Electronic Journal of Differential Equations, Vol. 2013 (2013), No. 22, pp. 1–9.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu

PERMANENCE FOR A COMPETITION AND COOPERATION MODEL OF ENTERPRISE CLUSTER WITH DELAYS AND FEEDBACK CONTROLS

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Abstract. In this article, based on population ecology theory, we present a competition and cooperation system of the enterprise cluster with time delays and feedback controls. By using differential inequalities, we obtain sufficient conditions for the permanence of the system, which shows that the time delay, feedback control and initial production have an influence on the persistent properties of the system. We further interpret our result from the economic point of view. Some suggestions about enterprise cluster are put forward through the analysis of our results.

1. Introduction

Enterprise cluster refers to the concentration of similar or related enterprises in a specific area, which form fixed economic outputs and have certain economic influence on outsides [8]. After a large number of observations, it is found that there is a similarity between the enterprise cluster and the ecological population system. Recently, some researchers have presented some models about enterprise clusters based on ecology theory, which arouse growing interest in applying the methods of ecology and dynamic system theory to study enterprise clusters, for example [3, 4, 5, 6, 9, 11, 12, 13, 14] and references cited therein. For an example, in [9], the authors considered the following competition and cooperation of enterprise cluster based on the ecosystem

$$\begin{aligned} x_1'(t) &= r_1x_1(t)[1-\frac{1}{K}x_1(t)-\frac{1}{K}\alpha(x_2(t)-b_2)^2],\\ x_2'(t) &= r_2x_2(t)[1-\frac{1}{K}x_2(t)+\frac{1}{K}\beta(x_1(t)-b_1)^2], \end{aligned}$$

where $x_1(t)$, $x_2(t)$ represent the outputs of enterprise A and enterprise B, respectively, $r_i, b_i, K, \alpha, \beta$ are positive constants, $i = 1, 2$. r_1, r_2 are the intrinsic growth rates, K denotes the carrying capacity of market under the natural conditions, α, β are the competitive power coefficients of the two enterprises, and b_1, b_2 are the initial productions of the enterprises, respectively.

2000 *Mathematics Subject Classification.* 34C11, 91B55, 92D25, 93B52.
Key words and phrases. Enterprise cluster; permanence; competition and cooperation model; time delay; feedback control.
©2013 Texas State University - San Marcos.
Submitted October 4, 2012. Published January 25, 2013.
Supported by grant 10971183 from the National Natural Sciences Foundation of China.

Let $a_1 = \frac{r_1}{K}$, $a_2 = \frac{r_2}{K}$, $c_1 = \frac{\alpha}{K}$, $c_2 = \frac{\beta}{K}$, the system above becomes

$$
\begin{aligned}
x_1'(t) &= x_1(t)[r_1 - a_1x_1(t) - c_1(x_2 - b_2)^2],\\
x_2'(t) &= x_2(t)[r_2 - a_2x_2(t) + c_2(x_1 - b_1)^2].
\end{aligned}
$$

In real world, competitors always invade the core assets of enterprises by counterplans and bring the actual loss, which is not transient happened, there is a time delay. On the other hand, enterprises in the real world are continuously distributed by unpredictable forces which can result in changes in the economic parameters such as intrinsic growth rates. Of practical interest in economics is the question of whether or not an enterprise cluster can withstand those unpredictable disturbances which persist for a finite period of time. In the language of control variables, we call the disturbance functions as control variables.

Motivated by above, in this paper, we propose a competitive and cooperation model of n satellite enterprises and a dominant enterprise under center halfback model with time-varying delays and feedback controls as follows:

$$
\begin{aligned}
\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} &= x_1(t)\Big[r_1(t) - \sum_{i=0}^{m} a_1^i(t)x_1(t-i\tau) - \gamma_1(t)(x_2(t)-b_2)^2\\
&\quad - q_1(t)\int_{-\delta_1}^{0} F_1(s)u_1(t+s)\mathrm{d}s\Big],\\
\frac{\mathrm{d}x_2(t)}{\mathrm{d}t} &= x_2(t)\Big[r_2(t) - \sum_{j=0}^{n} a_2^j(t)x_2(t-j\tau) + \gamma_2(t)\int_{-\sigma}^{0} H(s)(x_1(t+s)-b_1)^2\mathrm{d}s\\
&\quad - q_2(t)\int_{-\delta_2}^{0} F_2(s)u_2(t+s)\mathrm{d}s\Big],\\
\frac{\mathrm{d}u_k(t)}{\mathrm{d}t} &= -d_k(t)u_k(t) + e_k(t)x_k(t) + f_k(t)\int_{-\eta_k}^{0} G_k(s)x_k(t+s)\mathrm{d}s,\ k=1,2
\end{aligned}
\tag{1.1}
$$

with initial conditions

$$
\begin{gathered}
x_1(t) = \phi_1(t) \ge 0, \quad \text{for } t\in[-\gamma,0) \text{ and } \phi_1(0)>0,\\
x_2(t) = \phi_2(t) \ge 0, \quad \text{for } t\in[-\gamma,0) \text{ and } \phi_2(0)>0,\\
u_k(t) = \phi_{k+2}(t) \ge 0, \quad \text{for } t\in[-\gamma,0) \text{ and } \phi_{k+2}(0)>0,\ k=1,2,
\end{gathered}
\tag{1.2}
$$

where $\xi = \max\{\delta_1,\delta_2,\eta_1,\eta_2,\sigma,m\tau,n\tau\}$, $\phi_1(t),\phi_2(t),\phi_{k+2}(t)(k=1,2)$ are continuous on $[-\xi,0]$, $x_1(t)$ and $x_2(t)$ denote the outputs of enterprises A and B in cluster respectively, $r_1(t)$ and $r_2(t)$ are their intrinsic growth rates at time t, $a_1^i(t)$ and $a_2^j(t)$ account for their self-regulation coefficients, $\gamma_1(t)$ and $\gamma_2(t)$ represent their contribution coefficients to the other, b_1,b_2 are the initial productions of the enterprises respectively, $\delta_k,\eta_k,\sigma,\tau,m,n$ are positive constants, $F_k(s),G_k(s),H(s)$ are all nonnegative continuous functions such that

$$
\int_{-\delta_k}^{0} F_k(s)\mathrm{d}s = 1,\quad \int_{-\eta_k}^{0} G_k(s)\mathrm{d}s = 1,\quad \int_{-\sigma}^{0} H(s)\mathrm{d}s = 1\ (k=1,2),
$$

the above two equations describe the process of interactions between enterprises A and B, the latter two equations are control equations, $u_1(t)$ and $u_2(t)$ are feedback control variables, $a_1^i(t)$, $a_2^j(t)$ $(i=0,1,\dots,m;\ j=0,1,\dots,n)$, $r_k(t)$, $\gamma_k(t)$,

$q_k(t)$, $d_k(t)$, $e_k(t)$, $f_k(t)$ $(k = 1, 2)$ are continuous, bounded and positive real-valued functions on $[0, +\infty)$.

Since the competition and cooperation among inter-members in a cluster is the driving force for the evolution of enterprise cluster, a nature question is that under what conditions an enterprise cluster can attain the goal of co-existence, co-evolution and common prosperity? Our main purpose of this paper is to study the permanence of (1.1). Our result shows that not only the time delay and feedback control but also the initial production have influence on the permanence of system (1.1).

2. Main results

In this section, we establish the permanence of system (1.1). It is not difficult to see that solutions of system (1.1) and (1.2) are well defined for all $t \geq 0$ and satisfy

$$x_k(t) > 0, \quad u_k(t) > 0, \quad t \geq 0,\ k = 1, 2.$$

For convenience, we shall introduce some notations, definition and lemmas which will be useful for our main results. For a continuous bounded function $g(t)$ defined on $[0, +\infty)$, we denote

$$g^M = \sup_{0 \leq t < +\infty} g(t), \quad g^L = \inf_{0 \leq t < +\infty} g(t).$$

Definition 2.1 ([10])**.** System (1.1) is said to be permanent if there exists two positive constants m, M such that

$$\begin{aligned} m \leq \liminf_{t\to\infty} x_i(t) \leq \limsup_{t\to\infty} x_i(t) \leq M, \quad i = 1, 2;\\ m \leq \liminf_{t\to\infty} u_i(t) \leq \limsup_{t\to\infty} u_i(t) \leq M, \quad i = 1, 2; \end{aligned}$$

for any solution $(x_1(t), x_2(t), u_1(t), u_2(t))^T$ of system (1.1).

As a direct consequence of [10, Lemma 2.2], we have the following result.

Lemma 2.2. *Assume that for $y(t) > 0$, it holds that*

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} \leq y(t)\Big[\lambda - \sum_{l=0}^{k} \mu^l y(t - l\tau)\Big]$$

with initial conditions $y(t) = \phi(t) \geq 0$ for $t \in [-k\tau, 0)$ and $\phi(0) > 0$, where

$$\lambda > 0, \quad \mu^l \geq 0, \quad l = 0, 1, \dots, k, \quad \mu = \sum_{l=0}^{k} \mu^l > 0,$$

are constants. Then there exists a positive constant $K_y < +\infty$ such that

$$\limsup_{t\to+\infty} y(t) \leq K_y = \frac{\lambda}{\mu} \exp\{\lambda k\tau\} < +\infty. \tag{2.1}$$

Lemma 2.3 ([7])**.** *Assume that for $y(t) > 0$, it holds that*

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} \geq y(t)\Big[\lambda - \sum_{l=0}^{k} \mu^l y(t - l\tau)\Big].$$

If (2.1) holds, then there exists a positive constant $k_y > 0$ such that

$$\liminf_{t\to+\infty} y(t) \geq k_y = \frac{\lambda}{\mu} \exp\{(\lambda - \mu K_y)k\tau\} > 0, \tag{2.2}$$

where $\mu = \sum_{l=0}^{k} \mu^l > 0$, $\lambda > 0$.

Lemma 2.4 ([1]). *Let* $a > 0, b > 0$,

(I) *If* $\frac{\mathrm{d}x}{\mathrm{d}t} \geq b - ax$, *then* $\liminf_{t\to+\infty} x(t) \geq \frac{b}{a}$ *for* $t \geq 0$ *and* $x(0) > 0$.

(II) *If* $\frac{\mathrm{d}x}{\mathrm{d}t} \leq b - ax$, *then* $\limsup_{t\to+\infty} x(t) \leq \frac{b}{a}$ *for* $t \geq 0$ *and* $x(0) > 0$.

Lemma 2.5. [2] *Assume that* $a > 0$, $b(t) > 0$ *is a bounded continuous function and* $x(0) > 0$. *Further suppose that*

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} \leq b(t) - ax(t),$$

then for all $t \geq s \geq 0$,

$$x(t) \leq x(t-s)\exp\{-as\} + \int_{t-s}^{t} b(r)\exp\{a(r-t)\}\mathrm{d}r.$$

Lemma 2.6. *Assume that* $a_1^{iL} > 0, a_2^{jL} > 0$ $(i = 0, 1, \dots, m; j = 0, 1, \dots, n)$, $d_k^L > 0$ $(k = 1, 2)$. *Let* $(x(t), u(t))^T = (x_1(t), x_2(t), u_1(t), u_2(t))^T$ *be any positive solution of system* (1.1), *then there exists a positive constant* $\overline{M}$ *which is independent of the solution of system* (1.1) *such that*

$$\limsup_{t\to+\infty} x_k(t) \leq \overline{M}, \quad \limsup_{t\to+\infty} u_k(t) \leq \overline{M}, \quad k = 1, 2.$$

Proof. Let $(x(t), u(t))^T = (x_1(t), x_2(t), u_1(t), u_2(t))^T$ be a solution of system (1.1) satisfying the initial condition (1.2). For $t \geq 0$, from the first equation of system (1.1), it follows that

$$\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} \leq x_1(t)\Big[r_1^M - \sum_{i=0}^{m} a_1^{iL} x_1(t - i\tau)\Big], \quad t \geq 0. \tag{2.3}$$

Applying Lemma 2.2 to (2.3) leads to

$$\limsup_{t\to+\infty} x_1(t) \leq \frac{r_1^M}{\sum_{i=0}^{m} a_1^{iL}} \exp\{r_1^M m\tau\} := M_1. \tag{2.4}$$

Next, we show that $x_2(t)$ is bounded above. By (2.4), there exists a positive constant $T_1 > 0$ such that $x_1(t) \leq 2M_1$ for $t > T_1$. Then, from the second equation of system (1.1), we obtain

$$\frac{\mathrm{d}x_2(t)}{\mathrm{d}t} \leq x_2(t)\Big[r_2^M + \gamma_2^M(2M_1 - b_1)^2 - \sum_{j=0}^{n} a_2^{jL} x_2(t - j\tau)\Big], \quad t \geq T_1.$$

By Lemma 2.2, it follows that

$$\limsup_{t\to+\infty} x_2(t) \leq \frac{r_2^M + \gamma_2^M(2M_1 - b_1)^2}{\sum_{j=0}^{n} a_2^{jL}} \exp\{(r_2^M + \gamma_2^M(2M_1 - b_1)^2)n\tau\} := M_2. \tag{2.5}$$

Thus, there exists a $T_2 > T_1 + \xi$ such that $x_1(t) \leq 2M_1, x_2(t) \leq 2M_2$ for $t \geq T_2$. It follows from system (1.1) that

$$\frac{\mathrm{d}u_k(t)}{\mathrm{d}t} \leq 2(e_k^M + f_k^M)M_k - d_k^L u_k(t), \quad k = 1, 2,\ t \geq T_2,$$

applying Lemma 2.4 (II) to the differential inequalities above, we obtain

$$\limsup_{t\to+\infty} u_k(t) \leq \frac{2(e_k^M + f_k^M)M_k}{d_k^L} := M_{k+2}, \quad k = 1, 2. \tag{2.6}$$

Combined with (2.4),(2.5) and (2.6), we set

$$\overline{M} := \max\{M_1, M_2, M_3, M_4\}. \tag{2.7}$$

Obviously, $\overline{M}$ is independent of the solution of(1.1) and

$$\limsup_{t\to+\infty} x_k(t) \le \overline{M}, \quad \limsup_{t\to+\infty} u_k(t) \le \overline{M}, \quad k = 1, 2.$$

The proof is complete. □

Lemma 2.7. *Assume that $r_2^L > 0$, $\gamma_k^L > 0$, $d_k^L > 0$, $e_k^L > 0, f_k^L > 0$ $(k = 1, 2)$, $2q_2^M\overline{M} < \frac{1}{2}r_2^L$, $\gamma_1^M(2\overline{M} - b_2)^2 < \frac{r_1^L}{2}$. Let $(x(t), u(t))^T = (x_1(t), x_2(t), u_1(t), u_2(t))^T$ be any positive solution of* (1.1)*, then there exists a positive constant $\overline{m}$, which is independent of the solution of* (1.1) *such that*

$$\liminf_{t\to+\infty} x_k(t) \ge \overline{m}, \quad \liminf_{t\to+\infty} u_k(t) \ge \overline{m}, \quad k = 1, 2,$$

where $\overline{M}$ is defined by (2.7)*.*

Proof. Let $(x(t), u(t))^T = (x_1(t), x_2(t), u_1(t), u_2(t))^T$ be a solution of (1.1) satisfying the initial condition (1.2). From the first equation of system (1.1) and Lemma 2.6, there exists a positive constant $T_3 > T_2 + \xi$ such that $x_k(t) \le 2\overline{M}, u_k(t) \le 2\overline{M}, k = 1, 2$ for $t \ge T_3$, then we have

$$\begin{aligned}
\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} &\ge x_1(t)\Big[r_1^L - \sum_{i=0}^{m} a_1^{iM} 2\overline{M} - \gamma_1^M(2\overline{M} - b_2)^2 - 2q_1^M\overline{M}\Big], \quad t > T_3 \\
&\ge x_1(t)\Big[-\sum_{i=0}^{m} a_1^{iM} 2\overline{M} - \gamma_1^M(2\overline{M} - b_2)^2 - 2q_1^M\overline{M}\Big] = x_1(t)\cdot\theta,
\end{aligned} \tag{2.8}$$

where $\theta = -2\sum_{i=0}^{m} a_1^{iM}\overline{M} - \gamma_1^M(2\overline{M} - b_2)^2 - 2q_1^M\overline{M} < 0$. Integrating (2.8) from α to $t(\alpha \le t)$, we obtain

$$x_1(\alpha) \le x_1(t)\exp\{-\theta(t - \alpha)\}, \tag{2.9}$$

then

$$x_1(t + s) \le x_1(t)\exp\{\theta s\}, \quad s \le 0. \tag{2.10}$$

By the third equation of system (1.1), we obtain

$$\begin{aligned}
\frac{\mathrm{d}u_1(t)}{\mathrm{d}t} &\le -d_1^L u_1(t) + e_1^M x_1(t) + f_1^M \int_{-\eta_1}^{0} G_1(s)x_1(t + s)\mathrm{d}s \\
&\le -d_1^L u_1(t) + e_1^M x_1(t) + f_1^M \int_{-\eta_1}^{0} G_1(s)x_1(t)\exp\{\theta s\}\mathrm{d}s \\
&\le -d_1^L u_1(t) + e_1^M x_1(t) + f_1^M \exp\{-\theta\eta_1\}x_1(t) \\
&= (e_1^M + f_1^M \exp\{-\theta\eta_1\})x_1(t) - d_1^L u_1(t).
\end{aligned} \tag{2.11}$$

Applying Lemma 2.5 to (2.11), for $t \geq \alpha > T_3 + \xi$, we have

$$\begin{aligned}
u_1(t) &\leq u_1(t-\alpha)\exp\{-d_1^L\alpha\} + \int_{t-\alpha}^{t} (e_1^M + f_1^M \exp\{-\theta\eta_1\})x_1(r)\exp\{d_1^L(r-t)\}\mathrm{d}r \\
&\leq u_1(t-\alpha)\exp\{-d_1^L\alpha\} + (e_1^M + f_1^M \exp\{-\theta\eta_1\}) \\
&\quad \times \int_{t-\alpha}^{t} x_1(t)\exp\{-\theta(t-r)\}\exp\{d_1^L(r-t)\}\mathrm{d}r \\
&\leq u_1(t-\alpha)\exp\{-d_1^L\alpha\} + (e_1^M + f_1^M \exp\{-\theta\eta_1\})\frac{1}{\theta}(1-\exp\{-\theta\alpha\})x_1(t) \\
&= u_1(t-\alpha)\exp\{-d_1^L\alpha\} + \rho x_1(t),
\end{aligned} \tag{2.12}$$

where $\rho = \frac{1}{\theta}(e_1^M + f_1^M \exp\{-\theta\eta_1\})(1-\exp\{-\theta\alpha\}) > 0$. Notice that for large enough t, α and $t-\alpha > T_3$, then $u_1(t-\alpha) \leq 2\overline{M}$. Thus, for $t > T_3 + \alpha$, we obtain

$$u_1(t) \leq 2\overline{M}\exp\{-d_1^L\alpha\} + \rho x_1(t).$$

Combined with (2.10), for $t > T_3 + \alpha + \xi$, we have

$$\begin{aligned}
u_1(t+s) &\leq 2\overline{M}\exp\{-d_1^L\alpha\} + \rho x_1(t+s), \quad s \leq 0 \\
&\leq 2\overline{M}\exp\{-d_1^L\alpha\} + \rho x_1(t)\exp\{\theta s\}.
\end{aligned} \tag{2.13}$$

Take (2.13) into the first equation of system (1.1), for all $t > T_3 + \alpha + 2\xi$, we obtain

$$\begin{aligned}
\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} &\geq x_1(t)\Big[r_1^L - \sum_{i=0}^{m} a_1^{iM} x_1(t-i\tau) - \gamma_1^M(2\overline{M} - b_2)^2 \\
&\quad - q_1^M \int_{-\delta_1}^{0} F_1(s)\Big(2\overline{M}\exp\{-d_1^L\alpha\} + \rho x_1(t)\exp\{\theta s\}\Big)\mathrm{d}s\Big] \\
&\geq x_1(t)\Big[r_1^L - \sum_{i=0}^{m} a_1^{iM} x_1(t-i\tau) - \gamma_1^M(2\overline{M} - b_2)^2 \\
&\quad - q_1^M\Big(2\overline{M}\exp\{-d_1^L\alpha\} + \rho\exp\{-\theta\delta_1\}x_1(t)\Big)\Big] \\
&= x_1(t)\Big[r_1^L - (q_1^M\rho\exp\{-\theta\delta_1\})x_1(t) - \sum_{i=0}^{m} a_1^{iM} x_1(t-i\tau) \\
&\quad - \gamma_1^M(2\overline{M} - b_2)^2 - 2q_1^M\overline{M}\exp\{-d_1^L\alpha\}\Big].
\end{aligned}$$

Notice that for large enough $t, \exp\{-d_1^L\alpha\} \to 0$ as $\alpha \to +\infty$. Then, there exists a positive constant $\alpha_0 = \max\{\frac{1}{d_1^L} ln\frac{8q_1^M\overline{M}}{r_1^L} + 1, T_3 + \xi\}$ such that

$$2q_1^M\overline{M}\exp\{-d_1^L\alpha\} < \frac{r_1^L}{4} \quad \text{for} \quad \alpha \geq \alpha_0,$$

then, for $t > T_3 + \alpha_0 + 2\xi = T_4$, we have

$$\begin{aligned}
\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} &\geq x_1(t)\Big[\frac{r_1^L}{4} - (q_1^M\rho'\exp\{-\theta\delta_1\} + a_1^{0M})x_1(t) \\
&\quad - a_1^{1M}x_1(t-\tau) - \dots - -a_1^{mM}x_1(t-m\tau)\Big],
\end{aligned} \tag{2.14}$$

where $\rho' = \frac{1}{\theta}(e_1^M + f_1^M \exp\{-\theta\eta_1\})(1-\exp\{-\theta\alpha_0\}) > 0$. Applying Lemma 2.3 to the differential inequality (2.14), it follows that

$$\liminf_{t\to+\infty} x_1(t) \ge m_1 = \frac{\frac{1}{4}r_1^L}{\mu}\exp\{(\frac{1}{4}r_1^L - \mu k_1)m\tau\} > 0, \tag{2.15}$$

where

$$\mu = q_1^M\rho'\exp\{-\theta\delta_1\} + \sum_{i=0}^{m} a_1^{iM} > 0, \quad k_1 = \frac{r_1^L}{4\mu}\exp\{\frac{r_1^L}{4}m\tau\} > 0.$$

From (2.15), there exists a positive constant $T_5 > T_4 + \xi$ such that $x_1(t) \ge \frac{m_1}{2}$ for $t \ge T_5$. Then, by the second equation of system (1.1), we have

$$\frac{\mathrm{d}x_2(t)}{\mathrm{d}t} \ge x_2(t)\Big[r_2^L + \gamma_2^L(\frac{m_1}{2} - b_1)^2 - 2q_2^M\overline{M} - \sum_{j=0}^{n} a_2^{jM}x_2(t-j\tau)\Big], \quad t \ge T_5.$$

Applying Lemma 2.3 to the inequality above, we have

$$\begin{aligned}
&\liminf_{t\to+\infty} x_2(t) \ge m_2 \\
&= \frac{\frac{1}{2}r_2^L + \gamma_2^L(\frac{m_1}{2} - b_1)^2}{\sum_{j=0}^{n} a_2^{jM}}\exp\Big\{\Big[\frac{1}{2}r_2^L + \gamma_2^L(\frac{m_1}{2} - b_1)^2 - \sum_{j=0}^{n} a_2^{jM}k_2\Big]n\tau\Big\},
\end{aligned} \tag{2.16}$$

where $k_2 = \frac{\frac{1}{2}r_2^L + \gamma_2^L(\frac{m_1}{2} - b_1)^2}{\sum_{j=0}^{n} a_2^{jM}}\exp\{[\frac{1}{2}r_2^L + \gamma_2^L(\frac{m_1}{2} - b_1)^2]n\tau\}$. From the above discussion, there exists a $T_6 > T_5 + \xi$ such that

$$x_k(t) \ge \frac{1}{2}m_k, \quad k = 1, 2, \text{ for } t \ge T_6.$$

By system (1.1), we obtain

$$\frac{\mathrm{d}u_k(t)}{\mathrm{d}t} \ge \frac{1}{2}(e_k^L + f_k^L)m_k - d_k^M u_k(t), \quad k = 1, 2, \ t \ge T_6.$$

Applying Lemma 2.4(I) to the above differential inequalities, we obtain

$$\liminf_{t\to+\infty} u_k(t) \ge m_{k+2} = \frac{(e_k^L + f_k^L)m_k}{2d_k^M} > 0, \quad k = 1, 2. \tag{2.17}$$

Combined with (2.15),(2.16) and (2.17), we set $\overline{m} := \min\{m_1, m_2, m_3, m_4\}$. Then,

$$\liminf_{t\to+\infty} x_k(t) \ge \overline{m}, \quad \liminf_{t\to+\infty} u_k(t) \ge \overline{m}, \quad k = 1, 2.$$

The proof is complete. □

Theorem 2.8. *Assume that $a_1^{iL} > 0$, $a_2^{jL} > 0$ ($i = 0, 1, \dots, m; j = 0, 1, \dots, n$), $r_2^L > 0$, $\gamma_k^L > 0$, $d_k^L > 0$, $e_k^L > 0$, $f_k^L > 0$ ($k = 1, 2$), $2q_2^M\overline{M} < \frac{1}{2}r_2^L$, $\gamma_1^M(2\overline{M} - b_2)^2 < \frac{r_1^L}{2}$. Let $(x(t), u(t))^T = (x_1(t), x_2(t), u_1(t), u_2(t))^T$ be any positive solution of system (1.1), then system (1.1) is permanent, where $\overline{M}$ is defined by (2.7).*

Proof. Combining Lemma 2.6 and Lemma 2.7, the conclusion is obvious. □

Conclusion

In economic phenomena, time delays and feedback controls can not be ignored due to the effect of factors such as information, technology, patent protection, institutional arrangement and so on. In this paper, our result shows that time delays and feedback controls have an influence on the permanence of corporation and competition system (1.1) if $2q_2^M\overline{M} < \frac{1}{2}r_2^L$. The limited time delay is one of the most important conditions to guarantee the permanence of the system. Furthermore, the persistent property of the system not only relies on time delays and feedback controls, but also relies on the initial production of enterprise if $\gamma_1^M(2\overline{M}-b_2)^2 < \frac{r_1^L}{2}$, which means that one enterprise with small initial production in a cluster should expand its competitiveness and stimulate its production. Otherwise, the other enterprise in the cluster will search for a new co-operative enterprise which will add the new transaction cost and will be not good for its long term development. On the other hand, One enterprise with large initial production in the cluster should reduce its competitiveness. Otherwise, it will annex the other enterprise in the cluster leading to the extinction.

This prompts us that enterprises in a cluster should keep the moderate competition, moreover, share resource, improve the efficiency of cooperation, develop jointly and confront the change of external factors of enterprise cluster, which will guide enterprise cluster to become a healthy ecosystem of reciprocal regulation and mutual dependence.

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