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ITERATIVE TECHNIQUE FOR A THIRD-ORDER THREE-POINT BVP WITH SIGN-CHANGING GREEN'S FUNCTION

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Abstract. In this article, by applying iterative technique, we study the third-order three-point boundary value problem

$$u'''(t) = f(t, u(t)), \quad t \in [0, 1],$$
$$u'(0) = u''(\eta) = u(1) = 0,$$

where $f \in C([0,1] \times [0,+\infty), [0,+\infty))$ and $\eta \in [2-\sqrt{2}, 1)$. The emphasis is mainly that although the corresponding Green's function is sign-changing, the solution obtained is still positive. Moreover, our iterative scheme starts off with zero function, which implies that the iterative scheme is feasible. An example is also included to illustrate the main results.

1. Introduction

Third-order differential equations arise from a variety of areas of applied mathematics and physics, e.g., in the deflection of a curved beam having a constant or varying cross section, a three-layer beam, electromagnetic waves or gravity driven flows and so on [6].

Recently, the existence of single or multiple positive solutions to some third-order three-point boundary value problems (BVPs for short) has received much attention from many authors. For example, in 1998, by using the Leggett-Williams fixed point theorem, Anderson [1] proved the existence of at least three positive solutions to the BVP

$$-x'''(t) + f(x(t)) = 0, \quad t \in [0, 1],$$
$$x(0) = x'(t_2) = x''(1) = 0,$$

where $t_2 \in [1/2, 1)$. In 2003, Anderson [2] obtained some existence results of positive solutions for the BVP

$$x'''(t) = f(t, x(t)), \quad t_1 \le t \le t_3,$$
$$x(t_1) = x'(t_2) = 0, \quad \gamma x(t_3) + \delta x''(t_3) = 0.$$

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The main tools used were the Guo-Krasnosel'skii and Leggett-Williams fixed point theorems. In 2005, Sun [14] studied the existence of single and multiple positive solutions for the singular BVP

$$\begin{gathered} u'''(t) - \lambda a(t) F(t, u(t)) = 0, \ t \in (0,1), \\ u(0) = u'(\eta) = u''(1) = 0, \end{gathered}$$

where $\eta \in [1/2, 1)$, λ was a positive parameter and $a(t)$ was a nonnegative continuous function defined on $(0, 1)$. His main tool was the Guo-Krasnosel'skii fixed point theorem. In 2008, by using the Guo-Krasnosel'skii fixed point theorem, Guo, Sun and Zhao [7] obtained the existence of at least one positive solution for the BVP

$$\begin{gathered} u'''(t) + h(t) f(u(t)) = 0, \quad t \in (0,1), \\ u(0) = u'(0) = 0, \quad u'(1) = \alpha u'(\eta), \end{gathered}$$

where $0 < \eta < 1$ and $1 < \alpha < \frac{1}{\eta}$. For more results concerning the existence of positive solutions to third-order three-point BVPs, one can refer to [3, 4, 5, 9, 11, 15, 16].

We want to point out that all the above-mentioned works are achieved when the corresponding Green's functions are nonnegative, which is a very important condition. A natural question is that whether we can obtain the existence of positive solutions to some third-order three-point BVPs when the corresponding Green's functions are sign-changing. It is worth mentioning that Palamides and Smyrlis [8] discussed the existence of at least one positive solution to the singular third-order three-point BVP with an indefinitely signed Green's function

$$\begin{gathered} u'''(t) = a(t) f(t, u(t)), \quad t \in (0,1), \\ u(0) = u(1) = u''(\eta) = 0, \quad \eta \in (\frac{17}{24}, 1). \end{gathered}$$

Their technique was a combination of the Guo-Krasnosel'skii fixed point theorem and properties of the corresponding vector field. For some related results, one can refer to [10].

Very recently, inspired greatly by [8], the authors [12, 13] studied the following third-order three-point BVP

$$\begin{gathered} u'''(t) = f(t, u(t)), \quad t \in [0,1], \\ u'(0) = u''(\eta) = u(1) = 0, \end{gathered} \tag{1.1}$$

where $\eta \in (1/2, 1)$. Although the corresponding Green's function was sign-changing, the existence of single or multiple positive solutions for the BVP (1.1) was still obtained. The main tools used were the Guo-Krasnosel'skii and Leggett-Williams fixed point theorems.

In this article, we continue to study the BVP (1.1). Throughout this paper, we always assume that $f \in C([0, 1] \times [0, +\infty), [0, +\infty))$ and $\eta \in [2 - \sqrt{2}, 1)$. Although the corresponding Green's function is sign-changing, we still obtain the existence of a decreasing positive solution under some suitable conditions on f. Our main method is iterative technique. It is worth mentioning that the iterative scheme starts off with zero function, which is feasible for computational purpose. An example is also included to illustrate our main results.

2. Main results

Let the space $E = C[0,1]$ be equipped with the norm $\|u\| = \max_{t\in[0,1]} |u(t)|$. To obtain the existence of a positive solution for (1.1), we need to construct a suitable cone in the Banach E. Let u be a solution of (1.1). Then it is easy to verify that $u(t) \geq 0$ for $t \in [0,1]$ provided that $u'(1) \leq 0$. In fact, since f is nonnegative, we know that $u'''(t) \geq 0$ for $t \in [0,1]$, which together with $u''(\eta) = 0$ implies that

$$u''(t) \leq 0 \text{ for } t \in [0,\eta] \quad \text{and} \quad u''(t) \geq 0 \text{ for } t \in [\eta,1]. \tag{2.1}$$

In view of (2.1) and $u'(0) = 0$, we have

$$u'(t) \leq 0 \text{ for } t \in [0,\eta] \quad \text{and} \quad u'(t) \leq u'(1) \text{ for } t \in [\eta,1]. \tag{2.2}$$

If $u'(1) \leq 0$, then it follows from (2.2) that $u'(t) \leq 0$ for $t \in [0,1]$, which together with $u(1) = 0$ implies that $u(t) \geq 0$ for $t \in [0,1]$. Therefore, we define a cone in E as follows:

$$K = \{u \in E : u(t) \text{ is nonnegative and decreasing on } [0,1]\}.$$

Note that this induces an order relation $\lesssim$ in E by defining $u \lesssim v$ if and only if $v - u \in K$.

For any $y \in E$, we consider the boundary value problem

$$\begin{gathered} u'''(t) = y(t), \quad t \in [0,1], \\ u'(0) = u''(\eta) = u(1) = 0. \end{gathered} \tag{2.3}$$

After a simple computation, we may obtain the expression of Green's function $G(t,s)$ for (2.3) as follows: for $s \geq \eta$,

$$G(t,s) = \begin{cases} -\frac{(1-s)^2}{2}, & t \leq s, \\ \frac{t^2-2st+2s-1}{2}, & s \leq t \end{cases}$$

and for $s < \eta$,

$$G(t,s) = \begin{cases} \frac{-t^2-s^2+2s}{2}, & t \leq s, \\ (1-t)s, & s \leq t. \end{cases}$$

Obviously,

$$G(t,s) \geq 0 \text{ for } 0 \leq s < \eta \quad \text{and} \quad G(t,s) \leq 0 \text{ for } \eta \leq s \leq 1.$$

Moreover, for $s \geq \eta$,

$$\begin{gathered} \max\{G(t,s) : t \in [0,1]\} = G(1,s) = 0, \\ \min\{G(t,s) : t \in [0,1]\} = G(s,s) = -\frac{(1-s)^2}{2} \geq -\frac{(1-\eta)^2}{2} \end{gathered}$$

and for $s < \eta$,

$$\begin{gathered} \max\{G(t,s) : t \in [0,1]\} = G(0,s) = s - \frac{s^2}{2} \leq \eta - \frac{\eta^2}{2}, \\ \min\{G(t,s) : t \in [0,1]\} = G(1,s) = 0. \end{gathered}$$

So,

$$\max\{|G(t,s)| : t,s \in [0,1]\} = \max\big\{\frac{(1-\eta)^2}{2}, \eta - \frac{\eta^2}{2}\big\} = \eta - \frac{\eta^2}{2} =: M.$$

It is obvious that $\sqrt{2} - 1 \leq M < 1/2$.

In the remaining of this article, we assume that $f : [0, 1] \times [0, +\infty) \to [0, +\infty)$ is continuous and satisfies the following two conditions:

(C1) For each $u \in [0, +\infty)$, the mapping $t \mapsto f(t, u)$ is decreasing;
(C2) For each $t \in [0, 1]$, the mapping $u \mapsto f(t, u)$ is increasing.

Now, we define an operator T on K by

$$(Tu)(t) = \int_0^1 G(t, s)f(s, u(s))ds, \quad t \in [0, 1].$$

Obviously, if u is a fixed point of T in K, then u is a decreasing nonnegative solution of (1.1).

Lemma 2.1. *The operator $T : K \to K$ is completely continuous.*

Proof. Let $u \in K$. Then for $0 \le t \le \eta$, we obtain

$$\begin{aligned}(Tu)(t) = (1 - t)\int_0^t sf(s, u(s))ds + \int_t^\eta \frac{-t^2 - s^2 + 2s}{2} f(s, u(s))ds \\ - \int_\eta^1 \frac{(1 - s)^2}{2} f(s, u(s))ds,\end{aligned}$$

which shows that

$$(Tu)'(t) = -\Big(\int_0^t sf(s, u(s))ds + t\int_t^\eta f(s, u(s))ds\Big) \le 0.$$

For $\eta \le t \le 1$, we have

$$\begin{aligned}(Tu)(t) = (1 - t)\int_0^\eta sf(s, u(s))ds + \int_\eta^t \frac{t^2 - 2st + 2s - 1}{2} f(s, u(s))ds \\ - \int_t^1 \frac{(1 - s)^2}{2} f(s, u(s))ds,\end{aligned}$$

which together with (C1) and (C2) imply that

$$(Tu)'(t) = -\int_0^\eta sf(s, u(s))ds + \int_\eta^t (t - s)f(s, u(s))ds \le -f(\eta, u(\eta))(\eta - \frac{t}{2})t \le 0.$$

So, $(Tu)(t)$ is decreasing on $[0, 1]$. At the same time, since $(Tu)(1) = 0$, $(Tu)(t)$ is nonnegative on $[0, 1]$. This indicates that $Tu \in K$.

Now, we assume that $D \subset K$ is a bounded set. Then there exists a constant $M_1 > 0$ such that $\|u\| \le M_1$ for any $u \in D$. In what follows, we will prove that $T(D)$ is relatively compact. Let

$$M_2 = \sup\{f(t, u) : (t, u) \in [0, 1] \times [0, M_1]\}.$$

Then for any $y \in T(D)$, there exists $u \in D$ such that $y = Tu$, and so,

$$\begin{aligned}|y(t)| = |(Tu)(t)| &= \Big|\int_0^1 G(t, s)f(s, u(s))ds\Big| \\ &\le \int_0^1 |G(t, s)|f(s, u(s))ds \\ &\le M\int_0^1 f(s, u(s))ds \le MM_2, \quad t \in [0, 1],\end{aligned}$$

which implies that $T(D)$ is uniformly bounded. On the other hand, when $\varepsilon > 0$, if we choose $0 < \xi < \min\{1-\eta, \frac{\varepsilon}{12M_2(M+1)}\}$, then, for any $u \in D$,

$$\int_{\eta-\xi}^{\eta+\xi} f(s,u(s))ds \le 2M_2\xi < \frac{\varepsilon}{6(M+1)}. \tag{2.4}$$

Since $G(t,s)$ is uniformly continuous on $[0,1]\times[0,\eta-\xi]$ and $[0,1]\times[\eta+\xi,1]$, there exists $\delta > 0$ such that for any $t_1, t_2 \in [0,1]$ with $|t_1 - t_2| < \delta$,

$$|G(t_1,s) - G(t_2,s)| < \frac{\varepsilon}{3(M_2+1)(\eta-\xi)}, \quad s \in [0,\eta-\xi] \tag{2.5}$$

and

$$|G(t_1,s) - G(t_2,s)| < \frac{\varepsilon}{3(M_2+1)(1-\eta-\xi)}, \quad s \in [\eta+\xi,1]. \tag{2.6}$$

In view of (2.4), (2.5) and (2.6), for any $y \in T(D)$ and $t_1, t_2 \in [0,1]$ with $|t_1 - t_2| < \delta$, we have

$$\begin{aligned}
|y(t_1) - y(t_2)| &= |(Tu)(t_1) - (Tu)(t_2)| \\
&= |\int_0^1 (G(t_1,s) - G(t_2,s)) f(s,u(s))ds| \\
&\le \int_0^1 |G(t_1,s) - G(t_2,s)| f(s,u(s))ds \\
&= \int_0^{\eta-\xi} |G(t_1,s) - G(t_2,s)| f(s,u(s))ds \\
&\quad + \int_{\eta-\xi}^{\eta+\xi} |G(t_1,s) - G(t_2,s)| f(s,u(s))ds \\
&\quad + \int_{\eta+\xi}^{1} |G(t_1,s) - G(t_2,s)| f(s,u(s))ds \\
&\le \frac{M_2\varepsilon}{3(M_2+1)} + \frac{M\varepsilon}{3(M+1)} + \frac{M_2\varepsilon}{3(M_2+1)} < \varepsilon,
\end{aligned}$$

which implies that $T(D)$ is equicontinuous. By Arzela-Ascoli theorem, we know that $T(D)$ is relatively compact. Thus, we have shown that T is a compact operator.

Finally, we prove that T is continuous. Suppose that u_m $(m = 1,2,\dots)$, $u_0 \in K$ and $\|u_m - u_0\| \to 0 (m \to \infty)$. Then there exists $M_3 > 0$ such that for any m, $\|u_m\| \le M_3$. Let

$$M_4 = \sup\{f(t,u) : (t,u) \in [0,1]\times[0,M_3]\}.$$

Then for any m and $t \in [0,1]$, we have

$$G(t,s)f(s,u_m(s)) \le MM_4, \ s \in [0,1].$$

By applying Lebesgue Dominated Convergence theorem, we obtain

$$\begin{aligned}
\lim_{m\to\infty}(Tu_m)(t) &= \lim_{m\to\infty}\int_0^1 G(t,s)f(s,u_m(s))ds \\
&= \int_0^1 G(t,s)\lim_{m\to\infty} f(s,u_m(s))ds \\
&= \int_0^1 G(t,s)f(s,u_0(s))ds = (Tu_0)(t), \ t \in [0,1],
\end{aligned}$$

which indicates that T is continuous. Therefore, $T : K \to K$ is completely continuous. $\square$

Theorem 2.2. *Assume that $f(t, 0) \not\equiv 0$ for $t \in [0, 1]$ and there exist two positive constants a and b such that the following conditions are satisfied:*

- (C3) $f(0, a) \le 2a$;
- (C4) $b(u_2 - u_1) \le f(t, u_2) - f(t, u_1) \le 2b(u_2 - u_1)$, $0 \le t \le 1$, $0 \le u_1 \le u_2 \le a$.

If we construct a iterative sequence $v_{n+1} = Tv_n$, $n = 0, 1, 2, \dots$, where $v_0(t) \equiv 0$ for $t \in [0, 1]$, then $\{v_n\}_{n=1}^{\infty}$ converges to v^* in E and v^* is a decreasing positive solution of the BVP (1.1).

Proof. Let $K_a = \{u \in K : \|u\| \le a\}$. Then we may assert that $T : K_a \to K_a$. In fact, if $u \in K_a$, then it follows from Lemma 2.1 that $Tu \in K$. In view of (C3) and $0 \le u(s) \le a$ for $s \in [0, 1]$, we have

$$0 \le (Tu)(t) = \int_0^1 G(t, s) f(s, u(s)) ds \le 2aM \le a, \ t \in [0, 1],$$

which shows that $\|Tu\| \le a$. So, $T : K_a \to K_a$.

Now, we prove that $\{v_n\}_{n=1}^{\infty}$ converges to v^* in E and v^* is a decreasing positive solution of (1.1). Indeed, in view of $v_0 \in K_a$ and $T : K_a \to K_a$, we have $v_n \in K_a$, $n = 1, 2, \dots$. Since the set $\{v_n\}_{n=0}^{\infty}$ is bounded and T is completely continuous, we know that the set $\{v_n\}_{n=1}^{\infty}$ is relatively compact. In what follows, we prove that $\{v_n\}_{n=0}^{\infty}$ is monotone by induction. First, it is obvious that $v_1 - v_0 = v_1 \in K$, which shows that $v_0 \lesssim v_1$. Next, we assume that $v_{k-1} \lesssim v_k$. Then it follows from (C4) that for $0 \le t \le \eta$,

$$\begin{aligned}
&v'_{k+1}(t) - v'_k(t) \\
&= (Tv_k)'(t) - (Tv_{k-1})'(t) \\
&= \int_0^1 \frac{\partial G(t, s)}{\partial t} [f(s, v_k(s)) - f(s, v_{k-1}(s))] ds \\
&= -\Big\{ \int_0^t s[f(s, v_k(s)) - f(s, v_{k-1}(s))] ds + t \int_t^\eta [f(s, v_k(s)) - f(s, v_{k-1}(s))] ds \Big\} \\
&\le 0,
\end{aligned}$$

and for $\eta \le t \le 1$,

$$\begin{aligned}
&v'_{k+1}(t) - v'_k(t) \\
&= (Tv_k)'(t) - (Tv_{k-1})'(t) \\
&= \int_0^1 \frac{\partial G(t, s)}{\partial t} [f(s, v_k(s)) - f(s, v_{k-1}(s))] ds \\
&= -\int_0^\eta s[f(s, v_k(s)) - f(s, v_{k-1}(s))] ds + \int_\eta^t (t - s)[f(s, v_k(s)) - f(s, v_{k-1}(s))] ds \\
&\le b[v_k(\eta) - v_{k-1}(\eta)]\big(t^2 - 2\eta t + \frac{\eta^2}{2}\big) \le 0,
\end{aligned}$$

hence

$$v'_{k+1}(t) - v'_k(t) \le 0, \ t \in [0, 1]; \tag{2.7}$$

that is, $v_{k+1}(t) - v_k(t)$ is decreasing on $[0,1]$. At the same time, it is easy to see that

$$v_{k+1}(1) - v_k(1) = \int_0^1 G(1,s)[f(s, v_k(s)) - f(s, v_{k-1}(s))]ds = 0,$$

therefore,

$$v_{k+1}(t) - v_k(t) \geq v_{k+1}(1) - v_k(1) = 0, \quad t \in [0,1]. \tag{2.8}$$

It follows from (2.7) and (2.8) that $v_{k+1} - v_k \in K$, which indicates that $v_k \lesssim v_{k+1}$. Thus, we have shown that $v_n \lesssim v_{n+1}$, $n = 0, 1, 2 \dots$. Since $\{v_n\}_{n=1}^{\infty}$ is relatively compact and monotone, there exists a $v^* \in K_a$ such that $\|v_n - v^*\| \to 0 (n \to \infty)$, which together with the continuity of T and the fact that $v_{n+1} = Tv_n$ implies that $v^* = Tv^*$. This indicates that v^* is a decreasing nonnegative solution of (1.1). Moreover, in view of $f(t,0) \not\equiv 0$, $t \in [0,1]$, we know that zero function is not a solution of (1.1), which shows that v^* is a positive solution of (1.1). □

3. An example

Consider the boundary value problem

$$\begin{gathered} u'''(t) = \frac{1}{4}u^2(t) + u(t) + (1-t), \quad t \in [0,1], \\ u'(0) = u''(\frac{2}{3}) = u(1) = 0. \end{gathered} \tag{3.1}$$

If we let $\eta = 2/3$ and $f(t,u) = \frac{1}{4}u^2 + u + (1-t)$, $(t,u) \in [0,1] \times [0, +\infty)$, then all the hypotheses of Theorem 2.2 are fulfilled with $a = 2$ and $b = 1$. It follows from Theorem 2.2 that (3.1) has a decreasing positive solution v^*. Moreover, the iterative scheme is $v_0(t) \equiv 0$ for $t \in [0,1]$,

$$v_{n+1}(t) = \begin{cases} \int_0^t s(1-t)\big[\frac{1}{4}(v_n(s))^2 + v_n(s) + (1-s)\big]ds \\ + \int_t^{2/3} \frac{-t^2-s^2+2s}{2}\big[\frac{1}{4}(v_n(s))^2 + v_n(s) + (1-s)\big]ds \\ - \int_{2/3}^1 \frac{(1-s)^2}{2}[\frac{1}{4}(v_n(s))^2 + v_n(s) + (1-s)]ds, \\ \quad \text{if } t \in [0, 2/3],\ n = 0, 1, 2, \dots, \\ \int_0^{2/3} s(1-t)[\frac{1}{4}(v_n(s))^2 + v_n(s) + (1-s)]ds \\ + \int_{2/3}^t \frac{t^2-2st+2s-1}{2}[\frac{1}{4}(v_n(s))^2 + v_n(s) + (1-s)]ds \\ - \int_t^1 \frac{(1-s)^2}{2}[\frac{1}{4}(v_n(s))^2 + v_n(s) + (1-s)]ds, \\ \quad \text{if } t \in [2/3, 1],\ n = 0, 1, 2, \dots. \end{cases}$$

The first, second, third and fourth terms of the scheme are as follows:

$$\begin{gathered} v_0(t) \equiv 0, \quad v_1(t) = -\frac{2}{9}t^2 + \frac{1}{6}t^3 - \frac{1}{24}t^4 + \frac{7}{72}, \\ v_2(t) = -\frac{18305}{62208}t^2 + \frac{26425}{124416}t^3 + \frac{1915}{62208}t^4 - \frac{779}{7776}t^5 + \frac{1291}{31104}t^6 + \frac{23}{62208}t^7 \\ - \frac{95}{13824}t^8 + \frac{1}{324}t^9 - \frac{5}{6912}t^{10} + \frac{1}{13824}t^{11} + \frac{14161}{124416}, \end{gathered}$$

$$
\begin{aligned}
v_3(t) = &-\frac{58154370371}{185752092672}t^2 + \frac{27688166033}{123834728448}t^3 + \frac{1099507181}{15479341056}t^4 - \frac{4628786297}{30958682112}t^5 \\
&+ \frac{3441194273}{123834728448}t^6 + \frac{223703119}{3869835264}t^7 - \frac{169069657}{3869835264}t^8 + \frac{1231643521}{371504185344}t^9 \\
&+ \frac{89491399}{7739670528}t^{10} - \frac{1210466063}{185752092672}t^{11} - \frac{824305}{92876046336}t^{12} + \frac{52150655}{30958682112}t^{13} \\
&- \frac{153634915}{185752092672}t^{14} + \frac{3356471}{92876046336}t^{15} + \frac{18718607}{123834728448}t^{16} \\
&- \frac{15792949}{185752092672}t^{17} + \frac{1212829}{61917364224}t^{18} + \frac{223969}{123834728448}t^{19} \\
&- \frac{10753}{3439853568}t^{20} + \frac{1087}{859963392}t^{21} - \frac{157}{509607936}t^{22} + \frac{169}{3439853568}t^{23} \\
&- \frac{11}{2293235712}t^{24} + \frac{1}{4586471424}t^{25} + \frac{43366416289}{371504185344}.
\end{aligned}
$$

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