

## EXISTENCE OF POSITIVE ALMOST PERIODIC SOLUTIONS FOR DELAY LOTKA-VOLTERRA COOPERATIVE SYSTEMS

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ABSTRACT. In this article, we study a Lotka-Volterra cooperative system of equations with time-varying delays and distributed delays. By using Mawhin's continuation theorem of coincidence degree theory, we obtain sufficient conditions for the existence of positive almost periodic solutions. Also we present an example to illustrate our results.

### 1. INTRODUCTION

The well-known Lotka-Volterra model for ecological population modeling was proposed by Lotka [11] and Volterra [25], and have been extensively investigated. In recent years, it has found applications in epidemiology, physics, chemistry, economics, biological science, and other areas (see [1, 6, 7]). Due to their theoretical and practical significance, Lotka-Volterra systems have an extensive literature; see for example the references in this article.

Since biological and environmental parameters are naturally subject to fluctuation in time, periodically and almost periodically varying effects are important selective forces for biological systems. Models should take into account the seasonality or periodic changing conditions [9, 10, 13, 14, 15, 20, 23]. However, models are more realistic when considering almost periodic conditions. In both cases, models should include the effects of time delays.

There are many works on the study of the Lotka-Volterra type periodic systems [12, 13, 14, 15, 27, 28]. However, there are only a few articles on the existence of almost periodic solutions. Recently, by using: almost periodic functions, contraction mappings, fixed point theory, appropriate Lyapunov functionals, and almost periodic functional hull theory, some authors have published works in the theory on almost periodic systems [2, 3, 16, 17, 18, 19, 20, 22, 26, 30].

Motivated by this, in this article, we apply the coincidence theory to study the existence of positive almost periodic solutions for the delay Lotka-Volterra type

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system

$$\begin{aligned} \dot{u}_i(t) = & u_i(t) \left( r_i(t) - F_i(t, u_i(t)) - f_i(t, u_i(t - \tau_{ii}(t))) \right) \\ & - \int_{-\sigma_{ii}}^0 \mu_{ii}(t, s) u_i(t + s) ds + \sum_{j=1, j \neq i}^n G_{ij}(t, u_j(t)) \\ & + \sum_{j=i, j \neq i}^n g_{ij}(t, u_j(t - \tau_{ij}(t))) + \sum_{j=1}^n \int_{-\sigma_{ij}}^0 \mu_{ij}(t, s) u_j(t + s) ds, \end{aligned} \quad (1.1)$$

where  $i = 1, 2, \dots, n$ ,  $u_i(t)$  stands for the  $i$ th species population density at time  $t \in \mathbb{R}$ ,  $r_i(t)$  is the natural reproduction rate,  $F_i, f_i$  and  $\mu_{ii}$  represent the inner-specific competition,  $G_{ij}, g_{ij}$  and  $\mu_{ij}$  ( $i \neq j$ ) stand for the interspecific cooperation,  $\tau_{ij}(t) > 0$  are all continuous almost periodic functions on  $\mathbb{R}$ ,  $\mu_{ij}(t, s)$  are positive almost periodic functions on  $\mathbb{R} \times [-\sigma_{ij}, 0]$  and continuous with respect to  $t \in \mathbb{R}$  and integrable with respect to  $s \in [-\sigma_{ij}, 0]$ , where  $\sigma_{ij}$  are nonnegative constants, moreover  $\int_{-\sigma_{ij}}^0 \mu_{ij}(t, s) ds = 1, i, j = 1, 2, \dots, n$ . Throughout this paper, we always assume that  $r_i, F_i, f_i, G_{ij}$  and  $g_{ij}$  are nonnegative almost periodic functions with respect to  $t \in \mathbb{R}$  and satisfy the following conditions for each  $i, j = 1, 2, \dots, n$

$$\frac{\partial F_i(t, x)}{\partial x} > 0, \quad \frac{\partial f_i(t, x)}{\partial x} > 0, \quad \frac{\partial G_{ij}(t, x)}{\partial x} > 0, \quad \frac{\partial g_{ij}(t, x)}{\partial x} > 0, \quad (1.2)$$

and for each  $t \in \mathbb{R}$ , there exist positive constants  $\alpha_i, \beta_i, \gamma_{ij}, \delta_{ij}$  such that

$$F_i(t, \alpha_i) = 0, \quad f_i(t, \beta_i) = 0, \quad G_{ij}(t, \gamma_{ij}) = 0, \quad g_{ij}(t, \delta_{ij}) = 0. \quad (1.3)$$

The initial condition of (1.1) is of the form

$$u_i(s) = \phi_i(s), \quad i = 1, 2, \dots, n, \quad (1.4)$$

where  $\phi_i(s)$  is positive bounded continuous function on  $[-\tau, 0]$  and  $\tau = \max_{1 \leq i, j \leq n} \{\sup_{t \in \mathbb{R}} : \tau_{ij}(t), \sigma_{ij}\}$ .

The organization of the rest of this paper is as follows. In Section 2, we introduce some preliminary results which are needed in later sections. In Section 3, we establish our main results for the existence of almost-periodic solutions of (1.1). Finally, an example is given to illustrate the effectiveness of our results in Section 4.

## 2. PRELIMINARIES

To obtain the existence of an almost periodic solution of system (1.1), we first make the following preparations:

**Definition 2.1** ([5]). Let  $u(t) : \mathbb{R} \rightarrow \mathbb{R}$  be continuous in  $t$ .  $u(t)$  is said to be almost periodic on  $\mathbb{R}$ , if, for any  $\epsilon > 0$ , the set  $K(u, \epsilon) = \{\delta : |u(t + \delta) - u(t)| < \epsilon, \text{ for any } t \in \mathbb{R}\}$  is relatively dense, that is for any  $\epsilon > 0$ , it is possible to find a real number  $l(\epsilon) > 0$ , for any interval with length  $l(\epsilon)$ , there exists a number  $\delta = \delta(\epsilon)$  in this interval such that  $|u(t + \delta) - u(t)| < \epsilon$ , for any  $t \in \mathbb{R}$ .

**Definition 2.2.** A solution  $u(t) = (u_1(t), u_2(t), \dots, u_n(t))^T$  of (1.1) is called an almost periodic solution if and only if for each  $i = 1, 2, \dots, n, u_i(t)$  is almost periodic.

For convenience, we denote  $AP(\mathbb{R})$  is the set of all real valued, almost periodic functions on  $\mathbb{R}$ . For each  $j = 1, 2, \dots, n$ , let

$$\begin{aligned} \wedge(f_j) &= \{ \tilde{\lambda} \in \mathbb{R} : \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T f_j(s) e^{-i\tilde{\lambda}s} ds \neq 0 \}, \\ \text{mod}(f_j) &= \left\{ \sum_{i=1}^N n_i \tilde{\lambda}_i : n_i \in \mathbb{Z}, N \in \mathbb{N}^+, \tilde{\lambda}_i \in \wedge(f_j) \right\} \end{aligned}$$

be the set of Fourier exponents and the module of  $f_j$ , respectively, where  $f_j(\cdot)$  is almost periodic. Suppose  $f_j(t, \phi_j)$  is almost periodic in  $t$ , uniformly with respect to  $\phi_j \in C([-\tau, 0], \mathbb{R})$ . Let  $K_j(f_j, \epsilon, \phi_j)$  denote the set of  $\epsilon$ -almost periods for  $f_j$  uniformly with respect to  $\phi_j \in C([-\tau, 0], \mathbb{R})$ .  $l_j(\epsilon)$  denote the length of inclusion interval.  $m(f_j) = \frac{1}{T} \int_0^T f_j(s) ds$  be the mean value of  $f_j$  on interval  $[0, T]$ , where  $T > 0$  is a constant. clearly,  $m(f_j)$  depends on  $T$ .  $m[f_j] = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T f_j(s) ds$ .

**Lemma 2.3** ([5]). *Suppose that  $f$  and  $g$  are almost periodic. Then the following statements are equivalent*

- (i)  $\text{mod}(f) \supset \text{mod}(g)$ ,
- (ii) *for any sequence  $\{t_n^*\}$ , if  $\lim_{n \rightarrow \infty} f(t + t_n^*) = f(t)$  for each  $t \in \mathbb{R}$ , then there exists a subsequence  $\{t_n\} \subseteq \{t_n^*\}$  such that  $\lim_{n \rightarrow \infty} g(t + t_n) = f(t)$  for each  $t \in \mathbb{R}$ .*

**Lemma 2.4** ([4]). *Let  $u \in AP(\mathbb{R})$ . Then  $\int_{t-\tau}^t u(s) ds$  is almost periodic.*

Let  $X$  and  $Z$  be Banach spaces. A linear mapping  $L : \text{dom}(L) \subset X \rightarrow Z$  is called Fredholm if its kernel, denoted by  $\ker(L) = \{X \in \text{dom}(L) : Lx = 0\}$ , has finite dimension and its range, denoted by  $\text{Im}(L) = \{Lx : x \in \text{dom}(L)\}$ , is closed and has finite codimension. The index of  $L$  is defined by the integer  $\dim \ker(L) - \text{codim } \text{Im}(L)$ . If  $L$  is a Fredholm mapping with index zero, then there exists continuous projections  $P : X \rightarrow X$  and  $Q : Z \rightarrow Z$  such that  $\text{Im}(P) = \ker(L)$  and  $\ker(Q) = \text{Im}(L)$ . Then  $L|_{\text{dom}(L) \cap \ker(P)} : \text{Im}(L) \cap \ker(P) \rightarrow \text{Im}(L)$  is bijective, and its inverse mapping is denoted by  $K_P : \text{Im}(L) \rightarrow \text{dom}(L) \cap \ker(P)$ . Since  $\ker(L)$  is isomorphic to  $\text{Im}(Q)$ , there exists a bijection  $J : \ker(L) \rightarrow \text{Im}(Q)$ . Let  $\Omega$  be a bounded open subset of  $X$  and let  $N : X \rightarrow Z$  be a continuous mapping. If  $QN(\overline{\Omega})$  is bounded and  $K_P(I - Q)N : \overline{\Omega} \rightarrow X$  is compact, then  $N$  is called  $L$ -compact on  $\Omega$ , where  $I$  is the identity.

Let  $L$  be a Fredholm linear mapping with index zero and let  $N$  be a  $L$ -compact mapping on  $\overline{\Omega}$ . Define mapping  $F : \text{dom}(L) \cap \overline{\Omega} \rightarrow Z$  by  $F = L - N$ . If  $Lx \neq Nx$  for all  $x \in \text{dom}(L) \cap \partial\Omega$ , then by using  $P, Q, K_P, J$  defined above, the coincidence degree of  $F$  in  $\Omega$  with respect to  $L$  is defined by

$$\text{deg}_L(F, \Omega) = \text{deg}(I - P - (J^{-1}Q + K_P(I - Q))N, \Omega, 0)$$

where  $\text{deg}(g, \Gamma, p)$  is the Leray-Schauder degree of  $g$  at  $p$  relative to  $\Gamma$ .

Then the Mawhin's continuous theorem is reads as follows.

**Lemma 2.5** ([8]). *Let  $\Omega \subset X$  be an open bounded set and let  $N : X \rightarrow Z$  be a continuous operator which is  $L$ -compact on  $\overline{\Omega}$ . Assume*

- (a) *for each  $\lambda \in (0, 1), x \in \partial\Omega \cap \text{dom}(L), Lx \neq \lambda Nx$ ;*
- (b) *for each  $x \in \partial\Omega \cap L, QNx \neq 0$ ;*
- (c)  $\text{deg}(JNQ, \Omega \cap \ker(L), 0) \neq 0$ .

Then  $Lx = Nx$  has at least one solution in  $\bar{\Omega} \cap \text{dom}(L)$ .

### 3. EXISTENCE OF POSITIVE ALMOST-PERIODIC SOLUTIONS

In this section, we state and prove our main results of our this paper. By making the substitution

$$u_i(t) = e^{y_i(t)}, \quad i = 1, 2, \dots, n,$$

Equation (1.1) can be reformulated as

$$\begin{aligned} \dot{y}_i(t) &= r_i(t) - F_i(t, e^{y_i(t)}) - f_i(t, e^{y_i(t-\tau_{ii}(t))}) - \int_{-\sigma_{ii}}^0 \mu_{ii}(t, s) e^{y_i(t+s)} ds \\ &+ \sum_{j=1, j \neq i}^n G_{ij}(t, e^{y_j(t)}) + \sum_{j=1, i \neq j}^n g_{ij}(t, e^{y_j(t-\tau_{ij}(t))}) \\ &- \sum_{j=1, j \neq i}^n \int_{-\sigma_{ij}}^0 \mu_{ij}(t, s) e^{y_j(t+s)} ds, \quad i = 1, 2, \dots, n. \end{aligned} \tag{3.1}$$

The initial condition (1.4) can be rewritten as

$$y_i(s) = \ln \phi_i(s) =: \psi_i(s), \quad i = 1, 2, \dots, n. \tag{3.2}$$

Set  $X = Z = V_1 \oplus V_2$ , where

$$\begin{aligned} V_1 &= \left\{ y(t) = (y_1(t), y_2(t), \dots, y_n(t))^T \in C(\mathbb{R}, \mathbb{R}^n) : y_i(t) \in AP(\mathbb{R}), \right. \\ &\quad \left. \text{mod}(y_i(t)) \subset \text{mod}(H_i(t)), \forall \tilde{\lambda}_i \in \wedge(y_i(t)), |\tilde{\lambda}_i| > \beta, i = 1, 2, \dots, n \right\}, \\ V_2 &= \left\{ y(t) \equiv (h_1, h_2, \dots, h_n)^T \in \mathbb{R}^n \right\} \end{aligned}$$

$$\begin{aligned} H_i(t) &= r_i(t) - F_i(t, e^{\psi_i(t)}) - f_i(t, e^{\psi_i(-\tau_{ii}(0))}) - \int_{-\sigma_{ii}}^0 \mu_{ii}(t, s) e^{\psi_i(s)} ds \\ &+ \sum_{j=1, j \neq i}^n [G_{ij}(t, e^{\psi_j(t)}) + g_{ij}(t, e^{\psi_j(-\tau_{ij}(0))})] - \sum_{j=1}^n \int_{-\sigma_{ij}}^0 \mu_{ij}(t, s) e^{\psi_j(s)} ds \end{aligned}$$

and  $\psi(\cdot)$  is defined as (3.2),  $i = 1, 2, \dots, n, \beta > 0$  is a given constant. For  $y = (y_1, y_2, \dots, y_n)^T \in Z$ , define  $\|y\| = \max_{1 \leq i \leq n} \sup_{t \in \mathbb{R}} |y_i(t)|$ .

**Lemma 3.1.**  $Z$  is a Banach space equipped with the norm  $\|\cdot\|$ .

*Proof.* Let  $y^{\{k\}} = (y_1^{\{k\}}, y_2^{\{k\}}, \dots, y_n^{\{k\}})^T \in V_1$ , and let  $y^{\{k\}}$  converge to  $\bar{y} = (\bar{y}_1, \bar{y}_2, \dots, \bar{y}_n)^T$ ; that is,  $y_j^{\{k\}} \rightarrow \bar{y}_j$ , as  $k \rightarrow \infty, j = 1, 2, \dots, n$ . Then it is easy to show that  $\bar{y}_j \in AP(\mathbb{R})$  and  $\text{mod}(\bar{y}_j) \in \text{mod}(H_j)$ . For any  $|\tilde{\lambda}_j| \leq \beta$ , we have that

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T y_j^{\{k\}}(t) e^{-i\tilde{\lambda}_j t} dt = 0, \quad j = 1, 2, \dots, n;$$

therefore,

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \bar{y}_j(t) e^{-i\tilde{\lambda}_j t} dt = 0, \quad j = 1, 2, \dots, n,$$

which implies  $\bar{y} \in V_1$ . Then it is not difficult to see that  $V_1$  is a Banach space equipped with the norm  $\|\cdot\|$ . Thus, we can easily verify that  $X$  and  $Z$  are Banach spaces equipped with the norm  $\|\cdot\|$ . The proof is complete.  $\square$

**Lemma 3.2.** *Let  $L : X \rightarrow Z, Ly = \dot{y}$ , then  $L$  is a Fredholm mapping of index zero.*

*Proof.* Clearly,  $L$  is a linear operator and  $\ker(L) = V_2$ . We claim that  $\text{Im}(L) = V_1$ . Firstly, we suppose that  $z(t) = (z_1(t), z_2(t), \dots, z_n(t))^T \in \text{Im}(L) \subset Z$ . Then there exist  $z^{\{1\}}(t) = (z_1^{\{1\}}(t), z_2^{\{1\}}(t), \dots, z_n^{\{1\}}(t))^T \in V_1$  and constant vector  $z^{\{2\}} = (z_1^{\{2\}}, z_2^{\{2\}}, \dots, z_n^{\{2\}})^T \in V_2$  such that

$$z(t) = z^{\{1\}}(t) + z^{\{2\}};$$

that is,

$$z_i(t) = z_i^{\{1\}}(t) + z_i^{\{2\}}, \quad i = 1, 2, \dots, n.$$

From the definition of  $z_i(t)$  and  $z_i^{\{1\}}(t)$ , we can easily see that  $\int_{t-\tau}^t z_i(s)ds$  and  $\int_{t-\tau}^t z_i^{\{1\}}(s)ds$  are almost periodic function. So we have  $z_i^{\{2\}} \equiv 0, i = 1, 2, \dots, n$ , then  $z^{\{2\}} \equiv (0, 0, \dots, 0)^T$ , which implies  $z(t) \in V_1$ , that is  $\text{Im}(L) \subset V_1$ .

On the other hand, if  $u(t) = (u_1(t), u_2(t), \dots, u_n(t))^T \in V_1 \setminus \{0\}$ , then we have  $\int_0^t u_j(s)ds \in AP(\mathbb{R}), j = 1, 2, \dots, n$ . If  $\tilde{\lambda}_j \neq 0$ , then we obtain

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \left( \int_0^t u_j(s)ds \right) e^{-i\tilde{\lambda}_j t} dt = \frac{1}{i\tilde{\lambda}_j} \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T u_j(t) e^{-i\tilde{\lambda}_j t} dt,$$

for  $j = 1, 2, \dots, n$ . It follows that

$$\wedge \left[ \int_0^t u_j(s)ds - m \left( \int_0^t u_j(s)ds \right) \right] = \wedge(u_j(t)), \quad j = 1, 2, \dots, n;$$

hence

$$\int_0^t u(s)ds - m \left( \int_0^t u(s)ds \right) \in V_1 \subset X$$

Note that  $\int_0^t u(s)ds - m \left( \int_0^t u(s)ds \right)$  is the primitive of  $u(t)$  in  $X$ , we have  $u(t) \in \text{Im}(L)$ , that is  $V_1 \subset \text{Im}(L)$ . Therefore,  $\text{Im}(L) = V_1$ .

Furthermore, one can easily show that  $\text{Im}(L)$  is closed in  $Z$  and

$$\dim \ker(L) = n = \text{codim } \text{Im}(L);$$

therefore,  $L$  is a Fredholm mapping of index zero. The proof is complete. □

**Lemma 3.3.** *Let  $N : X \rightarrow Z, Ny = (G_1^y, G_2^y, \dots, G_n^y)^T$ , where*

$$\begin{aligned} G_i^y &= r_i(t) - F_i(t, \exp\{y_i(t)\}) - f_i(t, \exp\{y_i(t - \tau_{ii}(t))\}) \\ &\quad - \int_{-\sigma_{ii}}^0 \mu_{ii}(t, s) \exp\{y_i(t + s)\} ds + \sum_{j=1, j \neq i}^n G_{ij}(t, \exp\{y_j(t)\}) \\ &\quad + \sum_{j=1, i \neq j}^n g_{ij}(t, \exp\{y_j(t - \tau_{ij}(t))\}) + \sum_{j=1, j \neq i}^n \int_{-\sigma_{ij}}^0 \mu_{ij}(t, s) \exp\{y_j(t + s)\} ds, \end{aligned}$$

for  $j = 1, 2, \dots, n$ . Set

$$\begin{aligned} P : X &\rightarrow Z, \quad Py = (m(y_1), m(y_2), \dots, m(y_n))^T, \\ Q : Z &\rightarrow Z, \quad Qz = (m[z_1], m[z_2], \dots, m[z_n])^T. \end{aligned}$$

Then  $N$  is  $L$ -compact on  $\bar{\Omega}$ , where  $\Omega$  is an open bounded subset of  $X$ .

*Proof.* Obviously,  $P$  and  $Q$  are continuous projectors such that

$$\operatorname{Im} P = \ker(L), \quad \operatorname{Im}(L) = \ker(Q).$$

It is clear that  $(I - Q)V_2 = \{(0, 0, \dots, 0)\}$ ,  $(I - Q)V_1 = V_1$ . Hence

$$\operatorname{Im}(I - Q) = V_1 = \operatorname{Im}(L).$$

Then in view of

$$\operatorname{Im}(P) = \ker(L), \quad \operatorname{Im}(L) = \ker(Q) = \operatorname{Im}(I - Q),$$

we obtain that the inverse  $K_P : \operatorname{Im}(L) \rightarrow \ker(P) \cap \operatorname{dom}(L)$  of  $L_P$  exists and is given by

$$K_P(z) = \int_0^t z(s) ds - m \left[ \int_0^t z(s) ds \right].$$

Thus,

$$QNy = (m[G_1^y], m[G_2^y], \dots, m[G_n^y])^T,$$

$$K_P(I - Q)Ny = (f(y_1) - Q(f(y_1)), f(y_2) - Q(f(y_2)), \dots, f(y_n) - Q(f(y_n)))^T,$$

where

$$f(y_i) = \int_0^t (G_i^y - m[G_i^y]) ds, \quad i = 1, 2, \dots, n.$$

Clearly,  $QN$  and  $(I - Q)N$  are continuous. Now we will show that  $K_P$  is also continuous. By assumptions, for any  $0 < \epsilon < 1$  and any compact set  $\phi_i \subset C([- \tau, 0], \mathbb{R})$ , let  $l_i(\epsilon_i)$  be the length of the inclusion interval of  $K_i(H_i, \epsilon_i, \phi_i)$ ,  $i = 1, 2, \dots, n$ . Suppose that  $\{z^k(t)\} \subset \operatorname{Im}(L) = V_1$  and  $z^k(t) = (z_1^k(t), z_2^k(t), \dots, z_n^k(t))^T$  uniformly converges to  $\bar{z}(t) = (\bar{z}_1(t), \bar{z}_2(t), \dots, \bar{z}_n(t))^T$ ; that is,  $z_i^k \rightarrow \bar{z}_i$ , as  $k \rightarrow \infty$ ,  $i = 1, 2, \dots, n$ . Because of  $\int_0^t z^k(s) ds \in Z$ ,  $k = 1, 2, \dots, n$ , there exists  $\sigma_i(0 < \sigma_i < \epsilon_i)$  such that  $K_i(H_i, \sigma_i, \phi_i) \subset K_i(\int_0^t z_i^k(s) ds, \sigma_i, \phi_i)$ ,  $i = 1, 2, \dots, n$ . Let  $l_i(\sigma_i)$  be the length of the inclusion interval of  $K_i(H_i, \sigma_i, \phi_i)$  and

$$l_i = \max\{l_i(\epsilon_i), l_i(\sigma_i)\}, \quad i = 1, 2, \dots, n.$$

It is easy to see that  $l_i$  is the length of the inclusion interval of  $K_i(H_i, \sigma_i, \phi_i)$  and  $K_i(H_i, \epsilon_i, \phi_i)$ ,  $i = 1, 2, \dots, n$ . Hence, for any  $t \notin [0, l_i]$ , there exists  $\xi_t \in K_i(H_i, \sigma_i, \phi_i) \subset K_i(\int_0^t z_i^k(s) ds, \sigma_i, \phi_i)$  such that  $t + \xi_t \in [0, l_i]$ ,  $i = 1, 2, \dots, n$ . Hence, by the definition of almost periodic function we have

$$\begin{aligned} & \left\| \int_0^t z^k(s) ds \right\| \\ &= \max_{1 \leq i \leq n} \sup_{t \in \mathbb{R}} \left| \int_0^t z_i^k(s) ds \right| \\ &\leq \max_{1 \leq i \leq n} \sup_{t \in [0, l_i]} \left| \int_0^t z_i^k(s) ds \right| + \max_{1 \leq i \leq n} \sup_{t \notin [0, l_i]} \left| \int_0^t z_i^k(s) ds \right. \\ &\quad \left. - \int_0^{t+\xi_t} z_i^k(s) ds + \int_0^{t+\xi_t} z_i^k(s) ds \right| \\ &\leq 2 \max_{1 \leq i \leq n} \sup_{t \in [0, l_i]} \left| \int_0^t z_i^k(s) ds \right| + \max_{1 \leq i \leq n} \sup_{t \notin [0, l_i]} \left| \int_0^t z_i^k(s) ds - \int_0^{t+\xi_t} z_i^k(s) ds \right| \\ &\leq 2 \max_{1 \leq i \leq n} \left| \int_0^{l_i} z_i^k(s) ds \right| + \max_{1 \leq i \leq n} \epsilon_i. \end{aligned} \tag{3.3}$$

From this inequality, we can conclude that  $\int_0^t z(s)ds$  is continuous, where  $z(t) = (z_1(t), z_2(t), \dots, z_n(t))^T \in \text{Im}(L)$ . Consequently,  $K_P$  and  $K_P(I-Q)Ny$  are continuous. From (3.3), we also have  $\int_0^t z(s)ds$  and  $K_P(I-Q)Ny$  are uniformly bounded in  $\bar{\Omega}$ . Further, it is not difficult to verify that  $QN(\bar{\Omega})$  is bounded and  $K_P(I-Q)Ny$  is equicontinuous in  $\bar{\Omega}$ . By the Arzela-Ascoli theorem, we have immediately conclude that  $K_P(I-Q)N(\bar{\Omega})$  is compact. Thus  $N$  is  $L$ -compact on  $\bar{\Omega}$ . The proof is complete.  $\square$

By (1.2),  $F_i(t, x)$ ,  $f_i(t, x)$ ,  $G_{ij}(t, x)$  and  $g_{ij}(t, x)$  can be represented expansion into power-series at  $\alpha_i, \beta_i, \gamma_{ij}$  and  $\delta_{ij}$  of  $x$  in form of

$$\begin{aligned} F_i(t, x) &= F_i(t, \alpha_i) + \frac{\partial F_i}{\partial x}|_{(t, \alpha_i)}x + o(x^2), \quad i = 1, 2, \dots, n; \\ f_i(t, x) &= f_i(t, \beta_i) + \frac{\partial f_i}{\partial x}|_{(t, \beta_i)}x + o(x^2), \quad i = 1, 2, \dots, n; \\ G_{ij}(t, x) &= G_{ij}(t, \gamma_{ij}) + \frac{\partial G_{ij}}{\partial x}|_{(t, \gamma_{ij})}x + o(x^2), \quad i \neq j, i, j = 1, 2, \dots, n; \\ g_{ij}(t, x) &= g_{ij}(t, \delta_{ij}) + \frac{\partial g_{ij}}{\partial x}|_{(t, \delta_{ij})}x + o(x^2), \quad i \neq j, i, j = 1, 2, \dots, n; \end{aligned}$$

respectively, where  $o(x^2)$  is a higher-order infinitely small quantity of  $x^2$ . By (1.3), we can conclude that  $F_i(t, \alpha_i) = 0, f_i(t, \beta_i) = 0, G_{ij}(t, \gamma_{ij}) = 0, g_{ij}(t, \delta_{ij}) = 0$ . For convenience, we denote

$$\begin{aligned} \frac{\partial F_i}{\partial x}|_{(t, \alpha_i)} &:= b_{ii}(t), & \frac{\partial f_i}{\partial x}|_{(t, \beta_i)} &:= c_{ii}(t), \\ \frac{\partial G_{ij}}{\partial x}|_{(t, \gamma_{ij})} &:= b_{ij}(t), & \frac{\partial g_{ij}}{\partial x}|_{(t, \delta_{ij})} &:= c_{ij}(t), \end{aligned}$$

for  $i \neq j, i, j = 1, 2, \dots, n$ . By (1.3),  $b_{ij}(t) > 0, c_{ij}(t) > 0$  for  $i, j = 1, 2, \dots, n$ .

**Theorem 3.4.** *Assume the following conditions hold:*

- (H1)  $e_i := m[r_i(t)] > 0, m[b_{ij}(t)] > 0, m[c_{ij}(t)] > 0, i, j = 1, 2, \dots, n;$
- (H2)  $d_{ii} > \sum_{i \neq j, j=1}^n d_{ij}$ , where  $d_{ii} = m[b_{ii}(t)] + m[c_{ii}(t)] + 1, d_{ij} = m[b_{ij}(t)] + m[c_{ij}(t)] + 1, i \neq j, i, j = 1, 2, \dots, n.$

*Then (1.1) has at least one positive almost periodic solution.*

*Proof.* To use the continuation theorem of coincidence degree theorem to establish the existence of a solution of (1.1), we set Banach space  $X$  and  $Z$  the same as those in Lemma 3.1 and set mappings  $L, N, P, Q$  the same as those in Lemma 3.2 and Lemma 3.3, respectively. Then we can obtain that  $L$  is a Fredholm mapping of index zero and  $N$  is a continuous operator which is  $L$ -compact on  $\bar{\Omega}$ .

Now, we are in the position of searching for an appropriate open, bounded subset  $\Omega$  for the application of the continuation theorem. Corresponding to the operator equation

$$Ly = \lambda Ny, \quad \lambda \in (0, 1),$$

we obtain

$$\begin{aligned}
\dot{y}_i(t) = & \lambda \left[ r_i(t) - b_{ii}(t)e^{y_i(t)} - c_{ii}(t)e^{y_i(t-\tau_{ii}(t))} + \sum_{j=1, j \neq i}^n b_{ij}(t)e^{y_j(t)} \right. \\
& + \sum_{j=1, i \neq j}^n c_{ij}(t)e^{y_j(t-\tau_{ij}(t))} - \int_{-\sigma_{ii}}^0 \mu_{ii}(t, s)e^{y_j(t+s)} ds \\
& + \sum_{j=1}^n \int_{-\sigma_{ij}}^0 \mu_{ij}(t, s)e^{y_j(t+s)} ds - o(e^{2y_i(t)}) - o(e^{2y_i(t-\tau_{ii}(t))}) \\
& \left. + \sum_{j=1, j \neq i}^n (o(e^{2y_j(t)}) + o(e^{2y_j(t-\tau_{ij}(t))})) \right], \quad i = 1, 2, \dots, n.
\end{aligned} \tag{3.4}$$

Assume that  $y(t) = (y_1(t), y_2(t), \dots, y_n(t))^T \in X$  is a solution of (3.4) for some  $\lambda \in (0, 1)$ . Denote

$$\bar{\theta} = \max_{1 \leq i \leq n} \sup_{t \in \mathbb{R}} y_i(t), \quad \underline{\theta} = \min_{1 \leq i \leq n} \inf_{t \in \mathbb{R}} y_i(t).$$

On the both sides of (3.4), integrating from 0 to  $T$  and applying the mean value theorem of integral calculus, we have

$$\begin{aligned}
0 \leq & \lambda \left[ m(r_i(t)) - m(b_{ii}(t))e^{y_i(\xi_{ii})} - m(c_{ii}(t))e^{y_i(\eta_{ii}-\tau_{ii}(\eta_{ii}))} \right. \\
& + \sum_{j=1, j \neq i}^n m(b_{ij}(t))e^{y_j(\xi_{ij})} + \sum_{j=1, i \neq j}^n m(c_{ij}(t))e^{y_j(\eta_{ij}-\tau_{ij}(\eta_{ij}))} \\
& - \int_{-\sigma_{ii}}^0 m(\mu_{ii}(t, s))e^{y_j(\zeta_{ii}+s)} ds + \sum_{j=1}^n \int_{-\sigma_{ij}}^0 m(\mu_{ij}(t, s))e^{y_j(\zeta_{ij}+s)} ds \\
& - m(o(e^{2y_i(t)})) - m(o(e^{2y_i(t-\tau_{ii}(t)})) + \sum_{j=1, j \neq i}^n m(o(e^{2y_j(t)})) \\
& \left. + \sum_{j=1, j \neq i}^n m(o(e^{2y_j(t-\tau_{ij}(t)})) \right] + m(|\dot{y}_i(t)|), \quad i = 1, 2, \dots, n,
\end{aligned} \tag{3.5}$$

where  $\xi_{ij} \in [0, T]$ ,  $\eta_{ij} \in [0, T]$ ,  $\zeta_{ij} \in [0, T]$ ,  $i, j = 1, 2, \dots, n$ . In the light of (3.5), we have

$$\begin{aligned}
& \lambda \left[ m(b_{ii}(t))e^{y_i(\xi_{ii})} + m(c_{ii}(t))e^{y_i(\eta_{ii}-\tau_{ii}(\eta_{ii}))} + m(o(e^{2y_i(t)})) \right. \\
& \left. + \int_{-\sigma_{ii}}^0 m(\mu_{ii}(t, s))e^{y_j(\zeta_{ii}+s)} ds + m(o(e^{2y_i(t-\tau_{ii}(t)})) \right] \\
& \leq \lambda \left[ m(r_i(t)) + \sum_{j=1, j \neq i}^n m(b_{ij}(t))e^{y_j(\xi_{ij})} + \sum_{j=1, i \neq j}^n m(c_{ij}(t))e^{y_j(\eta_{ij}-\tau_{ij}(\eta_{ij}))} \right. \\
& \left. + \sum_{j=1}^n \int_{-\sigma_{ij}}^0 m(\mu_{ij}(t, s))e^{y_j(\zeta_{ij}+s)} ds + \sum_{j=1, j \neq i}^n m(o(e^{2y_j(t)})) \right. \\
& \left. + \sum_{j=1, j \neq i}^n m(o(e^{2y_j(t-\tau_{ij}(t)})) \right] + m(|\dot{y}_i(t)|), \quad i = 1, 2, \dots, n.
\end{aligned} \tag{3.6}$$



On both sides of (3.6), taking the supremum with respect to  $\xi_{ij}, \eta_{ij}, \zeta_{ij}$  and letting  $T \rightarrow +\infty$ , we obtain

$$\left(m[b_{ii}(t)] + m[c_{ii}(t)] + 1\right)e^{\bar{\theta}} \leq m[r_i(t)] + \sum_{j=1, j \neq i}^n \left(m[b_{ij}(t)] + m[c_{ij}(t)] + 1\right)e^{\bar{\theta}};$$

that is,

$$d_{ii}e^{\bar{\theta}} - \sum_{j=1, j \neq i}^n d_{ij}e^{\bar{\theta}} \leq e_i, \quad i = 1, 2, \dots, n,$$

which imply

$$\bar{\theta} \leq \ln B, \quad (3.7)$$

where  $B = \max_{1 \leq i \leq n} \{B_i\}$ ,

$$B_i = \frac{e_i}{d_{ii} - \sum_{j=1, j \neq i}^n d_{ij}}.$$

On both sides of (3.6), taking the infimum with respect to  $\xi_{ij}, \eta_{ij}, \zeta_{ij}$  and letting  $T \rightarrow +\infty$ , we obtain

$$\underline{\theta} \leq \ln B. \quad (3.8)$$

On the other hand, according to (3.4), we derive

$$\begin{aligned} \lambda r_i(t) - \dot{y}_i(t) &\leq \lambda \left[ b_{ii}(t)e^{y_i(t)} + c_{ii}(t)e^{y_i(t-\tau_{ii}(t))} \right. \\ &\quad \left. + \int_{-\sigma_{ii}}^0 \mu_{ii}(t, s)e^{y_i(t+s)} ds + o(e^{2y_i(t)}) + o(e^{2y_i(t-\tau_{ii}(t))}) \right], \end{aligned} \quad (3.9)$$

for  $i = 1, 2, \dots, n$ . Integrating on both sides of (3.9), from 0 to  $T$ , and using the mean value theorem, we obtain

$$\begin{aligned} \lambda m(r_i(t)) &< m(\dot{y}_i(t)) + \lambda \left[ m(b_{ii}(t))e^{y_i(\xi_i)} + m(c_{ii}(t))e^{y_i(\eta_i - \tau_{ii}(\eta_i))} \right. \\ &\quad \left. + \int_{-\sigma_{ii}}^0 m(\mu_{ii}(t, s))e^{y_i(\zeta_i + s)} ds + m(o(e^{2y_i(t)})) \right. \\ &\quad \left. + m(o(e^{2y_i(t-\tau_{ii}(t))}) \right)], \quad i = 1, 2, \dots, n, \end{aligned} \quad (3.10)$$

where  $\xi_i \in [0, T]$ ,  $\eta_i \in [0, T]$ ,  $\zeta_i \in [0, T]$ , for  $i = 1, 2, \dots, n$ . On both sides of (3.10), take the supremum and infimum with respect to  $\xi_i, \eta_i, \zeta_i$ , respectively, and let  $T \rightarrow +\infty$ , then for  $i = 1, 2, \dots, n$ , we have

$$\begin{aligned} m[r_i(t)] &< \left(m[b_{ii}(t)] + m[c_{ii}(t)] + 1\right)e^{\bar{\theta}}, \\ m[r_i(t)] &< \left(m[b_{ii}(t)] + m[c_{ii}(t)] + 1\right)e^{\underline{\theta}}, \end{aligned}$$

namely,

$$\begin{aligned} e^{\bar{\theta}} &> \frac{m[r_i(t)]}{m[b_{ii}(t)] + m[c_{ii}(t)] + 1} = \frac{e_i}{d_{ii}}, \\ e^{\underline{\theta}} &> \frac{m[r_i(t)]}{m[b_{ii}(t)] + m[c_{ii}(t)] + 1} = \frac{e_i}{d_{ii}}, \end{aligned}$$

which imply that

$$\bar{\theta} > \ln C, \quad \underline{\theta} > \ln C, \quad (3.11)$$

where  $C = \min_{1 \leq i \leq n} \{e_i/d_{ii}\}$ . Combing with (3.7), (3.8) and (3.11), we derive that for all  $t \in \mathbb{R}, i = 1, 2, \dots, n$ ,

$$\ln C < \underline{\theta} \leq y_i(t) \leq \bar{\theta} < \ln B + 1. \quad (3.12)$$

Denote  $M = \max\{|\ln C|, |\ln B + 1|\}$ . Clearly,  $M$  is independent of  $\lambda$ . Take

$$\Omega = \{y = (y_1, y_2, \dots, y_n)^T \in X : \|y\| < M\}.$$

It is clear that  $\Omega$  satisfies the requirement (a) in Lemma 2.5. When  $y \in \partial\Omega \cap \ker(L), y = (y_1, y_2, \dots, y_n)^T$  is a constant vector in  $\mathbb{R}^n$  with  $\|y\| = M$ . Then

$$QNy = (m[G_1], m[G_2], \dots, m[G_n])^T, \quad y \in X$$

where

$$\begin{aligned} G_i &= r_i(t) - F_i(t, e^{y_i}) - f_i(t, e^{y_i}) - \int_{-\sigma_{ii}}^0 \mu_{ii}(t, s) e^{y_i} ds \\ &+ \sum_{j=1, j \neq i}^n G_{ij}(t, e^{y_j}) + \sum_{j=1, i \neq j}^n g_{ij}(t, e^{y_j}) \\ &+ \sum_{j=1, j \neq i}^n \int_{-\sigma_{ij}}^0 \mu_{ij}(t, s) e^{y_j} ds, \quad i = 1, 2, \dots, n, \end{aligned}$$

and

$$\begin{aligned} m[G_i] &= m[r_i(t)] - (m[b_{ii}(t)] + m[c_{ii}(t)] + 1)e^{y_i} \\ &+ \sum_{j=1, j \neq i}^n (m[b_{ij}(t)] + m[c_{ij}(t)] + 1)e^{y_j} \\ &= e_i - d_{ii}e^{y_i} + \sum_{j=1, j \neq i}^n d_{ij}e^{y_j}, \quad i = 1, 2, \dots, n. \end{aligned}$$

There exist positive integers  $l, k \in \{1, 2, \dots, n\}$  such that  $x_l = \underline{y} = \min_{1 \leq i \leq n} \{y_i\}$  and  $x_k = \bar{y} = \max_{1 \leq i \leq n} \{y_i\}$ . If  $QNy = (0, 0, \dots, 0)^T$ , then we have

$$\begin{aligned} d_{ll}e^{\underline{y}} &= d_{ll}e^{y_l} = e_l + \sum_{j=1, j \neq l}^n d_{lj}e^{y_j} \geq e_l, \\ d_{ll}e^{\bar{y}} &= d_{ll}e^{y_k} = e_k + \sum_{j=1, j \neq k}^n d_{kj}e^{y_j} \leq e_k + \sum_{j=1, j \neq k}^n d_{kj}e^{\bar{y}}, \end{aligned}$$

namely,

$$\begin{aligned} e^{\underline{y}} &\geq \frac{e_l}{d_{ll}} > C, \\ e^{\bar{y}} &\leq \frac{e_k}{d_{ll} - \sum_{j=1, j \neq k}^n d_{kj}} \leq B, \end{aligned}$$

which imply that  $\ln C < \underline{y} \leq y_i \leq \bar{y} \leq \ln B, i = 1, 2, \dots, n$ . Thus,  $y = (y_1, y_2, \dots, y_n)^T \in \Omega$ , this contradicts the fact that  $y \in \partial\Omega \cap \ker(L)$ . Therefore,  $QNy \neq (0, 0, \dots, 0)^T$ , which implies that the requirement (b) in Lemma 2.5 is satisfied. When  $y \in \partial\Omega \cap \ker(L), y = (y_1, y_2, \dots, y_n)^T$  is a constant vector in  $\mathbb{R}^n$  with  $\|y\| = M$ . Then

$$QNy = (m[G_1], m[G_2], \dots, m[G_n])^T \neq (0, 0, \dots, 0)^T,$$

which implies that the requirement (b) in Lemma 2.5 is satisfied. If necessary, we can let  $M$  be greater such that  $y^T Q N y < 0$ , for any  $y \in \partial\Omega \cap \ker(L)$ . Furthermore, take the isomorphism  $J : \text{Im}(Q) \rightarrow \ker(L)$ ,  $Jz \equiv z$  and let  $\Phi(\gamma; y) = -\gamma y + (1 - \gamma)JQNy$ , then for any  $y \in \partial\Omega \cap \ker(L)$ ,  $y^T \Phi(\gamma; y) < 0$ , we have

$$\text{deg}\{JQN, \Omega \cap \ker(L), 0\} = \text{deg}\{-y, \Omega \cap \ker(L), 0\} \neq 0.$$

So, the requirement (c) in Lemma 2.5 is satisfied. Hence, (3.1) has at least one almost periodic solution in  $\bar{\Omega}$ , that is (1.1) has at least one positive almost periodic solution. The proof is complete.  $\square$

#### 4. EXAMPLES

Consider the following two species Lotka-Volterra type cooperative system with time-varying delays and distributed delays:

$$\begin{aligned} \dot{x}(t) &= x(t) \left( r_1(t) - F_1(t, x(t)) - f_1(t, x(t - \tau_{11}(t))) \right. \\ &\quad \left. - \int_{-\sigma_{11}}^0 \mu_{11}(t, s)x(t + s)ds + G_{12}(t, y(t)) \right. \\ &\quad \left. + g_{12}(t, y(t - \tau_{12}(t))) + \int_{-\sigma_{12}}^0 \mu_{12}(t, s)y(t + s)ds \right), \\ \dot{y}(t) &= y(t) \left( r_2(t) - F_2(t, y(t)) - f_2(t, y(t - \tau_{22}(t))) \right. \\ &\quad \left. - \int_{-\sigma_{22}}^0 \mu_{22}(t, s)y(t + s)ds + G_{21}(t, x(t)) \right. \\ &\quad \left. + g_{21}(t, x(t - \tau_{21}(t))) + \int_{-\sigma_{21}}^0 \mu_{21}(t, s)x(t + s)ds \right), \end{aligned} \tag{4.1}$$

where  $r_1(t) = 2 + \sin \sqrt{2}t + \sin \sqrt{3}t$ ,  $F_1(t, x) = (2 + \sin \sqrt{3}t + \sin \sqrt{5}t) \sin x$ ,  $f_1(t, x) = (2 + \cos \sqrt{3}t + \cos \sqrt{5}t) \sin x$ ,  $G_{12}(t, x) = \frac{1}{4}(2 + \cos \sqrt{2}t + \cos \sqrt{3}t) \sin 2x$ ,  $g_{12}(t, x) = \frac{2 + \sin \sqrt{2}t + \sin \sqrt{3}t}{6} \sin 3x$ ,  $\tau_{11}(t) = e^{\sin \sqrt{2}t + \sin \sqrt{5}t}$ ,  $\tau_{12}(t) = e^{\sin t + \cos \sqrt{2}t}$ ,  $r_2(t) = 2 - \sin \sqrt{2}t - \sin \sqrt{3}t$ ,  $F_2(t, y) = (2 + \sin \sqrt{3}t - \sin \sqrt{5}t) \sin y$ ,  $f_2(t, y) = (2 + \cos \sqrt{3}t - \cos \sqrt{5}t) \sin y$ ,  $G_{21}(t, y) = \frac{2 - \cos \sqrt{2}t + \cos \sqrt{3}t}{8} \sin 4y$ ,  $g_{21}(t, y) = \frac{2 - \sin \sqrt{2}t + \sin \sqrt{3}t}{10} \sin 5y$ ,  $\tau_{22}(t) = e^{\sin \sqrt{2}t + \cos \sqrt{5}t}$ ,  $\tau_{21}(t) = e^{\sin t - \cos \sqrt{2}t}$ ,  $\mu_{ij}(t, s)$  are positive almost periodic functions on  $\mathbb{R} \times [-\sigma_{ij}, 0]$  and continuous with respect to  $t \in \mathbb{R}$  and integrable with respect to  $s \in [-\sigma_{ij}, 0]$ , where  $\sigma_{ij}$  are nonnegative constants, moreover  $\int_{-\sigma_{ij}}^0 \mu_{ij}(t, s)ds = 1$ ,  $i, j = 1, 2, \dots, n$ . Obviously,  $\alpha_i = \beta_i = \gamma_i = 2\pi$ . By a simple calculation, we have

$$\begin{aligned} b_{11}(t) &= 2 + \sin \sqrt{3}t + \sin \sqrt{5}t, & c_{11}(t) &= 2 + \cos \sqrt{3}t + \cos \sqrt{5}t, \\ b_{12}(t) &= \frac{2 + \cos \sqrt{2}t + \cos \sqrt{3}t}{2}, & c_{12}(t) &= \frac{2 + \sin \sqrt{2}t + \sin \sqrt{3}t}{2}, \\ b_{22}(t) &= 2 + \sin \sqrt{3}t - \sin \sqrt{5}t, & c_{22}(t) &= 2 + \cos \sqrt{3}t - \cos \sqrt{5}t, \\ b_{21}(t) &= \frac{2 - \cos \sqrt{2}t + \cos \sqrt{3}t}{2}, & c_{21}(t) &= \frac{2 - \sin \sqrt{2}t + \sin \sqrt{3}t}{2}, \\ e_1 &= m[r_1(t)] = 2, & d_{11} &= m[b_{11}(t)] + m[c_{11}(t)] + 1 = 5, \\ d_{12} &= m[b_{12}(t)] + m[c_{12}(t)] + 1 = 3, & e_2 &= m[r_1(t)] = 2, \end{aligned}$$

$$d_{22} = m[b_{11}(t)] + m[c_{22}(t)] + 1 = 5, \quad d_{21} = m[b_{21}(t)] + m[c_{21}(t)] + 1 = 3,$$

then, in the matrix  $d_{11} = 5 > 3 = d_{12}$ ,  $d_{22} = 5 > 3 = d_{21}$  and

$$M = \max\{|\ln C|, |\ln B + 1|\} = 1,$$

$$\Omega = \{y = (y_1, y_2, \dots, y_n)^T \in X : \|y\| < 1\}.$$

Therefore, all conditions of Theorem 3.4 are satisfied, and system (4.1) has at least one positive almost periodic solution.

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