial u(x,t)}{\partial t} = -u(x,t) + \int_{-\tau}^{\tau} e^{\frac{-1}{1-(x-y)^2}} (1 + e^{-u(y)})^{-1} dy + h. \tag{5.1}$$

Defining $\varphi : \mathbb{R} \to S^1$ by $\varphi(x) = e^{i\frac{\pi}{\tau}x}$ and, for $u \in \mathbb{P}_{2\tau}$, $v : S^1 \to \mathbb{R}$ by $v(\varphi(x)) = u(x)$ and writing $\widetilde{J}(\varphi(x)) = J^\tau(x)$, follows from Proposition 2.1 that equation (5.1) is equivalent to

$$\frac{\partial u(w,t)}{\partial t} = -u(w,t) + \int_{S^1} \widetilde{J}(wz^{-1})(1 + e^{-u(z)})^{-1} dz + h, \tag{5.2}$$

and $dz = \frac{\tau}{\pi} d\theta$, where $d\theta$ denotes integration with respect to arc length.

The functions f and J satisfy (H1)–(H3) with $k_1 = S_{\max} = 1$, $L = \ln 2$ and $k_2 = \frac{1}{2}$ in (1.4). In fact,

(I) Note that $f'(x) = (1 + e^{-x})^{-2} e^{-x} > 0$. Then, since $1 < (1 + e^{-x})^2 \leq 4$, for all $x \in \mathbb{R}$, it follows that

$$\frac{1}{4} \leq (1 + e^{-x})^{-2} < 1.$$

Furthermore, since $f''(x) = 2(1+e^{-x})^{-3} e^{-2x} - (1+e^{-x})^{-2} e^{-x}$, we have $|f''(x)| < 3$, $\forall\, x \in \mathbb{R}$. Hence f' is locally Lipschitz.

(II) It is easy see that $0 < |(1 + e^{-x})^{-1}| < 1$ and $f^{-1}(x) = -\ln(\frac{1-x}{x})$. Thus by a direct computation we obtain that, for $0 \leq s \leq 1$,

$$\Big| \int_0^s -\ln(\frac{1-x}{x}) dx \Big| \leq \ln 2.$$

(III) Since $0 \leq J(x) \leq e^{-1}$ follows that, for $k_1 = 1$,

$$k_1 \|J\|_{L1} = \int_{-1}^{1} e^{-\frac{1}{1-x^2}} dx \leq \frac{1}{e} \int_{-1}^{1} dx = \frac{2}{e} < 1.$$

Moreover, from (I) it follows that

$$|f(x) - f(y)| = |(1 + e^{-x})^{-1} - (1 + e^{-y})^{-1}| \leq |x - y|.$$

In particular, since $f(0) = 1/2$, we have

$$|f(x)| \leq |x| + \frac{1}{2}, \quad \forall x \in \mathbb{R}.$$

Therefore all results of Sections 3 and 4 (in particular Propositions 3.1 and 4.6) are valid for the flow generated by equation (5.2).

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