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SCHRÖDINGER SYSTEMS WITH A CONVECTION TERM FOR THE $(p_1, \dots, p_d)$-LAPLACIAN IN $\mathbb{R}^N$

DRAGOS-PATRU COVEI

Abstract. The main goal is to study nonlinear Schrödinger type problems for the $(p_1, \dots, p_d)$-Laplacian with nonlinearities satisfying Keller- Osserman conditions. We establish the existence of infinitely many positive entire radial solutions by an application of a fixed point theorem and the Arzela-Ascoli theorem. An important aspect in this article is that the solutions are obtained by successive approximations and hence the proof can be implemented in a computer program.

1. Introduction

Nonreactive scattering of atoms and molecules, and related bound state energy eigenvalue problems can be formulated by the radial Schrödinger system

$$\begin{gathered} U'' + \frac{N-1}{r} U' = A(r) U(r) \\ U(r) \to U_\infty \quad \text{as } r \to \infty \end{gathered} \tag{1.1}$$

where $r := |x|$ ($|\cdot|$ is the Euclidean norm), the wave function $U(r)$ is a $d \times 1$ vector and the potential function $A(r)$ is a $d \times d$ symmetric matrix. We refer the reader to [12, 7] for some additional details.

In recent years, much effort has been devoted to the problems which arise in connection with the system (1.1) and that are related to nonlinear differential equations. However, most of the treatments are either for coupled systems of equations or for scalar equations (see [1]-[22]).

The object of this work is to develop an existence theory for radial solutions of the basic nonlinear elliptic system

$$\begin{gathered} \Delta_{p_1} u_1 + h_1(|x|) |\nabla u_1|^{p_1 - 1} = a_1(|x|) g_1(u_1, \dots, u_d) \quad \text{for } x \in \mathbb{R}^N, \\ \dots \\ \Delta_{p_d} u_d + h_d(|x|) |\nabla u_d|^{p_d - 1} = a_d(|x|) g_d(u_1, \dots, u_d) \quad \text{for } x \in \mathbb{R}^N, \end{gathered} \tag{1.2}$$

where $d \geq 1$, $1 < p_i \leq N - 1$, $i = 1, \dots, d$ and Δ_{p_i} is the so called p_i-Laplacian operator defined by

$$\Delta_{p_i} u_i := \operatorname{div}(|\nabla u_i|^{p_i - 2} \nabla u_i).$$

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The operator Δ_{p_i} with $p_i \neq 2$ occurs in many mathematical models of physical processes: it is used in non-Newtonian fluids where the shear stress $\overrightarrow{\tau}$ and the velocity gradient ∇u_i of the fluid are related in the manner that $\overrightarrow{\tau}(x) = r_i(x)|\nabla u_i|^{p_i-2}\nabla u_i$ (for $p_i = 2$ (respectively, $p_i < 2$, $p_i > 2$) if the fluid is Newtonian (respectively, pseudoplastic, dilatant)), in some reaction-diffusion problems, in nonlinear elasticity ($p_i > 2$), glaciology ($1 < p_i < \frac{4}{3}$), of flow through porous media ($p_i = \frac{3}{2}$), in petroleum extraction as well as in torsional creep problems, see the book of Diaz [5] and Lions [15] where are collected detailed references on physical background and presented mathematical treatments of free boundary problem associated with the operator Δ_{p_i}.

In the mathematical context several interesting results about blowup theorems for solutions of nonlinear system like (1.2) are known and have been obtained by several authors. For further discussion, examples and references, in the particular case $d = 2$ and $p_1 = p_2 = 2$ we refer to [4], [22] and [20]. The case $p_1 \neq \dots \neq p_d \neq 2$ is not yet well understood, but is the subject of much current research. Some very recent existence results on the p_i-Laplacian can be found in a recent paper of Hamydy-Massar and Tsouli [9] where they study entire large solutions to the system (1.2) when $p_j \geq 2$ ($d = 2$, $j = 1, 2$) and for the functions h_j, a_j, g_j that satisfy

- (A1) h_j, $a_j : [0,\infty) \to [0,\infty)$ are radial continuous functions;
- (G1) $g_j : [0,\infty)^d \to [0,\infty)$ are continuous in all variables;
- (G2) g_j are non-decreasing on $[0,\infty)^d$ in all variables;
- (G3) for all $M > 0$

$$\lim_{t\to\infty} \frac{g_1(Mg_2(t)^{\frac{1}{p_2-1}})}{t^{p_1-1}} = 0.$$

Under these hypotheses and the integral condition

$$\int_1^\infty (r^{1-N}e^{-\int_0^r h_j(t)dt}\int_0^t r^{N-1}e^{\int_0^r h_j(t)dt}a_j(s)ds)^{\frac{1}{p_j-1}}dt = \infty, \quad j = 1, \dots, d$$

they proved that the system (1.2) has infinitely many positive entire large solutions.

Regarding the case $d = 2$ and $p_1 = p_2 = 2$, Zhang and Liu [22] studied the existence of entire large positive solutions of the system

$$\begin{aligned} \Delta u_1 + |\nabla u_1| &= a_1(r)g_1(u_1, u_2), \\ \Delta u_2 + |\nabla u_2| &= a_2(r)g_2(u_1, u_2), \end{aligned}$$

where $r := |x|$, $x \in \mathbb{R}^N$. They generalized the results of several authors by considering a_1, a_2, g_1 and g_2 satisfying (A1), (G1), (G2) and instead of (G3) the condition

$$\int_a^\infty \frac{ds}{g_1(s,s) + g_2(s,s)} = \infty \quad \text{for } r \geq a > 0. \tag{1.3}$$

It is interesting that for a single equation of the form $\Delta u = g(u)$ where $g(u)$ is positive, real continuous function defined for all real u and nondecreasing the existence of entire large solutions is equivalent to a condition on g known as the Keller-Osserman condition

$$\int_{u_0}^\infty \Big(\int_0^t g(s)ds\Big)^{-1/2} dt = \infty \quad \text{for } u_0 > 0, \tag{1.4}$$

(see [10, 18]) and that for systems, no such a result exists yet.

Motivated by the references mentioned above it is interested whether similar results can be obtained for nonlinearities g_i ($i = 1, \dots, d$) of the type (1.4), which includes, as a special case, a similar result of [4]. Also, we are interested in the existence results allowing any $p_1, \dots, p_d > 1$. The answers of these questions are certainly not trivial and seems to be applicable to more general nonlinearities e.g., those studied in [10, 17] or more suggestive in the works of [6] respectively [20].

Let us finish our presentation to announce our main result that can be stated as follows.

Theorem 1.1. *Suppose the functions a_j, h_j satisfy* (A1), *g_j satisfy* (G1), (G2) *and the "Keller-Osserman type" condition*

$$I(\infty) := \lim_{r\to\infty} I(r) = \infty \tag{1.5}$$

where

$$I(r) := \int_a^r [G(s)]^{-1/\min\{p_1,\dots,p_d\}} ds$$

for $r \geq a > 0$, and $G(s) := \int_0^s \sum_{i=1}^d g_i(t, \dots, t)dt + 1$. Under these hypotheses there are infinitely many positive entire radial solutions of (1.2)*. Suppose furthermore that*

$$\frac{p_j}{p_j - 1} s^{\frac{p_j(N-1)}{p_j-1}} e^{\frac{p_j}{p_j-1}\int_0^s h_j(t)dt} a_j(s) \quad \text{for } j = 1, \dots, d,$$

is nondecreasing for large s. Then

(i) *The solutions are bounded if there exists a positive number ε such that*

$$\int_0^\infty t^{1+\varepsilon}\big(e^{\frac{p_j}{p_j-1}\int_0^t h_j(t)dt} a_j(t)\big)^{2/p_j} dt < \infty \quad \text{for all } j = 1, \dots, d, \tag{1.6}$$

(ii) *The solutions are large if*

$$\int_0^\infty \Big(\frac{e^{-\int_0^t h_j(s)ds}}{t^{N-1}} \int_0^t s^{N-1} e^{\int_0^s h_j(t)dt} a_j(s)ds\Big)^{1/(p_j-1)} dt = \infty \tag{1.7}$$

for $j = 1, \dots, d$.

As far as we know, there is no such a result in any work from the literature, because no solutions have been detected yet for the system of the form (1.2) under the Keller-Osserman conditions (1.5).

2. Proof of the Theorem 1.1

In this section, we show the existence of positive radial solutions of (1.2). The proof is inspired by [4] with some new ideas. Now we remark that (1.2) has a solution $(u_1, \dots, u_d) := (u_1(r), \dots, u_d(r))$ if and only if $(u_1, \dots, u_d)$ solves the system of second-order ordinary differential equations

$$\begin{gathered}
(p_1 - 1)(u_1')^{p_1-2}u_1'' + \frac{N-1}{r}(u_1')^{p_1-1} + h_1(r)|u_1'|^{p_1-1} = a_1(r)g_1(u_1, \dots, u_d), \\
\dots \\
(p_d - 1)(u_d')^{p_d-2}u_d'' + \frac{N-1}{r}(u_d')^{p_d-1} + h_d(r)|u_d'|^{p_d-1} = a_d(r)g_d(u_1, \dots, u_d), \\
u_i'(0) = 0 \quad \text{for } i = 1, \dots, d
\end{gathered} \tag{2.1}$$

where we can assume in the next that $u_i'(r) \geq 0$ for $i = 1, \dots, d$.

However, in view of the symmetry of $(u_1,\dots,u_d)$, we have that radial solutions of (2.1) are positive solutions $(u_1,\dots,u_d)$ of the integral equations

$$
\begin{gathered}
u_1(r)=u_1(0)+\int_0^r\Big(\frac{e^{-\int_0^t h_1(s)ds}}{t^{N-1}}\int_0^t s^{N-1}e^{\int_0^s h_1(s)dt}a_1(s)g_1(u_1,\dots,u_d)ds\Big)^{\frac{1}{p_1-1}}dt,\\
\dots\\
u_d(r)=u_d(0)+\int_0^r\Big(\frac{e^{-\int_0^t h_d(s)ds}}{t^{N-1}}\int_0^t s^{N-1}e^{\int_0^s h_d(s)dt}a_d(s)g_d(u_1,\dots,u_d)ds\Big)^{\frac{1}{p_d-1}}dt.
\end{gathered}
\tag{2.2}
$$

Our first idea in the proof of the main result is to regard (2.2) as an operator equation

$$
S(u_1(r),\dots,u_d(r))=(u_1(r),\dots,u_d(r))
$$

with

$$
S:C[0,\infty)\times\dots\times C[0,\infty)\to C[0,\infty)\times\dots\times C[0,\infty)
$$

defined by

$$
\begin{aligned}
&S(u_1(r),\dots,u_d(r))\\
&=\begin{pmatrix}
u_1(0)+\int_0^r(\frac{e^{-\int_0^t h_1(s)ds}}{t^{N-1}}\int_0^t s^{N-1}e^{\int_0^s h_1(s)dt}a_1(s)g_1(u_1,\dots,u_d)ds)^{\frac{1}{p_1-1}}dt\\
\dots\\
u_d(0)+\int_0^r(\frac{e^{-\int_0^t h_d(s)ds}}{t^{N-1}}\int_0^t s^{N-1}e^{\int_0^s h_d(s)dt}a_d(s)g_d(u_1,\dots,u_d)ds)^{\frac{1}{p_d-1}}dt
\end{pmatrix}
\end{aligned}
\tag{2.3}
$$

where $u_1(0)=\dots=u_d(0)=b/d$ with $b\geq a>0$ are the central values for the system. The integration in this operator implies that a fixed point

$$
(u_1,\dots,u_d)\in C[0,\infty)\times\dots\times C[0,\infty)
$$

is in fact in the space $C^1[0,\infty)\times\dots\times C^1[0,\infty)$. Then a solution of (2.1) will be obtained as a fixed point of the operator (2.3). To establish a solution to this operator, we use successive approximation which constitutes an indispensable tool for solving nonlinear systems (1.2) at this point. We define, recursively, sequences $\{u_i^k\}_{i=\overline{1,\dots,d}}^{k\geq 1}$ on $[0,\infty)$ by

$$
u_1^0=\dots=u_d^0=\frac{b}{d}\quad\text{for all } r\geq 0 \text{ and } b\geq a>0
$$

and

$$
\begin{aligned}
&(u_1^k,\dots,u_d^k)=S(u_1^{k-1}(r),\dots,u_d^{k-1}(r))\\
&=\begin{pmatrix}
\frac{b}{d}+\int_0^r(\frac{e^{-\int_0^t h_1(s)ds}}{t^{N-1}}\int_0^t s^{N-1}e^{\int_0^s h_1(s)dt}a_1(s)g_1(u_1^{k-1},\dots,u_d^{k-1})ds)^{\frac{1}{p_1-1}}dt\\
\dots\\
\frac{b}{d}+\int_0^r(\frac{e^{-\int_0^t h_d(s)ds}}{t^{N-1}}\int_0^t s^{N-1}e^{\int_0^s h_d(s)dt}a_d(s)g_d(u_1^{k-1},\dots,u_d^{k-1})ds)^{\frac{1}{p_d-1}}dt
\end{pmatrix}^T.
\end{aligned}
\tag{2.4}
$$

It is easy to see that, for all $r\geq 0$, $i=1,\dots,d$ and $k\in N$ we have

$$
u_i^k(r)\geq\frac{b}{d},
$$

and that $\{u_i^k\}_{i=1,\dots,d}^{k\geq 1}$ is an increasing sequence of nonnegative and non-decreasing functions.

We note that $\{u_i^k\}_{i=1,\dots,d}^{k\geq 1}$ satisfy

$$
\begin{aligned}
&(p_1-1)[(u_1^k)']^{p_1-2}(u_1^k)'' + (\frac{N-1}{r} + h_1(r))[(u_1^k)']^{p_1-1} \\
&= a_1(r)g_1(u_1^{k-1}(r),\dots,u_d^{k-1}(r)), \\
&\quad \dots \\
&(p_d-1)[(u_d^k)']^{p_d-2}(u_d^k)'' + (\frac{N-1}{r} + h_1(r))[(u_d^k)']^{p_d-1} \\
&= a_d(r)g_d(u_1^{k-1}(r),\dots,u_d^{k-1}(r)).
\end{aligned}
\tag{2.5}
$$

Using the monotonicity of $\{u_i^k\}_{i=1,\dots,d}^{k\geq 1}$ we have

$$
\begin{aligned}
a_1(r)g_1(u_1^{k-1}(r),\dots,u_d^{k-1}(r)) &\leq a_1(r)g_1(u_1^k,\dots,u_d^k) \\
&\leq a_1(r)\sum_{i=1}^d g_i\Big(\sum_{i=1}^d u_i^k,\dots,\sum_{i=1}^d u_i^k\Big), \\
&\quad \dots \\
a_d(r)g_d(u_1^{k-1}(r),\dots,u_d^{k-1}(r)) &\leq a_d(r)g_d(u_1^k,\dots,u_d^k) \\
&\leq a_d(r)\sum_{i=1}^d g_i\Big(\sum_{i=1}^d u_i^k,\dots,\sum_{i=1}^d u_i^k\Big);
\end{aligned}
\tag{2.6}
$$

moreover,

$$
\begin{aligned}
&(p_1-1)[(u_1^k(r))']^{p_1-1}(u_1^k)'' + (\frac{N-1}{r} + h_1(r))[(u_1^k(r))']^{p_1} \\
&\leq a_1(r)\sum_{i=1}^d g_i\Big(\sum_{i=1}^d u_i^k,\dots,\sum_{i=1}^d u_i^k\Big)(u_1^k(r))', \\
&\quad \dots \\
&(p_d-1)[(u_d^k(r))']^{p_d-1}(u_d^k)'' + (\frac{N-1}{r} + h_d(r))[(u_d^k(r))']^{p_d} \\
&\leq a_d(r)\sum_{i=1}^d g_i\Big(\sum_{i=1}^d u_i^k,\dots,\sum_{i=1}^d u_i^k\Big)(u_d^k(r))',
\end{aligned}
\tag{2.7}
$$

which implies

$$
\begin{aligned}
&(p_1-1)[(u_1^k(r))']^{p_1-1}(u_1^k)'' + (\frac{N-1}{r} + h_1(r))[(u_1^k(r))']^{p_1} \\
&\leq a_1(r)\sum_{i=1}^d g_i\Big(\sum_{i=1}^d u_i^k,\dots,\sum_{i=1}^d u_i^k\Big)\Big(\sum_{i=1}^d u_i^k(r)\Big)', \\
&\quad \dots \\
&(p_d-1)[(u_d^k(r))']^{p_d-1}(u_d^k)'' + (\frac{N-1}{r} + h_d(r))[(u_d^k(r))']^{p_d} \\
&\leq a_d(r)\sum_{i=1}^d g_i\Big(\sum_{i=1}^d u_i^k,\dots,\sum_{i=1}^d u_i^k\Big)\Big(\sum_{i=1}^d u_i^k(r)\Big)'.
\end{aligned}
\tag{2.8}
$$

Now if we let

$$
a_i^R = \max\{a_i(r) : 0 \leq r \leq R\}, \quad i = 1,\dots,d,
\tag{2.9}
$$

we can prove that $u_i^k(R)$ and $(u_i^k(R))'$, both of them are nonnegative and bounded above independent of k. Using (2.9) and the fact that $(u_i^k)' \geq 0$ for $i = 1, \dots, d$, we observe that (2.8) yields

$$(p_1 - 1)[(u_1^k)']^{p_1-1}(u_1^k)'' \leq a_1^R \sum_{i=1}^d g_i\Big(\sum_{i=1}^d u_i^k, \dots, \sum_{i=1}^d u_i^k\Big)\Big(\sum_{i=1}^d u_i^k(r)\Big)'$$
$$\dots$$
$$(p_d - 1)[(u_d^k)']^{p_d-1}(u_d^k)'' \leq a_d^R \sum_{i=1}^d g_i\Big(\sum_{i=1}^d u_i^k, \dots, \sum_{i=1}^d u_i^k\Big)\Big(\sum_{i=1}^d u_i^k(r)\Big)'$$

or, equivalently

$$\begin{gathered}\frac{p_1 - 1}{p_1}\{[(u_1^k)']^{p_1}\}' \leq a_1^R \sum_{i=1}^d g_i\Big(\sum_{i=1}^d u_i^k, \dots, \sum_{i=1}^d u_i^k\Big)\Big(\sum_{i=1}^d u_i^k(r)\Big)',\\ \dots \\ \frac{p_d - 1}{p_d}\{[(u_d^k)']^{p_d}\}' \leq a_d^R \sum_{i=1}^d g_i\Big(\sum_{i=1}^d u_i^k, \dots, \sum_{i=1}^d u_i^k\Big)\Big(\sum_{i=1}^d u_i^k(r)\Big)'.\end{gathered} \tag{2.10}$$

An integration of (2.10) in $(0, r)$ gives

$$\begin{aligned}\big[(u_1^k(r))'\big]^{p_1} &\leq \frac{p_1}{p_1 - 1} a_1^R \int_b^{\sum_{i=1}^d u_i^k(r)} \sum_{i=1}^d g_i(s, \dots, s)ds \\ &\leq \frac{p_1}{p_1 - 1} a_1^R \int_0^{\sum_{i=1}^d u_i^k(r)} \sum_{i=1}^d g_i(s, \dots, s)ds,\end{aligned} \tag{2.11}$$
$$\dots$$
$$\begin{aligned}\big[(u_d^k(r))'\big]^{p_d} &\leq \frac{p_d}{p_d - 1} a_d^R \int_b^{\sum_{i=1}^d u_i^k(r)} \sum_{i=1}^d g_i(s, \dots, s)ds \\ &\leq \frac{p_d}{p_d - 1} a_d^R \int_0^{\sum_{i=1}^d u_i^k(r)} \sum_{i=1}^d g_i(s, \dots, s)ds.\end{aligned} \tag{2.12}$$

At this stage, it is clear that

$$(u_1^k(r))' \leq \sqrt[p_1]{\frac{p_1}{p_1 - 1} a_1^R}\Big(\int_0^{\sum_{i=1}^d u_i^k(r)} \sum_{i=1}^d g_i(s, \dots, s)ds + 1\Big)^{1/\min\{p_1, \dots, p_d\}}, \tag{2.13}$$
$$\dots$$
$$(u_d^k(r))' \leq \sqrt[p_d]{\frac{p_d}{p_d - 1} a_d^R}\Big(\int_0^{\sum_{i=1}^d u_i^k(r)} \sum_{i=1}^d g_i(s, \dots, s)ds + 1\Big)^{1/\min\{p_1, \dots, p_d\}}, \tag{2.14}$$

Summing (2.13)-(2.14) and simplifying, we obtain

$$\begin{aligned}&\Big(\sum_{i=1}^d u_i^k(r)\Big)'\Big(\int_0^{\sum_{i=1}^d u_i^k(r)} \sum_{i=1}^d g_i(s, \dots, s)ds + 1\Big)^{-1/\min\{p_1, \dots, p_d\}} \\ &\leq \sum_{j=1}^d \sqrt[p_j]{\frac{p_j}{p_j - 1} a_j^R} \quad \text{for } 0 \leq r \leq R.\end{aligned} \tag{2.15}$$

Integrating (2.15) between 0 and R, we have

$$\begin{aligned}
&\int_b^{\sum_{i=1}^d u_i^k(R)} \Big[\int_0^t \sum_{i=1}^d g_i(s,\dots,s)ds+1\Big]^{-1/\min\{p_1,\dots,p_d\}} dt\\
&= I\Big(\sum_{i=1}^d u_i^k(R)\Big) - I(b) \le R\sum_{j=1}^d \sqrt[p_j]{\frac{p_j}{p_j-1}a_j^R}.
\end{aligned}$$

Since I is a bijection with I^{-1} increasing we obtain

$$\sum_{i=1}^d u_i^k(R) \le I^{-1}\Big(R\sum_{j=1}^d \sqrt[p_j]{\frac{p_j}{p_j-1}a_j^R} + I(b)\Big) \quad \text{for all } r\ge 0, \tag{2.16}$$

as in [8]. We are now in the position to observe that from the Keller-Osserman condition (1.5) we can conclude that $\sum_{i=1}^d u_i^k(R)$ is uniformly bounded above independent of k and using this fact in (2.15) shows that the same is true of $\big(\sum_{i=1}^d u_i^k(R)\big)'$. Then, since $u_i^k(r) \le u_i^k(R)$ ($r\le R$ and $u_i^k(r)$ is non-decreasing sequence!) for $i=1,\dots,d$ we obtain the conclusion that the sequences $u_i^k(r)$ are uniformly bounded above independent of k. Also, we clearly have $u_i^k(r)>0$ for all $r\ge 0$ and so our sequence is equi-continuous on $[0,R]$ for arbitrary $R>0$. A recapitulation of the above information says that $u_i^k(r)$ ($i=1,\dots,d$) is a monotonic, uniformly bounded, equi-continuous sequence of functions on $[0,R]$ and then there exists a function

$$(u_1,\dots,u_d)\in C([0,R])\times\dots\times C([0,R])$$

such that $u_i^k(r)\to u_i(r)$ ($i=1,\dots,d$) uniformly. Therefore, by an argument of a Fixed Point Theorem, it follows that $(u_1,\dots,u_d)$ is a fixed point of (2.4) in $C([0,R])\times\dots\times C([0,R])$.

Next, we extend this result to show that S has a fixed point in $C^1([0,\infty))\times\dots\times C^1([0,\infty))$. Let $\{u_i^k(r)\}_{i=1,\dots,d}^{k\ge1}$ be a sequence of fixed points defined by

$$\begin{gathered}
(u_1^k(r),\dots,u_d^k(r)) = S(u_1^k(r),\dots,u_d^k(r)) \quad \text{on } [0,k],\\
(u_1^k(r),\dots,u_d^k(r)) \in C([0,k])\times\dots\times C([0,k]),
\end{gathered} \tag{2.17}$$

for $k=1,2,3,\dots$. As earlier, we may show that both $u_1^k(r),\dots$ and $u_d^k(r)$ are bounded and equi-continuous on $[0,1]$. Thus by applying the Arzela-Ascoli Theorem to each sequence separately, we can derive that $\{(u_1^k(r),\dots,u_d^k(r))\}^{k\ge1}$ contains a convergent subsequence, $(u_1^{k_1^1}(r),\dots,u_d^{k_d^1}(r))$, that converges uniformly on $[0,1]\times\dots\times[0,1]$. Let

$$(u_1^{k_1^1}(r),\dots,u_d^{k_d^1}(r)) \to (u_1^1,\dots,u_d^1) \quad \text{uniformly on } [0,1]\times\dots\times[0,1]$$

as $k_1^1,\dots,k_d^1\to\infty$. Likewise, the subsequences $u_1^{k_1^1}(r),\dots,u_d^{k_d^1}(r)$ are bounded and equi-continous on $[0,2]$ so there exists a subsequence $(u_1^{k_1^2}(r),\dots,u_d^{k_d^2}(r))$ of $(u_1^{k_1^1}(r),\dots,u_d^{k_d^1}(r))$ such that $(u_1^{k_1^2}(r),\dots,u_d^{k_d^2}(r))\to(u_1^2,\dots,u_d^2)$ uniformly on $[0,2]\times\dots\times[0,2]$ as $k_1^2,\dots,k_d^2\to\infty$. Note that

$$\{(u_1^{k_1^2}(r),\dots,u_d^{k_d^2}(r))\} \subseteq \{(u_1^{k_1^1}(r),\dots,u_d^{k_d^1}(r))\} \subseteq \{(u_1^k(r),\dots,u_d^k(r))\}_{k\ge1}^\infty$$

so

$$(u_1^2,\dots,u_d^2) = (u_1^1,\dots,u_d^1) \quad \text{on } [0,1]\times\dots\times[0,1].$$

Continuing this reasoning, we obtain a sequence, denoted $(u_1^k(r),\dots,u_d^k(r))$, such that

$$\begin{gathered}(u_1^k(r),\dots,u_d^k(r))\in C([0,k])\times\dots\times C([0,k]),\quad k=1,2,\dots\\ (u_1^k(r),\dots,u_d^k(r))=(u_1^1(r),\dots,u_d^1(r))\quad\text{for } r\in[0,1]\\ (u_1^k(r),\dots,u_d^k(r))=(u_1^2(r),\dots,u_d^2(r))\quad\text{for } r\in[0,2]\\ \dots\\ (u_1^k(r),\dots,u_d^k(r))=(u_1^{k-1}(r),\dots,u_d^{k-1}(r))\quad\text{for } r\in[0,k-1],\end{gathered}$$

and these functions are radially symmetric. Therefore $(u_1^k(r),\dots,u_d^k(r))$ converges pointwise to some $(u_1(r),\dots,u_d(r))$ which satisfies

$$(u_1(r),\dots,u_d(r))=(u_1^k(r),\dots,u_d^k(r))\quad\text{if } 0\le r\le k.$$

Hence, $(u_1(r),\dots,u_d(r))$ is radially symmetric. Further, since $(u_1^k(r),\dots,u_d^k(r))$ is in the form (2.17), we have that $(u_1^k(r),\dots,u_d^k(r))$ is also equi-continuous. Pointwise convergence and equi-continuity imply uniform convergence and thus the convergence is uniform on bounded sets. Thus

$$(u_1(r),\dots,u_d(r))\in C^1([0,\infty))\times\dots\times C^1([0,\infty))$$

is a fixed point of (2.4) and a solution to (1.2) with central value $(\frac{b}{d},\dots,\frac{b}{d})$. Since $b\ge a>0$ was chosen arbitrarily, it follows that (1.2) has infinitely many positive entire solutions and so the first part of our theorem is proved.

Proof of (i) Assume that (1.6) holds. Finally, we show that any entire positive radial solution $(u_1,\dots,u_d)$ of system (1.2) is bounded. We choose $R>0$ so that

$$\frac{p_j}{p_j-1}r^{\frac{p_j(N-1)}{p_j-1}}e^{\frac{p_j}{p_j-1}\int_0^r h_j(t)dt}a_j(r)$$

are non-decreasing for $r\ge R$ and $j=1,\dots,d$. Multiply each line of the system

$$\begin{aligned}&(p_1-1)[(u_1(r))']^{p_1-1}(u_1)''+(\frac{N-1}{r}+h_1(r))[(u_1(r))']^{p_1}\\ &\le a_1(r)\sum_{i=1}^d g_i\Big(\sum_{i=1}^d u_i,\dots,\sum_{i=1}^d u_i\Big)\Big(\sum_{i=1}^d u_i(r)\Big)',\\ &\dots\\ &(p_d-1)[(u_d(r))']^{p_d-1}(u_d)''+(\frac{N-1}{r}+h_d(r))[(u_d(r))']^{p_d}\\ &\le a_d(r)\sum_{i=1}^d g_i\Big(\sum_{i=1}^d u_i,\dots,\sum_{i=1}^d u_i\Big)\Big(\sum_{i=1}^d u_i(r)\Big)'.\end{aligned}$$

by

$$\frac{p_i}{p_i-1}r^{\frac{p_i(N-1)}{p_i-1}}e^{\frac{p_i}{p_i-1}\int_0^r h_i(t)dt}\quad i=1,\dots,d,$$

where i represent the equation of the system that will be multiplied by. Then summing we have

$$\begin{aligned}&\Big[r^{\frac{p_1(N-1)}{p_1-1}}e^{\frac{p_1}{p_1-1}\int_0^r h_1(t)dt}(u_1')^{p_1}\Big]'\\ &\le r^{\frac{p_1(N-1)}{p_1-1}}\frac{p_1}{p_1-1}e^{\frac{p_1}{p_1-1}\int_0^r h_1(t)dt}a_1(r)\sum_{i=1}^d g_i\Big(\sum_{i=1}^d u_i,\dots,\sum_{i=1}^d u_i\Big)\Big(\sum_{i=1}^d u_i\Big)'\end{aligned}$$

$$
\dots
$$
$$
\begin{aligned}
&\Big[r^{\frac{p_d(N-1)}{p_d-1}}e^{\frac{p_d}{p_d-1}\int_0^r h_d(t)dt}(u_d')^{p_d}\Big]' \\
&\leq r^{\frac{p_d(N-1)}{p_d-1}}\frac{p_d}{p_d-1}e^{\frac{p_d}{p_d-1}\int_0^r h_d(t)dt}a_d(r)\sum_{i=1}^d g_i\Big(\sum_{i=1}^d u_i,\dots,\sum_{i=1}^d u_i\Big)\Big(\sum_{i=1}^d u_i\Big)'.
\end{aligned}
$$

Integrating this gives

$$
\begin{aligned}
&\int_R^r\Big[s^{\frac{p_1(N-1)}{p_1-1}}(e^{\frac{1}{p_1-1}\int_0^s h_1(t)dt}u_1')^{p_1}\Big]'ds \\
&\leq \int_R^r s^{\frac{p_1(N-1)}{p_1-1}}\frac{p_1}{p_1-1}e^{\frac{p_1}{p_1-1}\int_0^s h_1(t)dt}a_1(s)\sum_{i=1}^d g_i\Big(\sum_{i=1}^d u_i,\dots,\sum_{i=1}^d u_i\Big)\Big(\sum_{i=1}^d u_i\Big)'ds,
\end{aligned}
\tag{2.18}
$$
$$
\dots
$$
$$
\begin{aligned}
&\int_R^r\Big[s^{\frac{p_d(N-1)}{p_d-1}}(e^{\frac{1}{p_d-1}\int_0^s h_d(t)dt}u_d')^{p_d}\Big]'ds \\
&\leq \int_R^r s^{\frac{p_d(N-1)}{p_d-1}}\frac{p_d}{p_d-1}e^{\frac{p_d}{p_d-1}\int_0^s h_d(t)dt}a_d(s)\sum_{i=1}^d g_i\Big(\sum_{i=1}^d u_i,\dots,\sum_{i=1}^d u_i\Big)\Big(\sum_{i=1}^d u_i\Big)'ds.
\end{aligned}
\tag{2.19}
$$

With the use of (2.18)-(2.19) we obtain

$$
\begin{aligned}
&r^{\frac{p_1(N-1)}{p_1-1}}\Big(e^{\frac{1}{p_1-1}\int_0^r h_1(t)dt}u_1'(r)\Big)^{p_1}-R^{\frac{p_1(N-1)}{p_1-1}}\Big(e^{\frac{1}{p_1-1}\int_0^R h_1(t)dt}(u_1'(R))\Big)^{p_1} \\
&\leq \int_R^r s^{\frac{p_1(N-1)}{p_1-1}}\frac{p_1}{p_1-1}e^{\frac{p_1}{p_1-1}\int_0^s h_1(t)dt}a_1(s)\sum_{i=1}^d g_i\Big(\sum_{i=1}^d u_i,\dots,\sum_{i=1}^d u_i\Big)\Big(\sum_{i=1}^d u_i\Big)'ds, \\
&\dots \\
&r^{\frac{p_d(N-1)}{p_d-1}}\Big(e^{\frac{1}{p_d-1}\int_0^r h_d(t)dt}u_d'(r)\Big)^{p_d}-R^{\frac{p_d(N-1)}{p_d-1}}\Big(e^{\frac{1}{p_d-1}\int_0^R h_d(t)dt}u_d'(R)\Big)^{p_d} \\
&\leq \int_R^r s^{\frac{p_d(N-1)}{p_d-1}}\frac{p_d}{p_d-1}e^{\frac{p_d}{p_d-1}\int_0^s h_d(t)dt}a_d(s)\sum_{i=1}^d g_i\Big(\sum_{i=1}^d u_i,\dots,\sum_{i=1}^d u_i\Big)\Big(\sum_{i=1}^d u_i\Big)'ds,
\end{aligned}
$$

for $r \geq R$.

Noting that, by the monotonicity of

$$
\frac{p_j}{p_j-1}s^{\frac{p_j(N-1)}{p_j-1}}e^{\frac{p_j}{p_j-1}\int_0^s h_j(t)dt}a_j(s)
$$

for $j = 1,\dots,d$ and $r \geq s \geq R$, we obtain

$$
\begin{aligned}
&r^{\frac{p_1(N-1)}{p_1-1}}\Big(e^{\frac{1}{p_1-1}\int_0^r h_1(t)dt}u_1'\Big)^{p_1} \\
&\leq C+\frac{p_1}{p_1-1}r^{\frac{p_1(N-1)}{p_1-1}}e^{\frac{p_1}{p_1-1}\int_0^r h_1(t)dt}a_1(r)G\Big(\sum_{i=1}^d u_i\Big), \\
&\dots, \\
&r^{\frac{p_d(N-1)}{p_d-1}}\Big(e^{\frac{1}{p_d-1}\int_0^r h_d(t)dt}u_d'\Big)^{p_d}
\end{aligned}
$$

$$\le C + \frac{p_d}{p_d-1} r^{\frac{p_d(N-1)}{p_d-1}} e^{\frac{p_d}{p_d-1}\int_0^r h_d(t)dt} a_d(r) G\Big(\sum_{i=1}^d u_i\Big),$$

which yields

$$\begin{aligned} &r^{\frac{N-1}{p_1-1}} e^{\frac{1}{p_1-1}\int_0^r h_1(t)dt} u_1' \\ &\le \Big[C + \frac{p_1}{p_1-1} r^{\frac{p_1(N-1)}{p_1-1}} e^{\frac{p_1}{p_1-1}\int_0^r h_d(t)dt} a_1(r) G\Big(\sum_{i=1}^d u_i(r)\Big)\Big]^{1/p_1} \end{aligned} \tag{2.20}$$

$$\dots$$

$$\begin{aligned} &r^{\frac{N-1}{p_d-1}} e^{\frac{1}{p_d-1}\int_0^r h_1(t)dt} u_d' \\ &\le \big[C + \frac{p_d}{p_d-1} r^{\frac{p_d(N-1)}{p_d-1}} e^{\frac{p_d}{p_d-1}\int_0^r h_d(t)dt} a_d(r) G\Big(\sum_{i=1}^d u_i(r)\Big)\Big]^{1/p_d} \end{aligned} \tag{2.21}$$

where

$$\begin{aligned} C = \max\Big\{ & R^{\frac{p_1(N-1)}{p_1-1}} \big[e^{\frac{1}{p_1-1}\int_0^R h_1(t)dt} (u_1(R))'\big]^{p_1}, \dots, \\ & R^{\frac{p_d(N-1)}{p_d-1}} \big[e^{\frac{1}{p_d-1}\int_0^R h_d(t)dt} (u_d(R))'\big]^{p_d}\Big\}. \end{aligned}$$

We need to recall an important inequality which is the key ingredient of our next proof. Since $(1/p_i) < 1$ we know that

$$(b_1 + b_2)^{1/p_i} \le b_1^{1/p_i} + b_2^{1/p_i}$$

for any non-negative constants b_i and $i = 1, 2$. Therefore, by applying these inequalities in (2.20) and (2.21) we obtain

$$\begin{aligned} u_1' &\le e^{\frac{1}{p_1-1}\int_0^r h_1(t)dt} u_1' \\ &\le \sqrt[p_1]{C} r^{\frac{1-N}{p_1-1}} + r^{\frac{1-N}{p_1-1}} \Big[\frac{p_1}{p_1-1} r^{\frac{p_1(N-1)}{p_1-1}} e^{\frac{p_1}{p_1-1}\int_0^r h_1(t)dt} a_1(r)\Big]^{1/p_1} \big[G(\sum_{i=1}^d u_i)\big]^{1/p_1} \\ &\dots \\ u_d' &\le e^{\frac{1}{p_d-1}\int_0^r h_d(t)dt} u_d' \\ &\le \sqrt[p_d]{C} r^{\frac{1-N}{p_d-1}} + r^{\frac{1-N}{p_d-1}} \Big[\frac{p_d}{p_d-1} r^{\frac{p_d(N-1)}{p_d-1}} e^{\frac{p_d}{p_d-1}\int_0^r h_d(t)dt} a_d(r)\Big]^{1/p_d} \big[G(\sum_{i=1}^d u_i)\big]^{1/p_d}. \end{aligned}$$

Summing the above inequalities and integrating, we obtain

$$\begin{aligned} &\frac{d}{dr}\int_{\sum_{i=1}^d u_i(R)}^{\sum_{i=1}^d u_i(r)} [G(t)]^{-1/\min\{p_1,\dots,p_d\}} dt \\ &\le \sum_{j=1}^d \sqrt[p_j]{C} r^{\frac{1-N}{p_j-1}} \Big[G(\sum_{i=1}^d u_i(r))\Big]^{-1/\min\{p_1,\dots,p_d\}} \\ &\quad + \sum_{i=1}^d \Big(\frac{p_i}{p_i-1} e^{\frac{p_i}{p_i-1}\int_0^r h_i(t)dt} a_i(r)\Big)^{1/p_i}. \end{aligned} \tag{2.22}$$

Inequality (2.22) combined with

$$\begin{aligned}\Big(e^{\frac{p_i}{p_i-1}\int_0^r h_i(t)dt}a_i(s)\Big)^{1/p_i} &= \Big(s^{p_i(1+\varepsilon)/2}e^{\frac{p_i}{p_i-1}\int_0^r h_i(t)dt}a_i(s)s^{-p_i(1+\varepsilon)/2}\Big)^{1/p_i}\\ &\leq (\frac{1}{2})^{1/p_i}\Big[s^{1+\varepsilon}(e^{\frac{p_i}{p_i-1}\int_0^r h_i(t)dt}a_i(r))^{2/p_i}+s^{-1-\varepsilon}\Big],\end{aligned}$$

for each $\varepsilon > 0$, yields

$$\begin{aligned}&\int_{\sum_{i=1}^d u_i(R)}^{\sum_{i=1}^d u_i(r)}[G(t)]^{-1/\min\{p_1,\dots,p_d\}}dt\\ &\leq \int_R^r \sum_{j=1}^d \sqrt[p_j]{C}t^{\frac{1-N}{p_j-1}}\Big[G\Big(\sum_{i=1}^d u_i(t)\Big)\Big]^{-1/\min\{p_1,\dots,p_d\}}dt\\ &\quad+\sum_{i=1}^d(\frac{1}{2})^{1/p_i}\sqrt[p_i]{\frac{p_i}{p_i-1}}\Big[\int_R^r t^{1+\varepsilon}\Big(e^{\frac{p_i}{p_i-1}\int_0^t h_i(s)ds}a_i(t)\Big)^{2/p_i}dt+\int_R^r t^{-1-\varepsilon}dt\Big]\\ &\leq \sum_{j=1}^d \sqrt[p_j]{C}\Big[G\Big(\sum_{i=1}^d u_i(R)\Big)\Big]^{-1/\min\{p_1,\dots,p_d\}}\frac{p_j-1}{p_j-N}R^{\frac{p_j-N}{p_j-1}}\\ &\quad+\sum_{i=1}^d(\frac{1}{2})^{1/p_i}\sqrt[p_i]{\frac{p_i}{p_i-1}}\Big[\int_R^r t^{1+\varepsilon}(e^{\frac{p_i}{p_i-1}\int_0^t h_i(s)ds}a_i(t))^{2/p_i}dt+\frac{1}{\varepsilon R^{\varepsilon}}\Big].\end{aligned} \tag{2.23}$$

Since the right side of this inequality is bounded (note that $u_i(t) \geq b/d$), so is the left side and hence, in light of Keller-Osserman condition, the sequence $\sum_{i=1}^d u_i(r)$ is bounded and finally $u_i(r)$ $(i = 1,\dots,d)$ is a bounded function. Thus, for every $x \in \mathbb{R}^N$ the function $(u_1(|x|),\dots,u_d(|x|))$ is a positive bounded solution of (1.2).

Proof of (ii) Suppose that a_i $(i = 1,\dots,d)$ satisfies 1.7). Now, let $(u_1,\dots,u_d)$ be any positive entire radial solution of (1.2) determined in the first step of the proof. Clearly

$$(u_1(r),\dots,u_d(r)) \geq (\frac{b}{d},\dots,\frac{b}{d})$$

and since g_j are non-decreasing on $[0,\infty)^d$ in all variables it follows

$$g_j(u_1(r),\dots,u_d(r)) \geq g_j(\frac{b}{d},\dots,\frac{b}{d}). \tag{2.24}$$

On the other hand, substituting (2.24) in the system (2.1) we obtain

$$\begin{gathered}(p_1-1)(u_1')^{p_1-2}u_1''+\frac{N-1}{r}(u_1')^{p_1-1}+h_1(r)|u_1'|^{p_1-1} \geq a_1(r)g_1(\frac{b}{d},\dots,\frac{b}{d}),\\ \dots\\ (p_d-1)(u_d')^{p_d-2}u_d''+\frac{N-1}{r}(u_d')^{p_d-1}+h_d(r)|u_d'|^{p_d-1} \geq a_d(r)g_d(\frac{b}{d},\dots,\frac{b}{d}),\end{gathered}$$

or, equivalently

$$\begin{gathered}\big[r^{N-1}e^{\int_0^r h_1(t)dt}(u_1')^{p_1-1}\big]' \geq r^{N-1}e^{\int_0^r h_1(t)dt}a_1(r)g_1(\frac{b}{d},\dots,\frac{b}{d}),\\ \dots\\ \big[r^{N-1}e^{\int_0^r h_d(t)dt}(u_d')^{p_d-1}\big]' \geq r^{N-1}e^{\int_0^r h_d(t)dt}a_d(r)g_d(\frac{b}{d},\dots,\frac{b}{d}).\end{gathered}$$

However, this system of inequalities may be written as

$$u_1(r) \geq \frac{b}{d} + g_1^{\frac{1}{p_1-1}}(\frac{b}{d},\dots,\frac{b}{d}) \int_0^r \Big(\frac{e^{-\int_0^t h_1(s)ds}}{t^{N-1}} \int_0^t s^{N-1} e^{\int_0^s h_1(s)ds} a_1(s) ds \Big)^{\frac{1}{p_1-1}} dt,$$

$$\dots$$

$$u_d(r) \geq \frac{b}{d} + g_d^{\frac{1}{p_d-1}}(\frac{b}{d},\dots,\frac{b}{d}) \int_0^r \Big(\frac{e^{-\int_0^t h_d(s)ds}}{t^{N-1}} \int_0^t s^{N-1} e^{\int_0^s h_d(s)ds} a_d(s) ds \Big)^{\frac{1}{p_d-1}} dt.$$

It is evident that $r \to \infty$ implies $(u_1(r),\dots,u_d(r)) \to (\infty,\dots,\infty)$. The proof is complete.

From the above proof and the work [4] we can easy obtain the following remark.

Remark 2.1. Under the same assumptions as in Theorem 1.1 on a_j, h_j and g_j except for (i)-(ii). If (2.2) has a nonnegative entire large solution, then a_j ($j = 1,\dots,d$) satisfy

$$\sum_{j=1}^{d} (\frac{1}{2})^{1/p_j} \sqrt[p_j]{\frac{p_j}{p_j-1}} \int_0^\infty t^{1+\varepsilon} \Big(e^{\frac{p_j}{p_j-1}\int_0^t h_j(s)ds} a_j(t) \Big)^{2/p_j} dt = \infty, \tag{2.25}$$

for every $\varepsilon > 0$.

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Dragos-Patru C. Covei
Department of Development, Constantin Brancusi University from Tg-Jiu, Str. Grivitei, Nr. 1, Tg-Jiu, Romania
E-mail address: coveipatru@yahoo.com