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ATTRACTORS FOR STOCHASTIC STRONGLY DAMPED PLATE EQUATIONS WITH ADDITIVE NOISE

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Abstract. We study the asymptotic behavior of stochastic plate equations with homogeneous Neumann boundary conditions. We show the existence of an attractor for the random dynamical system associated with the equation.

1. Introduction

Let Ω be a bounded open set of $\mathbb{R}^n$ $(n = 5)$ with a smooth boundary $\partial\Omega$. We consider the stochastic strongly damped plate equation with additive noise,

$$\begin{gathered}
du_t + du + (f(u) + \Delta^2 u + \Delta^2 u_t)dt = g dt + \sum_{j=1}^{m} h_j dW_j, \\
u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \\
u|_{\partial\Omega} = \frac{\partial u}{\partial n}|_{\partial\Omega} = 0, \quad t \geqslant 0,
\end{gathered} \tag{1.1}$$

for $(x,t) \in \Omega \times [0,+\infty)$, where $u_0 \in H_0^2(\Omega)$ and $u_1 \in L^2(\Omega)$. Here $u = u(x,t)$ is a real valued function on $\Omega \times [0,+\infty), g \in H_0^2(\Omega)$ is a given external force. The nonlinear term f is a C^1-function with $f(0) = 0$, that satisfies the following conditions:

$$\liminf_{|s|\to\infty} \frac{f(s)}{s} > -\lambda_1^2, \quad \forall s \in \mathbb{R}, \tag{1.2}$$

$$|f'(s)| \leqslant C(1 + |s|^8), \quad \forall s \in \mathbb{R}, \tag{1.3}$$

$$f(s + \varpi) = f(s), \quad \forall s \in \mathbb{R}, \varpi \geqslant 0, \tag{1.4}$$

where λ_1 is the first eigenvalue of Δ^2 on $H_0^2(\Omega)$ and C is a positive constant. $h_j \in H^4(\Omega) \cap H_0^2(\Omega)$ with $\frac{\partial h_j}{\partial n} = 0$ on $\partial\Omega, j = 1, \dots, m$, and $\{W_j\}_{j=1}^m$ are independent two-sided real-valued Wiener processes on a probability space $(\Theta, \mathscr{F}, \mathbb{P})$, where

$$\Theta = \{\omega = (\omega_1, \omega_2, \dots, \omega_m) \in C(\mathbb{R}, \mathbb{R}^m) : \omega(0) = 0\}$$

is endowed with compact open topology, $\mathbb{P}$ is the corresponding Wiener measure, and $\mathscr{F}$ is the $\mathbb{P}$-completion of Borel σ-algebra on Θ.

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We identify ω with $(W_1, W_2, \dots, W_m)$, as $\omega(t) = (W_1(t), W_2(t), \dots, W_m(t))$ for $t \in \mathbb{R}$. Define

$$\theta_t\omega(\cdot) = \omega(\cdot + t) - \omega(t), \quad t \in \mathbb{R}, \omega \in \Theta.$$

A random attractor of a random dynamical system is a measurable and compact invariant random set attracting all the orbits. When such an attracting set exists, it is the smallest attracting compact set and the largest invariant set [4]. This seems to be a good generalization of the now classical concept of a global attractor for deterministic dynamical systems [1]. The notion of a random attractor is very useful for many infinite-dimensional random dynamical systems (RDS), see [4, 5].

Many authors have studied the existence of a random attractor for an RDS. For instance, Crauel and Flandoli in [5] introduced the notion of a random attractor and obtained a general theorem on the existence of a random attractor for the RDS. Their theorem has been successfully applied to the stochastic reaction-diffusion equations and the stochastic Navier-Stokes equations. In [4] they generalized the notion of a random attractor for the stochastic dynamical system introduced previously and considered the stochastic nonlinear wave equations. The asymptotic behavior of solutions for stochastic wave equation has been studied by several authors (see [3, 6, 7, 10, 14]). The existence of global attractor for plate equation was studied in [8, 16]. And in [13], the author have investigated the existence of uniform attractor about the non-autonomous case. Recently, Yang and Kloeden in [15] studied the existence of a random attractor for a class of stochastic semilinear degenerate parabolic equations. But there were no results on the random attractor for the stochastic strongly damped plate equation with additive noise. It is therefore necessary to investigate this problem. In this article, we consider the asymptotic dynamics of the stochastic plate equation with homogeneous Neumann boundary condition.

This article is organized as follows. In section 2, we recall some basic concepts and properties for general random dynamical systems. In section 3, we first provide some basic settings about (1.1) and show that it generates a random dynamical system in proper function space, and then we prove the existence of a unique random attractor of the random dynamical system.

2. Random dynamical systems

In this section, we recall some basic knowledge about general random dynamical systems (see [1] for details).

Let $(X, \|\cdot\|_X)$ be a separable Hilbert space with Borel σ-algebra $\mathscr{B}(X)$ and let $(\Theta, \mathscr{F}, \mathbb{P}, (\theta_t)_{t\in\mathbb{R}})$ be a metric dynamical system.

Definition 2.1. Let $(\Theta, \mathscr{F}, \mathbb{P}, (\theta_t)_{t\in\mathbb{R}})$ be a metric dynamical system. Suppose that the mapping $\phi : \mathbb{R}^+ \times \Omega \times X \to X$ is $(\mathscr{B}(\mathbb{R}^+) \times \mathscr{F} \times \mathscr{B}(X), \mathscr{B}(X))$-measurable and satisfies the following two properties:

(1) $\phi(0, \omega)x = x$, and
(2) $\phi(s, \theta_t\omega) \circ \phi(t, \omega)x = \phi(s + t, \omega)x$

for all $s, t \in \mathbb{R}^+, x \in X$ and $\omega \in \Theta$. Then ϕ is called a random dynamical system (RDS). Moreover, ϕ is called a continuous RDS if ϕ is continuous with respect to x for $t \geqslant 0$ and $\omega \in \Theta$.

To study the asymptotic behavior of the RDS determined by (1.1), we first need to recall some definitions and properties.

A set-valued mapping $B : \Theta \to 2^X$ is called a random closed set if $B(\omega)$ is closed, nonempty, and $\omega \mapsto d(x, B(\omega))$ is measurable for all $x \in X$ for each $\omega \in \Theta$. A random set $\mathscr{B} := \{B(\omega)\}_{\omega\in\Theta}$ is said to tempered if

$$\lim_{t\to\infty} e^{-\eta t} \operatorname{diam}(B(\theta_{-t}\omega)) = 0$$

for a.e. $\omega \in \Theta$ and all $\eta > 0$, where $\operatorname{diam}(B) := \sup_{x,y\in B} d(x, y)$.

Let $\mathscr{D}$ be the collection of all tempered random sets in X. We will only deal with the system $\mathscr{D}$ of tempered random sets in this article.

Definition 2.2. A random set $\mathscr{A} := \{A(\omega)\}_{\omega\in\Theta} \in X$ is called a $\mathscr{D}$-random attractor for an RDS ϕ if

(1) $\mathscr{A}$ is a random compact set, i.e. $A(\omega)$ is nonempty and compact for a.e. $\omega \in \Theta$ and $\omega \mapsto d(x, A(\omega))$ is measurable for every $x \in X$;
(2) $\mathscr{A}$ is ϕ-invariant, i.e. $\phi(t, \omega, A(\omega)) = A(\theta_t\omega)$, for all $T \geqslant 0$ and a.e. $\omega \in \Theta$;
(3) $\mathscr{A}$ attracts all tempered random sets $B \in \mathscr{D}$ in the sense that

$$\lim_{t\to\infty} \operatorname{dist}(\phi(t, \theta_{-t}\omega, B(\theta_{-t}\omega)), A(\omega)) = 0, \quad \text{a.e. } \omega \in \Theta.$$

Theorem 2.3. *Let ϕ be a continuous random dynamical system over dynamical system $(\Omega, \mathscr{F}, \mathbb{P}, (\theta_t)_{t\in\mathbb{R}})$. Suppose that there exists a $\mathscr{D}$-random absorbing set $\{B(\omega)\}_{\omega\in\Omega}$ which absorbs every tempered random set $D \in \mathscr{D}$. Then, ϕ has a unique $\mathscr{D}$-random attractor $\mathscr{A} = \{A(\omega)\}_{\omega\in\Omega}$, which is unique in the class of tempered random sets with*

$$A(\omega) = \cap_{\tau\geqslant 0}\overline{\cup_{t\geqslant\tau}\phi(t, \theta_{-t}\omega, B(\theta_{-t}\omega))}, \quad \omega \in \Omega.$$

3. Attractor for the strongly damped plate equation

3.1. **Basic settings.** In this subsection, we give some basic settings about (1.1) and show that it generates a random dynamical system.

Let $A = \Delta^2$, then $D(A) = \{u \in H^4(\Omega) \cap H_0^2(\Omega) : \frac{\partial u}{\partial n}|_{\partial\Omega} = 0\}$. Clearly, A is a self-adjoint, positive linear operator with the eigenvalues $\{\lambda_i\}_{i\in\mathbb{N}}$:

$$0 = \lambda_0 < \lambda_1 \leqslant \lambda_2 \leqslant \dots \leqslant \lambda_i \leqslant \dots, \lambda_i \to +\infty \quad (i \to +\infty).$$

Let $E = H_0^2(\Omega) \times L^2(\Omega)$, which is a separable Hilbert space endowed with the usual norm

$$\|Y\|_{H_0^2\times L^2} = (\|\Delta u\|^2 + \|v\|^2)^{1/2} \quad \text{for } Y = (u, v)^\top, \tag{3.1}$$

where $\|\cdot\|$ denotes the usual norm in $L^2(\Omega)$ and $\top$ stands for the transposition.

For our purpose, it is convenient to convert the problem (1.1) into a deterministic system with a random parameter, and then show that it generates a random dynamical system. Consider Ornstein-Uhlenbeck equations

$$dz_j + z_j dt = dW_j(t), \quad j = \{1, 2, \dots, m\}, \tag{3.2}$$

and Ornstein-Uhlenbeck processes

$$z_j(\theta_t\omega_j) = -\int_{-\infty}^{0} e^s(\theta_t\omega_j)(s)ds, \quad t \in \mathbb{R}.$$

From [2], it is known that the random variable $|z_j(\omega_j)|$ is tempered, and there is a θ_t-invariant set $\widetilde{\Theta} \subset \Theta$ of full $\mathbb{P}$ measure such that $t \mapsto z_j(\theta_t\omega_j)$ is continuous in t

for every $\omega \in \Theta$ and $j = 1, 2, \dots, m$. Put

$$z(\theta_t\omega) = z(x, \theta_t\omega) = \sum_{j=1}^{m} h_j z_j(\theta_t\omega_j), \tag{3.3}$$

which is a solution to

$$dz + zdt = \sum_{j=1}^{m} h_j dW_j.$$

Lemma 3.1 ([11]). *For any $\epsilon > 0$, there exist tempered random variable $r, r^{(l)} : \Theta \mapsto \mathbb{R}^+, l = \frac{1}{2}, 1$, such that for all $t \in \mathbb{R}, \omega \in \Theta$,*

$$\|z(\theta_t\omega)\| \leqslant e^{\epsilon|t|} r(\omega), \quad e^{-\epsilon|t|} r(\omega) \leqslant r(\theta_t\omega) \leqslant e^{\epsilon|t|} r(\omega),$$
$$\|A^{(l)} z(\theta_t\omega)\| \leqslant e^{\epsilon|t|} r^{(l)}(\omega), \quad e^{-\epsilon|t|} r^{(l)}(\omega) \leqslant r^{(l)}(\theta_t\omega) \leqslant e^{\epsilon|t|} r^{(l)}(\omega),$$

where $r^{(l)}(\omega) = \sum_{j=1}^{m} r_j(\omega_j)\|A^{(l)} h_j\|$.

It is convenient to reduce (1.1) to a evolution equation of first order in time

$$\begin{gathered}\dot{u} = v, \\ \dot{v} = -Av - Au - v - f(u) + g + \sum_{j=1}^{m} h_j \dot{W}_j, \\ u(x,0) = u_0(x), v(x,0) = u_1(x), x \in \Omega,\end{gathered} \tag{3.4}$$

Let

$$Y = \begin{pmatrix} u \\ v \end{pmatrix}, \quad M = \begin{pmatrix} 0 & I \\ -A & -A - I \end{pmatrix},$$
$$F(t, \omega, Y) = \begin{pmatrix} 0 \\ -f(u) + g + \sum_{j=1}^{m} h_j \dot{W}_j \end{pmatrix}.$$

Then problem (3.4) has the simple matrix form

$$\dot{Y} = MY + F(t, \omega, Y). \tag{3.5}$$

Let $\psi_1 = u, \psi_2 = v - z(\theta_t\omega)$, then (3.4) can be rewritten as the equivalent system, in E,

$$\begin{gathered}\dot{\psi_1} = \psi_2 + z(\theta_t\omega), \\ \dot{\psi_2} = -A\psi_1 - A\psi_2 - \psi_2 - f(\psi_1) + g - Az(\theta_t\omega), \\ \psi_1(x,0) = u_0(x), \quad \psi_2(x,0) = u_1(x) - z(\omega), \quad x \in \Omega,\end{gathered} \tag{3.6}$$

which has the vector form

$$\dot{\psi} = M\psi + \overline{F}(\theta_t\omega, \psi), \tag{3.7}$$

where

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \overline{F}(\theta_t\omega, \psi) = \begin{pmatrix} z(\theta_t\omega) \\ -f(\psi_1) + g - Az(\theta_t\omega) \end{pmatrix} \tag{3.8}$$

We will consider (3.5) or (3.7) for $\omega \in \widetilde{\Theta}$ and write $\widetilde{\Theta}$ as Θ from now on. From [9], M is an unbounded closed operator on E with domain $D(M)$,

$$D(M) = \{(u,v)^\top : u, v \in H_0^2(\Omega), u + v \in D(A)\}. \tag{3.9}$$

Moreover, the spectral set of M consists of only following points

$$\mu_i^{\pm} = \frac{-(\lambda_i + 1) \pm \sqrt{(\lambda_i + 1)^2 - 4\lambda_i}}{2}, \quad i = 0, 1, 2, \dots \tag{3.10}$$

and M generates a C^0-semigroup of bounded linear operators $\{e^{Mt}\}_{t\geqslant 0}$ on E.

Let $\overline{F}^{\omega}(t, \psi) := \overline{F}(\theta_t\omega, \psi)$, it is easy to verify that $\overline{F}^{\omega}(\cdot, \cdot) : [0, +\infty) \times E \to E$ is continuous in t and globally Lipschitz continuous in ψ for each $\omega \in \Theta$. By the classical semigroup theory on the existence and uniqueness of the solutions [12], we have the following theorem.

Theorem 3.2. *Consider* (3.7)*. For each $\omega \in \Theta$ and each $\psi_0 \in E$, there exists a unique function $\psi(\cdot, \omega, \psi_0) \in C([0, +\infty); E)$ such that $\psi(0, \omega, \psi_0) = \psi_0$ and $\psi(t, \omega, \psi_0)$ satisfies the integral equation*

$$\psi(t, \omega, \psi_0) = e^{Mt}\psi_0 + \int_0^t e^{M(t-s)}\overline{F}(\theta_s\omega, \psi(s, \omega, \psi_0))ds, \tag{3.11}$$

$\psi(t, \omega, \psi_0)$ is jointly continuous in t, ψ_0, and is measurable in ω. Furthermore, if $\psi_0 \in D(M)$, there exists $\psi(\cdot, \omega, \psi_0) \in C([0, +\infty); D(M)) \cap C^1([0, +\infty); E)$, which satisfies (3.7)*. Hence the solution mapping*

$$\overline{S}(t, \omega) : \psi_0 \mapsto \psi(t, \omega, \psi_0) \tag{3.12}$$

generates a random dynamical system.

Define a mapping $S(t, \omega)$ by

$$S(t, \omega) : Y_0 = \psi_0 + (0, z(\omega))^\top \mapsto Y(t, \omega, Y_0) = \psi(t, \omega, \psi_0) + (0, z(\theta_t\omega))^\top, \tag{3.13}$$

where $Y_0 = (u_0, u_1)^\top$ and $\psi_0 = (u_0, u_1 - z(\omega))^\top$. Then $S(t, \omega)$ is a continuous random dynamical system associated with the problem (3.5) or (1.1) on E. $S(t, \omega)$ has the following relation with $\bar{S}(t, \omega)$

$$\overline{S}(t, \omega) = R(\theta_t\omega)S(t, \omega)R^{-1}(\theta_t\omega), \tag{3.14}$$

where $R(\theta_t\omega) : (a, b)^\top \mapsto (a, b - z(\theta_t\omega))^\top$ is a homeomorphism of E.

We will also use the transformation

$$\varphi_1 = u = \psi_1, \quad \varphi_2 = v + \varepsilon u - z(\theta_t\omega),$$

where ε is a given positive number. Then the (3.7) can be rewritten as

$$\dot{\varphi} = M_\varepsilon\varphi + \overline{F}_\varepsilon(\theta_t\omega, \varphi), \varphi_0(x, 0) = (u_0(x), u_1(x) + \varepsilon u_0(x) - z(\omega)), \tag{3.15}$$

where

$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}, \quad M_\varepsilon = \begin{pmatrix} -\varepsilon I & I \\ \varepsilon(1 - \varepsilon)I + \varepsilon A - A & (\varepsilon - 1)I - A \end{pmatrix}, \tag{3.16}$$

$$\overline{F}_\varepsilon(\theta_t\omega, \varphi) = \begin{pmatrix} z(\theta_t\omega) \\ -f(\varphi_1) + g + \varepsilon z(\theta_t\omega) - Az(\theta_t\omega) \end{pmatrix}. \tag{3.17}$$

Then the mapping

$$\bar{S}_\varepsilon(t, \omega) = T_\varepsilon\overline{S}(t, \omega)T_{-\varepsilon} : \varphi_0 \mapsto \varphi(t, \omega, \varphi_0), \tag{3.18}$$

generates a random dynamical system associated with (3.15), where $\varphi_0 = (u_0, u_1 + \varepsilon u_0 - z(\omega))^\top$, and $T_\varepsilon : (a, b)^\top \mapsto (a, b + \varepsilon a)^\top$ is a isomorphism of E.

Notice that all the above random dynamical systems $S(t, \omega), \overline{S}(t, \omega), \overline{S}_\varepsilon(t, \omega)$ are equivalent. Hence, we only need to consider the random dynamical system $\overline{S}(t, \omega)$.

Let $p_0 = (\varpi, 0)^\top = \varpi\eta_0 \in E_1$, then $Mp_0 = 0$. Thus, by the periodicity of function f, the random dynamical system $\overline{S}(t,\omega)$ is p_0-translation invariant in the sense that

$$\psi(t,\omega,\psi_0 + p_0) = \psi(t,\omega,\psi_0) + p_0, t \geqslant 0, \omega \in \Theta, \psi_0 \in E, \tag{3.19}$$

which implies that the average of the first component of $\psi(t,\omega,\psi_0+p_0)$ will be unbounded in E_1 (corresponding to the direction of η_0 with respect to 0 eigenvalue), hence $\overline{S}(t,\omega)$ is unbounded in the direction of η_0 in E_1, which means that it is impossible to obtain a bounded attractor for $\overline{S}(t,\omega)$ as usual. So we need to introduce a random dynamical system $\overline{\Phi}(t,\omega)$ defined on cylinder induced from $\overline{S}(t,\omega)$ according to p_0-translation invariance of $\bar{S}(t,\omega)$. To this end, we introduce some space and notation.

For any $u \in L^2(\Omega)$, define the spatial average of u as

$$\overline{u} = \frac{1}{|\Omega|}\int_\Omega u(x)dx. \tag{3.20}$$

Let

$$\dot{L}^2(\Omega) = \{u \in L^2(\Omega) : \bar{u} = 0\}, \quad \dot{H}_0^2(\Omega) = H_0^2(\Omega) \cap \dot{L}^2(\Omega), E_{22} = \dot{H}_0^2(\Omega) \times \dot{L}^2(\Omega)$$

By (3.10), M has two real eigenvalues 0 and -1 with eigenvectors $\eta_0 = (1,0)^\top$ and $\eta_{-1} = (1,-1)^\top$. Let

$$E_1 = \operatorname{span}\{\eta_0\}, \quad E_{-1} = \operatorname{span}\{\eta_{-1}\}, \quad E_{11} = E_1 \oplus E_{-1} = \mathbb{R}^2, \quad E_2 = E_{-1} \oplus E_{22}.$$

Then

$$E = E_{11} \oplus E_{22} = \mathbb{R}^2 \oplus E_{22} = E_1 \oplus E_{-1} \oplus E_{22} = E_1 \oplus E_2, \tag{3.21}$$

and E_1 is positive invariant under M.

Let $\mathbb{T}^1 = E_1/p_0\mathbb{Z}$ and $\mathbf{E} = \mathbb{T}^1 \oplus E_2 = \mathbb{T}^1 \oplus E_{-1} \oplus E_{22} = \mathbb{T}^1 \times E_{-1} \times E_{22}$. For $\Psi_0 := \psi_0(mod p_0) = \Psi_0 + p_0\mathbb{Z} \subset E$ denotes the equivalence class of Ψ_0, which is an element of $\mathbf{E}$. And the norm on $\mathbf{E}$ is denoted by

$$\|\Psi_0\|_{\mathbf{E}} = \inf_{y \in p_0\mathbb{Z}} \|\psi_0 + y\|_E.$$

Note that, $\psi(t,\omega,\psi_0 + kp_0) = \psi(t,\omega,\psi_0) + kp_0$, for all $k \in \mathbb{Z}$ for $t \geqslant 0, \omega \in \Theta$ and $\psi_0 \in E$. With this, we define

$$\overline{\Phi}(t,\omega) : \Psi_0 \mapsto \Psi(t,\omega,\Psi_0) = \psi(t,\omega,\psi_0)(mod p_0). \tag{3.22}$$

It is easy to see that $\overline{\Phi}(t,\omega)$ is a random dynamical system on $\mathbf{E}$.

Similarly, the random dynamical system $S(t,\omega)$ also induces a random dynamical system $\Phi(t,\omega)$ on $\mathbf{E}$ defined by

$$\Phi(t,\omega) : \mathbf{Y}_0 \mapsto \mathbf{Y}(t,\omega,\mathbf{Y}_0) = \Psi(t,\omega,\Psi_0) + \mathbf{Z}(\theta_t\omega)(mod p_0), \tag{3.23}$$

where $\mathbf{Y}_0 = Y_0(mod p_0), \mathbf{Z}(\theta_t\omega) = (0, Z(\theta_t\omega))^\top$ and $\Psi_0 = \mathbf{Y}_0 - \mathbf{Z}(\omega)(mod p_0)$.

We introduce a new norm which is equivalent to the usual norm $\|\cdot\|_{H_0^2\times L^2}$ on E in (3.1). For $Y_i = (u_i, v_i)^\top \in E_{11}, i = 1,2$, let

$$\begin{aligned} &\langle Y_1, Y_2\rangle_{E_{11}} \\ &= \frac{1}{4}\langle u_1, u_2\rangle + \langle \frac{1}{2}u_1 + v_1, \frac{1}{2}u_2 + v_2\rangle, \end{aligned} \tag{3.24}$$

where $\langle\cdot,\cdot\rangle$ denotes the inner product on $L^2(\Omega)$, and for $Y_i = (u_i, v_i)^\top \in E_{22}, i = 1,2$, let

$$\langle Y_1, Y_2\rangle_{E_{22}} = \mu\langle A^{1/2}u_1, A^{1/2}u_2\rangle + \langle v_1, v_2\rangle, \tag{3.25}$$

where $A^{1/2} = \Delta$ and μ is chosen such that $\mu = 1 - \varepsilon \in (\frac{1}{2}, 1)$ in which $\varepsilon \in (0, 1)$ is a small positive number. By the generalized Poincaré inequality

$$\|A^{1/2}u\|^2 \geqslant \lambda_1^{1/2}\|u\|^2, \quad \forall u \in \dot{H}_0^2(\Omega),$$

Expression (3.25) is then positive definite. A bilinear form on E can be induced from (3.24) and (3.25),

$$\langle X, Y\rangle_E = \langle \overline{X}, \overline{Y}\rangle_{E_{11}} + \langle X - \overline{X}, Y - \overline{Y}\rangle_{E_{22}}, \tag{3.26}$$

where $X = \overline{X} + X - \overline{X} \in E, Y = \overline{Y} + Y - \overline{Y} \in E$, with $\overline{X}, \overline{Y} \in E_{11}$ and $X - \overline{X}, Y - \overline{Y} \in E_{22}$. It is easy to obtain the following fact.

Lemma 3.3. *Expressions* (3.24) *and* (3.25) *define inner products on* E_{11} *and* E_{22}, *respectively. Meanwhile,* (3.26) *defines an inner products on* E, *and the corresponding norm* $\|\cdot\|_E$ *is equivalent to the usual norm* $\|\cdot\|_{H_0^2 \times L^2}$ *in* (3.1).

Under the inner product $\langle\cdot,\cdot\rangle_E, E_1 \perp E_{-1}, E_{11} \perp E_{22}, E_1 \perp E_2$. Denote by $P, \overline{Q}$ and Q the projections from E into E_1, E_{-1} and E_{22}, respectively:

$$PY = \begin{pmatrix} \overline{u} + \overline{v} \\ 0 \end{pmatrix} \in E_1, \quad \overline{Q}Y = \begin{pmatrix} -\overline{v} \\ \overline{v} \end{pmatrix} \in E_{-1}, \quad QY = Y - \overline{Y} = \begin{pmatrix} u - \overline{u} \\ v - \overline{v} \end{pmatrix} \in E_{22},$$

where $Y = (u, v)^\top \in E$. Sometimes we write $Qu = u - \overline{u}$ for $u \in L^2(\Omega)$.

3.2. **Random attractor.** First we consider the boundedness of the component $\overline{Q}\psi$ of solution ψ of (3.7) in E_{-1}. Taking the average of (3.6), by Green's formula and Neumann boundary condition (1.2), and take the second equation, we have

$$\dot{\overline{\psi}}_2 = -\overline{\psi}_2 - \overline{f(\psi_1)} + \overline{g}, \overline{\psi}_2(0) = \overline{u}_1 - \overline{z(\omega)}, \tag{3.27}$$

then

$$\frac{d}{dt}|\overline{\psi}_2(t, \omega)|^2 \leqslant -|\overline{\psi}_2(0, \omega)|^2 + (c_1 + |\overline{g}|)^2, t \geqslant 0, \tag{3.28}$$

thus,

$$|\overline{\psi}_2(t, \omega)|^2 \leqslant |\overline{\psi}_2(0, \omega)|^2 e^{-t} + (c_1 + |\overline{g}|)^2 = |\overline{u}_1 - \overline{z(\omega)}|^2 e^{-t} + (c_1 + |\overline{g}|)^2, \quad t \geqslant 0.$$

So if $|\overline{u}_1 - \overline{z(\omega)}|$ is tempered, then there exists $\overline{t}_0 \geqslant 0$ such that

$$|\overline{\psi}_2(t, \omega)| \leqslant 2(c_1 + |\overline{g}|), \quad t \geqslant \overline{t}_0. \tag{3.29}$$

This show the uniformly boundedness of $\overline{Q}\psi = (-\overline{\psi}_2, \overline{\psi}_2)$ of solution of (3.7) in one-dimensional subspace E_{-1} of $\mathbb{R}^2$, which implies that $\overline{Q}\psi$ possesses a compact absorbing set $\{B_{-1}(\omega)\}$ in E_{-1}.

Next we prove that $Q\psi$ of solution ψ of (3.7) possesses a compact attracting set in E_{22}.

Lemma 3.4. *There exists a small positive constant* $0 < \sigma < \varepsilon$ *such that*

$$\langle M_\varepsilon QY, QY\rangle_E \leqslant -\sigma\|QY\|_E^2 - \frac{1}{2}\|A^{1/2}Qv\|^2 - \frac{1}{2}\|Qv\|^2 \tag{3.30}$$

for $Y = (u, v)^\top \in E$, *and*

$$\begin{aligned}\langle M_\varepsilon QY, AQY\rangle_E \leqslant -\sigma\|A^{1/2}QY\|_E^2 - \frac{1}{2}\|AQv\|^2 - \frac{1}{2}\|A^{1/2}Qv\|^2 forY \\ = (u, v)^\top \in D(M) \cap E.\end{aligned}$$

The proof of the above lemma is similar to that of [17, Lemma 1], and it is omitted.

Lemma 3.5. *Assume that (1.2)–(1.4) and $g \in H_0^2(\Omega)$ hold. Then there exists a random ball $\{B_0(\omega)\} \in \mathscr{D}$ centered at 0 with random radius $\varrho(\omega) > 0$ such that for any $\{\widehat{B}(\omega)\} \in \mathscr{D}$, there is a $T_{\widehat{B}}(\omega, \varrho) > 0$ such that for any $\varphi_0(\theta_{-t}\omega) \in \widehat{B}(\theta_{-t}\omega)$ satisfies for a.e. $\omega \in \Theta$,*

$$\|Q\varphi(t, \theta_{-t}\omega, \varphi_0(\theta_{-t}\omega))\|_E \leqslant \varrho(\omega) \quad \forall t \geqslant T_{\widehat{B}}(\omega, \varrho). \tag{3.31}$$

Proof. By (3.15) and $QM_\varepsilon = M_\varepsilon Q$, we have

$$Q\dot{\varphi} = M_\varepsilon Q\varphi + Q\overline{F}_\varepsilon(\theta_t\omega, \varphi), \tag{3.32}$$

where

$$Q\overline{F}_\varepsilon(\theta_t\omega, \varphi) = \begin{pmatrix} Qz(\theta_t\omega) \\ Q[-f(u) + g + \varepsilon z(\theta_t\omega) - Az(\theta_t\omega)] \end{pmatrix}. \tag{3.33}$$

Taking the inner product $\langle \cdot, \cdot \rangle_E$ of (3.32) with $Q\varphi \in E_{22}$, we note that

$$\begin{aligned} \mu\langle A^{1/2}Qz(\theta_t\omega), A^{1/2}Q\varphi_1\rangle &\leqslant \mu\|A^{1/2}z(\theta_t\omega)\| \cdot \|A^{1/2}Q\varphi_1\| \\ &\leqslant \frac{\mu}{2\sigma}\|A^{1/2}z(\theta_t\omega)\|^2 + \frac{\sigma\mu}{2}\|A^{1/2}Q\varphi_1\|^2, \end{aligned}$$

$$\langle -(f(u) - \overline{f(u)}), Q\varphi_2\rangle \leqslant 2C(\varphi_1 + |\varphi_1|^9) \cdot \|Q\varphi_2\| \leqslant (2C(\varphi_1 + |\varphi_1|^9))^2 + \frac{1}{4}\|Q\varphi_2\|^2,$$

$$\langle g - \overline{g}, Q\varphi_2\rangle \leqslant \|g - \overline{g}\| \cdot \|Q\varphi_2\| \leqslant 4\|g\|^2 + \frac{1}{4}\|Q\varphi_2\|^2,$$

$$\begin{aligned} \langle \varepsilon(z(\theta_t\omega) - \overline{z(\theta_t\omega)}), Q\varphi_2\rangle &\leqslant \|\varepsilon(z(\theta_t\omega) - \overline{z(\theta_t\omega)})\| \cdot \|Q\varphi_2\| \\ &\leqslant \frac{\varepsilon^2}{\sigma}\|z(\theta_t\omega)\|^2 + \frac{\sigma}{2}\|Q\varphi_2\|^2, \end{aligned}$$

$$\langle Az(\theta_t\omega), Q\varphi_2\rangle \leqslant \|A^{1/2}z(\theta_t\omega)\| \cdot \|A^{1/2}Q\varphi_2\| \leqslant \frac{1}{2}\|A^{1/2}z(\theta_t\omega)\|^2 + \frac{1}{2}\|A^{1/2}Q\varphi_2\|^2,$$

$$\langle M_\varepsilon Q\varphi, Q\varphi\rangle_E \leqslant -\sigma\|Q\varphi\|_E^2 - \frac{1}{2}\|A^{1/2}Q\varphi_2\|^2 - \frac{1}{2}\|Q\varphi_2\|^2.$$

From the above inequalities, we have

$$\frac{d}{dt}\|Q\varphi\|_E^2 + 2\sigma\|Q\varphi\|_E^2 \leqslant 2R_0(\theta_t\omega), \tag{3.34}$$

where

$$R_0(\theta_t\omega) = \frac{\mu + \sigma}{2\sigma}\|A^{1/2}z(\theta_t\omega)\|^2 + (2C(|\varphi_1| + |\varphi_1|^9))^2 + 4\|g\|^2 + \frac{\varepsilon^2}{\sigma}\|z(\theta_t\omega)\|^2. \tag{3.35}$$

Applying the Gronwall lemma, for all $t \geqslant 0$, we have

$$\|Q\varphi(t, \omega, \varphi_0(\omega))\|_E^2 \leqslant e^{-2\sigma t}\|\varphi_0(\omega)\|_E^2 + 2\int_0^t R_0(\theta_s\omega)e^{-2\sigma(t-s)}ds. \tag{3.36}$$

By replacing ω by $\theta_{-t}\omega$, we get from (3.36) that, for all $t \geqslant 0$,

$$\begin{aligned} \|Q\varphi(t, \theta_{-t}\omega, \varphi_0(\theta_{-t}\omega))\|_E^2 &\leqslant e^{-2\sigma t}\|\varphi_0(\theta_{-t}\omega)\|_E^2 + 2\int_0^t R_0(\theta_{s-t}\omega)e^{-2\sigma(t-s)}ds \\ &= e^{-2\sigma t}\|\varphi_0(\theta_{-t}\omega)\|_E^2 + 2\int_{-t}^0 R_0(\theta_\tau\omega)e^{2\sigma\tau}d\tau. \end{aligned}$$

By Lemma 3.1 with $\epsilon = \frac{\sigma}{4}$, we have that

$$\int_{-t}^{0} R_0(\theta_\tau\omega)e^{2\sigma\tau}d\tau \leqslant \int_{-t}^{0} \widetilde{R_0}(\tau,\omega)e^{2\sigma\tau}d\tau \leqslant \int_{-\infty}^{0} \widetilde{R_0}(\tau,\omega)e^{2\sigma\tau}d\tau < +\infty, \quad (3.37)$$

where

$$\widetilde{R_0}(\tau,\omega) = \frac{\mu+\sigma}{2\sigma}(e^{\frac{\sigma}{4}|\tau|}r^{(1/2)}(\omega))^2 + (2C(|\varphi_1| + |\varphi_1|^9))^2 + 4\|g\|^2 + \frac{\varepsilon^2}{\sigma}(e^{\frac{\sigma}{4}|\tau|}r(\omega))^2.$$

Note that $\{\widehat{B}(\omega)\} \in \mathscr{D}$ is tempered, then for any $\varphi_0(\theta_{-t}\omega) \in \widehat{B}(\theta_{-t}\omega)$,

$$\lim_{t\to+\infty} e^{-2\sigma t}\|\varphi_0(\theta_{-t}\omega)\|_E^2 = 0.$$

Hence, there exists a $T_{\widehat{B}}(\omega,\varrho) > 0$ such that for any $\varphi_0(\theta_{-t}\omega) \in \widehat{B}(\theta_{-t}\omega)$ satisfies for a.e. $\omega \in \Theta$,

$$\|Q\varphi(t,\theta_{-t}\omega,\varphi_0(\theta_{-t}\omega))\|_E \leqslant \varrho(\omega) forallt \geqslant T_{\widehat{B}}(\omega,\varrho), \quad (3.38)$$

where

$$\varrho^2(\omega) = 2\int_{-\infty}^{0} \widetilde{R_0}(\tau,\omega)e^{2\sigma\tau}d\tau. \quad (3.39)$$

So, the proof is complete. □

We now construct a random compact attracting set for RDS $\bar{S}_\varepsilon(t,\omega)$. For this purpose, we split the solution φ of the system (3.7) with the initial value $\varphi_0 = (u_0, v_0 + \varepsilon u_0 - z(\omega))^\top$ into two parts $\varphi = \varphi^a + \varphi^b = (u^a, v^a + \varepsilon u^a)^\top + (u^b, v^b + \varepsilon u^b - z(\theta_t\omega))^\top$, where φ^a solves

$$\dot{\varphi}^a = M_\varepsilon\varphi^a, \varphi_0^a = (u_0, v_0 + \varepsilon u_0)^\top, \quad (3.40)$$

and φ^b solves

$$\dot{\varphi}^b = M_\varepsilon\varphi^b + \overline{F}_\varepsilon(\theta_t\omega,\varphi), \quad \varphi_0^b = (0, -z(\omega))^\top. \quad (3.41)$$

Lemma 3.6. *Assume that* (1.2)–(1.4) *and $g \in H_0^2(\Omega)$ hold. Then there exists a random variable $\varrho_1(\omega) > 0$ such that for any $\{\widehat{B}(\omega)\} \in \mathscr{D}$ and $\varphi_0(\omega) \in \widehat{B}(\omega)$, there is a $T_{\widehat{B}}(\omega,\varrho_1) > 0$ such that for any φ of the system* (3.7) *satisfies for a.e. $\omega \in \Theta$,*

$$\|Q\varphi^a(t,\theta_{-t}\omega,\varphi_0^a(\theta_{-t}\omega))\|_E \leqslant e^{-2\sigma t}\|\varphi_0^a(\theta_{-t}\omega)\|_E \to 0, \quad \text{as } t \to +\infty, \quad (3.42)$$

and

$$\|A^{1/2}Q\varphi^b(t,\theta_{-t}\omega,\varphi_0^b(\theta_{-t}\omega))\|_E \leqslant \varrho_1(\omega), \quad \forall t \geqslant T_{\widehat{B}}(\omega,\varrho_1), \quad (3.43)$$

where $Q\varphi^a$ and $Q\varphi^b$ satisfy (3.40) *and* (3.41).

Proof. By (3.40), we have

$$Q\dot{\varphi}^a = M_\varepsilon Q\varphi^a. \quad (3.44)$$

Take the inner product $\langle\cdot,\cdot\rangle_E$ of (3.44) with $Q\varphi^a$. By Lemma 3.5, we obtain

$$\|Q\varphi^a(t,\theta_{-t}\omega,\varphi_0^a(\theta_{-t}\omega))\|_E^2 \leqslant e^{-2\sigma t}\|\varphi_0^a(\theta_{-t}\omega)\|_E^2. \quad (3.45)$$

Then, the first assertion is valid.

From (3.41), we have

$$Q\dot{\varphi}^b = M_\varepsilon Q\varphi^b + Q\bar{F}_\varepsilon(\theta_t\omega,\varphi^b). \quad (3.46)$$

Take the inner product $\langle\cdot,\cdot\rangle_E$ of (3.46) with $AQ\varphi^b$. By Lemma 3.4, we have

$$\langle M_\varepsilon Q\varphi^b, AQ\varphi^b\rangle_E \leqslant -\sigma\|A^{1/2}Q\varphi^b\|_E^2 - \frac{1}{2}\|AQ\varphi_2^b\|^2 - \frac{1}{2}\|A^{1/2}Q\varphi_2^b\|^2. \quad (3.47)$$

By the Cauchy-Schwarz inequality, we obtain

$$
\begin{aligned}
\mu\langle A^{1/2}Qz(\theta_t\omega), A^{1/2}AQ\varphi_1^b\rangle &\leqslant \mu\|AQz(\theta_t\omega)\|\cdot\|AQ\varphi_1^b\| \\
&\leqslant \frac{\mu}{2\sigma}\|Az(\theta_t\omega)\|^2+\frac{\sigma\mu}{2}\|AQ\varphi_1^b\|^2,
\end{aligned}
$$

$$
\langle Qf(\varphi_1^b), AQ\varphi_2^b\rangle \leqslant \|A^{1/2}Qf(\varphi_1^b)\|\cdot\|A^{1/2}Q\varphi_2^b\| \leqslant \|A^{1/2}f(\varphi_1^b)\|^2+\frac{1}{4}\|A^{1/2}Q\varphi_2^b\|^2,
$$

$$
\langle Qg, AQ\varphi_2^b\rangle \leqslant \|A^{1/2}g\|\cdot\|A^{1/2}Q\varphi_2^b\| \leqslant \|A^{1/2}g\|^2+\frac{1}{4}\|A^{1/2}Q\varphi_2^b\|^2,
$$

$$
\begin{aligned}
\langle \varepsilon Qz(\theta_t\omega), AQ\varphi_2^b\rangle &\leqslant \|\varepsilon A^{1/2}Qz(\theta_t\omega)\|\cdot\|A^{1/2}Q\varphi_2^b\| \\
&\leqslant \frac{\varepsilon^2}{2\sigma}\|A^{1/2}z(\theta_t\omega)\|^2+\frac{\sigma}{2}\|A^{1/2}Q\varphi_2^b\|^2,
\end{aligned}
$$

$$
\langle QAz(\theta_t\omega), AQ\varphi_2^b\rangle \leqslant \|QAz(\theta_t\omega)\|\cdot\|AQ\varphi_2^b\| \leqslant \|Az(\theta_t\omega)\|^2+\frac{1}{2}\|QA\varphi_2^b\|^2.
$$

From the above inequalities and (3.47), we have

$$
\frac{d}{dt}\|A^{1/2}Q\varphi^b\|_E^2+2\sigma\|A^{1/2}Q\varphi^b\|_E^2 \leqslant 2R_1(\theta_t\omega), \tag{3.48}
$$

where

$$
R_1(\theta_t\omega)=\frac{\mu+2\sigma}{2\sigma}\|Az(\theta_t\omega)\|^2+\|A^{1/2}f(\varphi_1^b)\|^2+\|A^{1/2}g\|^2+\frac{\varepsilon^2}{2\sigma}\|A^{1/2}z(\theta_t\omega)\|^2.
$$

By Gronwall's lemma, for all $t\geqslant 0$,

$$
\begin{aligned}
&\|A^{1/2}Q\varphi^b(t,\omega,\varphi_0^b(\omega))\|_E^2 \\
&\leqslant e^{-2\sigma t}\|A^{1/2}\varphi_0^b(\omega)\|_E^2+2\int_0^t R_1(\theta_s\omega)e^{-2\sigma(t-s)}ds \\
&= e^{-2\sigma t}\|A^{1/2}z(\omega)\|_E^2+2\int_0^t R_1(\theta_s\omega)e^{-2\sigma(t-s)}ds.
\end{aligned} \tag{3.49}
$$

Replacing ω by $\theta_{-t}\omega$, in (3.49) we obtain that for all $t\geqslant 0$,

$$
\begin{aligned}
&\|A^{1/2}Q\varphi^b(t,\theta_{-t}\omega,\varphi_0^b(\theta_{-t}\omega))\|_E^2 \\
&\leqslant e^{-2\sigma t}\|A^{1/2}z(\theta_{-t}\omega)\|^2+2\int_0^t R_1(\theta_{s-t}\omega)e^{-2\sigma(t-s)}ds \\
&= e^{-2\sigma t}\|A^{1/2}z(\theta_{-t}\omega)\|_E^2+2\int_{-t}^0 R_1(\theta_\tau\omega)e^{2\sigma\tau}d\tau.
\end{aligned} \tag{3.50}
$$

By Lemma 3.1 with $\epsilon=\frac{\sigma}{4}$, we have

$$
\lim_{t\to+\infty}e^{-2\sigma t}\|A^{1/2}z(\theta_{-t}\omega)\|^2 \leqslant \lim_{t\to+\infty}e^{-2\sigma t}(e^{\frac{\sigma}{4}|\tau|}r^{(1/2)}(\omega))^2=0,
$$

$$
\int_{-t}^0 R_1(\theta_\tau\omega)e^{2\sigma\tau}d\tau \leqslant \int_{-t}^0 \widetilde{R_1}(\tau,\omega)e^{2\sigma\tau}d\tau \leqslant \int_{-\infty}^0 \widetilde{R_1}(\tau,\omega)e^{2\sigma\tau}d\tau<+\infty,
$$

where

$$
\widetilde{R_1}(\tau,\omega)=\frac{\mu+2\sigma}{2\sigma}(e^{\frac{\sigma}{4}|\tau|}r^1(\omega))^2+\|A^{1/2}f(\varphi_1^b)\|^2+\|A^{1/2}g\|^2+\frac{\varepsilon^2}{2\sigma}(e^{\frac{\sigma}{4}|\tau|}r^{(1/2)}(\omega))^2.
$$

Set

$$
\varrho_1^2(\omega)=2\int_{-\infty}^0 \widetilde{R_1}(\tau,\omega)e^{2\sigma\tau}d\tau, \tag{3.51}
$$

Hence, there exists a $T_{\widehat{B}}(\omega, \varrho) > 0$ such that for any φ of the system (3.7) satisfies for a.e. $\omega \in \Theta$,

$$\|A^{1/2}Q\varphi^b(t, \theta_{-t}\omega, \varphi_0^b(\theta_{-t}\omega))\|_E \leqslant \varrho_1(\omega), \quad \forall t \geqslant T_{\widehat{B}}(\omega, \varrho_1), \tag{3.52}$$

So, the second assertion is valid. $\square$

Notice that

$$\begin{aligned}\|A^{1/2}Q\varphi^b(t, \theta_{-t}\omega, \varphi_0^b(\theta_{-t}\omega))\|_E &= \Big\| \begin{pmatrix} A^{1/2}Q\varphi_1^b \\ A^{1/2}Q\varphi_2^b \end{pmatrix} \Big\|_E \geqslant \widetilde{c_1} \Big\| \begin{pmatrix} A^{1/2}Q\varphi_1^b \\ A^{1/2}Q\varphi_2^b \end{pmatrix} \Big\|_{H_0^2 \times L^2} \\ &= \widetilde{c_1}(\|AQ\varphi_1^b\|^2 + \|A^{1/2}Q\varphi_2^b\|^2)^{1/2},\end{aligned}$$

which along with (3.31), yields that for $\varphi_0(\omega) \in \widehat{B}(\omega) \in \mathscr{D}$,

$$\|Q\varphi^b(t, \theta_{-t}\omega, \varphi_0(\theta_{-t}\omega))\|_{H^4 \times H_0^2} \leqslant K_0(\varrho_1(\omega) + \varrho(\omega)), \tag{3.53}$$

for all $t \geqslant T_{\widehat{B}}(\omega, \varrho_1) + T_{\widehat{B}}(\omega, \varrho)$ for a constant $K_0 > 0$. Let $\{B_1(\omega)\}$ be a closed ball of E:

$$B_1(\omega) = \{b(\omega) \in E : \|Qb(\omega)\|_{H^4 \times H_0^2} \leqslant K_0(\varrho_1(\omega) + \varrho(\omega))\}. \tag{3.54}$$

By (3.42), (3.53), and

$$Q\varphi(t, \theta_{-t}\omega, \varphi_0(\theta_{-t}\omega)) = Q\varphi^a(t, \theta_{-t}\omega, \varphi_0(\theta_{-t}\omega)) + Q\varphi^b(t, \theta_{-t}\omega, \varphi_0(\theta_{-t}\omega)), \tag{3.55}$$

we have for a.e. $\omega \in \Theta$,

$$d_E(\varphi(t, \theta_{-t}\omega, B_0(\theta_{-t}\omega)), B_1(\omega)) \to 0 \quad \text{as } t \to +\infty, \tag{3.56}$$

this implies that for a.e. $\omega \in \Theta$,

$$d_E(T_{-\varepsilon}\varphi(t, \theta_{-t}\omega, B_0(\theta_{-t}\omega)), T_{-\varepsilon}B_1(\omega)) \to 0 ast \to +\infty, \tag{3.57}$$

where $QT_{-\varepsilon}B_1(\omega) \subset E_{22}$ is bounded in the norm of $H^4(\Omega) \times H_0^2(\Omega)$ by (3.53) and (3.54). By the compact embedding of $\widetilde{E} = H^4(\Omega) \times H_0^2(\Omega)$ into E, $\{QT_{-\varepsilon}B_1(\omega)\}$ is compact in E_{22}, which imply that $\omega \mapsto \mathbf{B}_0(\omega) := (B_1(\omega) + B_{-1}(\omega))(mod p_0)$ is a tempered random compact attracting set for $\overline{\Phi}(t, \omega)$. Thus for Theorem 2.3, we have the following result.

Theorem 3.7. *Assume that* (1.2)–(1.4) *and* $g \in H_0^2(\Omega)$ *hold. Then the random dynamical system* $\overline{\Phi}(t, \omega)$ *defined in* (3.5) *has a unique random attractor* $\{\mathscr{A}_0(\omega)\}$ *in* $\mathbf{E}$*, where*

$$\mathscr{A}_0(\omega) = \cap_{t>0}\overline{\cup_{\tau \geqslant t}\Psi(\tau, \theta_{-\tau}\omega, \mathbf{B}_0(\theta_{-\tau}\omega))}, \omega \in \Omega,$$

in which $\{\mathbf{B}_0(\omega)\}$ *is a tempered random compact attracting set for* $\overline{\Phi}(t, \omega)$.

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