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EXISTENCE OF POSITIVE ALMOST-PERIODIC SOLUTIONS FOR A NICHOLSON'S BLOWFLIES MODEL

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Abstract. This article concerns the existence of solutions to a first-order differential equation with time-varying delay, which is known as a Nicholson's blowflies model. By using fixed a point theorem and the Lyapunov functional method, we establish the existence and locally exponential stability of almost periodic solutions for the model. Also, we apply our results to two examples, for which some earlier results can not be applied.

1. Introduction and preliminaries

Nicholson [10] and Gurney [5] proposed the following delay differential equation as a population model

$$x'(t) = -\delta x(t) + Px(t-\tau)e^{-ax(t-\tau)}, \tag{1.1}$$

where $x(t)$ is the size of the population at time t, P is the maximum per capita daily egg production, $1/a$ is the size at which the population reproduces at its maximum rate, δ is the per capita daily adult death rate, τ is the generation time.

Nicholson's blowflies model and its analogous equations have attracted much attention. Especially, equation (1.1) and its variants have been of great interest for many mathematicians. As pointed out in [6], compared with periodic effects, almost periodic effects are more frequent in many biological dynamic systems. Therefore, recently, the existence and stability of almost periodic solutions for (1.1) and its variants have attracted more and more attention. We refer the reader to [1, 2, 3, 4, 7, 8, 9] and references therein for some recent development on such topic.

Motivated by the above works, we study further this topic, by investigate the Nicholson's blowflies model

$$x'(t) = -\alpha(t)x(t) + \sum_{j=1}^{m} \beta_j(t)x(t-\tau_j(t))e^{-\gamma_j(t)x(t-\tau_j(t))}, \tag{1.2}$$

where m is a given positive integer, and $\alpha, \beta_j, \gamma_j, \tau_j : \mathbb{R} \to (0, +\infty)$ are almost periodic functions, $j = 1, 2, \dots, m$.

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In this article, for a bounded continuous function g on $\mathbb{R}$, we denote

$$g^+ = \sup_{t\in\mathbb{R}} g(t), \quad g^- = \inf_{t\in\mathbb{R}} g(t).$$

In addition, for $j = 1, 2, \dots, m$, we assume that

$$\alpha^- > 0, \quad \beta_j^- > 0, \quad \gamma_j^- > 0, \quad \tau_j^- > 0.$$

Next, we recall some notation and basic results about almost periodic functions. For more details, we refer the reader to [6].

Definition 1.1. A continuous function f: $\mathbb{R} \to \mathbb{R}$ is called almost periodic if for each $\varepsilon > 0$, there exists $l(\varepsilon) > 0$ such that every interval I of length $l(\varepsilon)$ contains a number τ with the property that

$$\sup_{t\in\mathbb{R}} |f(t+\tau) - f(t)| < \varepsilon.$$

We denote the set of all such functions by $AP(\mathbb{R})$.

Definition 1.2. Let $x(\cdot)$ and $Q(\cdot)$ be $n \times n$ continuous matrix defined on $\mathbb{R}$. The linear system

$$x'(t) = Q(t)x(t) \tag{1.3}$$

is said to admit an exponential dichotomy on $\mathbb{R}$ if there exist positive constants k, α and a projection P such that

$$\|X(t)PX^{-1}(s)\| \le ke^{-\alpha(t-s)}, \quad t \ge s,$$
$$\|X(t)(I-P)X^{-1}(s)\| \le ke^{-\alpha(s-t)}, \quad t \le s,$$

for a fundamental solution matrix $X(t)$ of (1.3).

Lemma 1.3. *If the linear system* (1.3) *admits an exponential dichotomy, then the almost periodic system*

$$x'(t) = Q(t)x(t) + g(t)$$

has a unique almost periodic solution $x(t)$ given by

$$x(t) = \int_{-\infty}^{t} X(t)PX^{-1}(s)g(s)ds - \int_{t}^{+\infty} X(t)(I-P)X^{-1}(s)g(s)ds.$$

2. Main results

2.1. **Existence of almost periodic solutions.** For the next theorem we use the following two assumptions:

(H1) $n_2 \ge n_1 \ge \frac{1}{\min_{1\le j\le m}\{\gamma_j^+\}}$, where

$$n_2 = \sum_{j=1}^{m} \frac{\beta_j^+}{\alpha^-\gamma_j^- e}, \quad n_1 = \sum_{j=1}^{m} \frac{n_2\beta_j^- e^{-\gamma_j^+ n_2}}{\alpha^+}.$$

(H2) $\sum_{j=1}^{m} \frac{\beta_j^+}{\alpha^-} < \min\{\frac{e^k}{|1-k|}, e^2\}$, where $k = n_1 \min_{1\le j\le m}\{\gamma_j^-\}$. Moreover, if $k = 1$, we denote

$$\min\Big\{\frac{e^k}{|1-k|}, e^2\Big\} = e^2.$$

Theorem 2.1. *Under assumptions* (H1)–(H2), *Equation* (1.2) *has a unique almost periodic solution in*

$$\Omega = \{\varphi \in AP(\mathbb{R}) : n_1 \le \varphi(t) \le n_2, \forall t \in \mathbb{R}\}.$$

Proof. For each $\varphi \in AP(\mathbb{R})$, we consider the almost periodic differential equation

$$x'(t) = -\alpha(t)x(t) + \sum_{j=1}^{m} \beta_j(t)\varphi(t-\tau_j(t))e^{-\gamma_j(t)\varphi(t-\tau_j(t))}. \tag{2.1}$$

Since $\alpha^- > 0$, we know that

$$x'(t) = -\alpha(t)x(t)$$

admits an exponential dichotomy on $\mathbb{R}$ with $P = I$. Combining this with Lemma 1.3, we conclude that (2.1) has a unique almost periodic solution:

$$x^{\varphi}(t) = \int_{-\infty}^{t} e^{-\int_s^t \alpha(u)du}\Big(\sum_{j=1}^{m} \beta_j(s)\varphi(s-\tau_j(s))e^{-\gamma_j(s)\varphi(s-\tau_j(s))}\Big)ds. \tag{2.2}$$

Now, we define a mapping T on $AP(\mathbb{R})$ by

$$(T\varphi)(t) = x^{\varphi}(t), \quad t \in \mathbb{R}.$$

Next, we show that $T(\Omega) \subset \Omega$. It suffices to prove that

$$n_1 \le (T\varphi)(t) \le n_2$$

for all $t \in \mathbb{R}$ and $\varphi \in \Omega$. Noticing that

$$\sup_{x\ge 0} xe^{-\gamma_j^- x} = \frac{1}{\gamma_j^- e}, \quad 1 \le j \le m, \tag{2.3}$$

for all $t \in \mathbb{R}$ and $\varphi \in \Omega$, we have

$$\begin{aligned}
(T\varphi)(t) = x^{\varphi}(t) &= \int_{-\infty}^{t} e^{-\int_s^t \alpha(u)du}\Big(\sum_{j=1}^{m} \beta_j(s)\varphi(s-\tau_j(s))e^{-\gamma_j(s)\varphi(s-\tau_j(s))}\Big)ds \\
&\le \int_{-\infty}^{t} e^{-\int_s^t \alpha(u)du}\Big(\sum_{j=1}^{m} \beta_j^+\varphi(s-\tau_j(s))e^{-\gamma_j^-\varphi(s-\tau_j(s))}\Big)ds \\
&\le \int_{-\infty}^{t} e^{-\int_s^t \alpha(u)du}\Big(\sum_{j=1}^{m} \frac{\beta_j^+}{\gamma_j^- e}\Big)ds \\
&\le \sum_{j=1}^{m} \frac{\beta_j^+}{\gamma_j^- e}\int_{-\infty}^{t} e^{-\int_s^t \alpha^- du}ds \\
&= \sum_{j=1}^{m} \frac{\beta_j^+}{\gamma_j^- e}\int_{-\infty}^{t} e^{\alpha^-(s-t)}ds \\
&= \sum_{j=1}^{m} \frac{\beta_j^+}{\alpha^-\gamma_j^- e} = n_2.
\end{aligned}$$

On the other hand, by (H1), we know that $n_1 \ge \frac{1}{\min_{1\le j\le m}\{\gamma_j^+\}}$. Thus, we have

$$\inf_{n_1\le x\le n_2} xe^{-\gamma_j^+ x} = n_2 e^{-\gamma_j^+ n_2}, \quad 1 \le j \le m, \tag{2.4}$$

which yields

$$
\begin{aligned}
(T\varphi)(t) = x^{\varphi}(t) &= \int_{-\infty}^{t} e^{-\int_s^t \alpha(u)du}\Big(\sum_{j=1}^{m}\beta_j(s)\varphi(s-\tau_j(s))e^{-\gamma_j(s)\varphi(s-\tau_j(s))}\Big)ds \\
&\geq \int_{-\infty}^{t} e^{-\int_s^t \alpha(u)du}\Big(\sum_{j=1}^{m}\beta_j^-\varphi(s-\tau_j(s))e^{-\gamma_j^+\varphi(s-\tau_j(s))}\Big)ds \\
&\geq \int_{-\infty}^{t} e^{-\int_s^t \alpha(u)du}\Big(\sum_{j=1}^{m}\beta_j^- n_2 e^{-\gamma_j^+ n_2}\Big)ds \\
&\geq \sum_{j=1}^{m}\beta_j^- n_2 e^{-\gamma_j^+ n_2}\int_{-\infty}^{t} e^{-\int_s^t \alpha^+ du}ds \\
&\geq \sum_{j=1}^{m}\beta_j^- n_2 e^{-\gamma_j^+ n_2}\int_{-\infty}^{t} e^{\alpha^+(s-t)}ds \\
&= \sum_{j=1}^{m}\frac{n_2\beta_j^- e^{-\gamma_j^+ n_2}}{\alpha^+} = n_1
\end{aligned}
$$

for all $t \in \mathbb{R}$ and $\varphi \in \Omega$.

By a direct calculation, one can obtain

$$
|xe^{-x} - ye^{-y}| \leq \max\Big\{\frac{|1-k|}{e^k}, \frac{1}{e^2}\Big\}|x-y|, \quad x, y \geq k. \tag{2.5}
$$

Denoting

$$
\Gamma_j(s) = \gamma_j(s)\varphi(s-\tau_j(s))e^{-\gamma_j(s)\varphi(s-\tau_j(s))} - \gamma_j(s)\psi(s-\tau_j(s))e^{-\gamma_j(s)\psi(s-\tau_j(s))},
$$

for $\varphi, \psi \in \Omega$, we have

$$
\begin{aligned}
&\|T\varphi - T\psi\| \\
&= \sup_{t\in\mathbb{R}} |(T\varphi)(t) - (T\psi)(t)| \\
&= \sup_{t\in\mathbb{R}} \Big|\int_{-\infty}^{t} e^{-\int_s^t \alpha(u)du}\sum_{j=1}^{m}\frac{\beta_j(s)}{\gamma_j(s)}\Gamma_j(s)ds\Big| \\
&\leq \max\Big\{\frac{|1-k|}{e^k}, \frac{1}{e^2}\Big\}\sup_{t\in\mathbb{R}}\int_{-\infty}^{t} e^{-\int_s^t \alpha^- du}\sum_{j=1}^{m}\beta_j^+|\varphi(s-\tau_j(s)) - \psi(s-\tau_j(s))|ds \\
&\leq \max\Big\{\frac{|1-k|}{e^k}, \frac{1}{e^2}\Big\}\sum_{j=1}^{m}\beta_j^+\|\varphi-\psi\|\cdot\sup_{t\in\mathbb{R}}\int_{-\infty}^{t} e^{\alpha^-(s-t)}ds \\
&= \max\Big\{\frac{|1-k|}{e^k}, \frac{1}{e^2}\Big\}\sum_{j=1}^{m}\frac{\beta_j^+}{\alpha^-}\|\varphi-\psi\| \\
&= \frac{\sum_{j=1}^{m}\frac{\beta_j^+}{\alpha^-}}{\min\{\frac{e^k}{|1-k|}, e^2\}}\|\varphi-\psi\|.
\end{aligned}
$$

Now, by (H2), T has a unique fixed point in Ω, i.e., Equation (1.2) has a unique almost periodic solution in Ω. □

Remark 2.2. Compared with some earlier results (cf. [4, 9]), in Theorem 2.1, we removed the following two restrictive conditions:

$$\gamma_j^- > 1, \quad n_1 \geq \frac{1}{\min_{1\leq j\leq m}\{\gamma_j^-\}}.$$

For the next corollary we use the assumption

(H3) there exists $\lambda > 0$ such that
$$\frac{\sum_{j=1}^m \|\beta_j\|_{S^\lambda}}{1-e^{-\lambda\alpha^-}} < \min\{\frac{e^k}{|1-k|}, e^2\},$$
where
$$k = n_1 \cdot \min_{1\leq j\leq m}\{\gamma_j^-\}, \quad \text{and} \quad \|\beta_j\|_{S^\lambda} = \sup_{t\in\mathbb{R}} \int_t^{t+\lambda} \beta_j(s)ds.$$

Corollary 2.3. *Under assumptions* (H1), (H3), *Equation* (1.2) *has a unique almost periodic solution in* Ω.

Proof. Let T be the self mapping on Ω in Theorem 2.1, i.e.,
$$(T\varphi)(t) = \int_{-\infty}^t e^{-\int_s^t \alpha(u)du}\Big(\sum_{j=1}^m \beta_j(s)\varphi(s-\tau_j(s))e^{-\gamma_j(s)\varphi(s-\tau_j(s))}\Big)ds$$
for $\varphi \in \Omega$ and $t \in \mathbb{R}$. For all $t \in \mathbb{R}$ and $\varphi, \psi \in \Omega$, by (2.5), we obtain
$$\begin{aligned}
&|(T\varphi)(t) - (T\psi)(t)| \\
&\leq \max\big\{\frac{|1-k|}{e^k}, \frac{1}{e^2}\big\} \int_{-\infty}^t e^{-\alpha^-(t-s)} \sum_{j=1}^m \beta_j(s)|\varphi(s-\tau_j(s)) - \psi(s-\tau_j(s))|ds \\
&\leq \max\big\{\frac{|1-k|}{e^k}, \frac{1}{e^2}\big\}\|\varphi-\psi\| \cdot \sum_{j=1}^m \int_{-\infty}^t e^{-\alpha^-(t-s)}\beta_j(s)ds \\
&= \max\big\{\frac{|1-k|}{e^k}, \frac{1}{e^2}\big\}\|\varphi-\psi\| \sum_{j=1}^m \int_0^{+\infty} e^{-\alpha^- s}\beta_j(t-s)ds \\
&= \max\big\{\frac{|1-k|}{e^k}, \frac{1}{e^2}\big\}\|\varphi-\psi\| \cdot \sum_{j=1}^m \sum_{k=0}^{+\infty} \int_{k\lambda}^{(k+1)\lambda} e^{-\alpha^- s}\beta_j(t-s)ds \\
&\leq \max\big\{\frac{|1-k|}{e^k}, \frac{1}{e^2}\big\}\|\varphi-\psi\| \sum_{j=1}^m \sum_{k=0}^{+\infty} e^{-\alpha^- k\lambda} \int_{k\lambda}^{(k+1)\lambda} \beta_j(t-s)ds \\
&= \max\big\{\frac{|1-k|}{e^k}, \frac{1}{e^2}\big\}\|\varphi-\psi\| \sum_{j=1}^m \sum_{k=0}^{+\infty} e^{-\alpha^- k\lambda} \int_{t-k\lambda-\lambda}^{t-k\lambda} \beta_j(s)ds \\
&\leq \max\big\{\frac{|1-k|}{e^k}, \frac{1}{e^2}\big\}\|\varphi-\psi\| \sum_{j=1}^m \sum_{k=0}^{+\infty} e^{-\alpha^- k\lambda}\|\beta_j\|_{S^\lambda} \\
&= \max\big\{\frac{|1-k|}{e^k}, \frac{1}{e^2}\big\}\|\varphi-\psi\| \frac{\sum_{j=1}^m \|\beta_j\|_{S^\lambda}}{1-e^{-\lambda\alpha^-}}.
\end{aligned}$$
Then, by (H3), T is a contraction, and thus (1.2) has a unique almost periodic solution in Ω. $\square$

Remark 2.4. Note that in some cases, (H3) is weaker than (H2) (see for example 2.8).

2.2. **Locally exponential stability of the almost periodic solution.**

Theorem 2.5. *Suppose that* (H1), (H2) *are satisfied, $x(t)$ is the unique almost periodic solution of* (1.2) *in Ω, and $y(t)$ is an arbitrary solution of* (1.2) *with*

$$y(t) \geq n_1, \quad \forall t \in [-r, +\infty), \tag{2.6}$$

where $r = \max_{1\leq j\leq m}\{\tau_j^+\}$. Then, there exists a constant $\lambda > 0$ such that

$$|x(t) - y(t)| \leq Me^{-\lambda t}, \quad \forall t \in [-r, +\infty),$$

where $M = \max_{-r\leq t\leq 0} |x(t) - y(t)|$.

Proof. Let $\widetilde{k} = \max\{\frac{|1-k|}{e^k}, \frac{1}{e^2}\}$. Then, by (H2), we have

$$\sum_{j=1}^m \frac{\beta_j^+}{\alpha^-} < \frac{1}{\tilde{\tilde{k}}}\,.$$

Thus, there exists $\lambda > 0$ such that

$$\lambda - \alpha^- + \widetilde{k}\sum_{j=1}^m \beta_j^+ e^{\lambda r} < 0. \tag{2.7}$$

Next, setting $z(t) = x(t) - y(t)$, we consider Lyapunov functional

$$V(t) = |x(t) - y(t)|e^{\lambda t} = |z(t)|e^{\lambda t}.$$

It can see easily that

$$V(t) \leq M, \quad t \in [-r, 0].$$

Now, we claim that

$$V(t) \leq M, \quad t \in (0, +\infty). \tag{2.8}$$

Otherwise, the set

$$S = \{t > 0 : V(t) > M\} \neq \emptyset.$$

We denote $t_1 = \inf S$. It is not difficult to prove that

$$V(t_1) = M, \quad V(t) \leq M, \quad \forall t \in (0, t_1).$$

In addition, for each $\delta > 0$, there exists $t' \in (t_1, t_1 + \delta)$ such that $V(t') \geq M$. It follows that

$$D^+V(t_1) = \inf_{\delta>0} \sup_{t\in(t_1,t_1+\delta)} \frac{V(t) - V(t_1)}{t - t_1} \geq 0.$$

On the other hand, by using (2.5), we obtain

$$\begin{aligned}
D^+V(t_1) &= D^+(|z(t_1)|e^{\lambda t_1}) \\
&\leq e^{\lambda t_1} D^+|z(t_1)| + \lambda|z(t_1)|e^{\lambda t_1} \\
&\leq e^{\lambda t_1} sgn(z(t_1))z'(t_1) + \lambda|z(t_1)|e^{\lambda t_1} \\
&\leq e^{\lambda t_1}\big\{ - \alpha(t_1)z(t_1)sgn(z(t_1)) + \sum_{j=1}^m \frac{\beta_j(t_1)}{\gamma_j(t_1)}|\Gamma_j(t_1)|\big\} + \lambda|z(t_1)|e^{\lambda t_1} \\
&\leq -\alpha(t_1)e^{\lambda t_1}|z(t_1)| + \widetilde{k}\sum_{j=1}^m \beta_j^+|z(t_1 - \tau_j(t_1))|e^{\lambda t_1} + \lambda|z(t_1)|e^{\lambda t_1}
\end{aligned}$$

$$\begin{aligned}
&\le -\alpha^- M + \widetilde{k}\sum_{j=1}^{m} \beta_j^+ M e^{\lambda\tau_j(t_1)} + \lambda M \\
&\le (\lambda - \alpha^- + \widetilde{k}\sum_{j=1}^{m} \beta_j^+ e^{\lambda r})M,
\end{aligned}$$

where

$\Gamma_j(t_1) = \gamma_j(t_1)x(t_1-\tau_j(t_1))e^{-\gamma_j(t_1)x(t_1-\tau_j(t_1))} - \gamma_j(t_1)y(s-\tau_j(t_1))e^{-\gamma_j(t_1)y(t_1-\tau_j(t_1))}$. Noting that $D^+V(t_1) \ge 0$, we obtain

$$\lambda - \alpha^- + \widetilde{k}\sum_{j=1}^{m} \beta_j^+ e^{\lambda r} \ge 0,$$

which contradicts (2.7). Hence, (2.8) holds; i.e.,

$$|x(t) - y(t)| \le Me^{-\lambda t}, \quad t \in [-r, +\infty).$$

This completes the proof. □

2.3. **Examples.** In this section, we give two examples to illustrate our results.

Example 2.6. We consider the following Nicholson's blowflies model with multiple time-varying delays:

$$x'(t) = -\alpha(t)x(t) + \sum_{j=1}^{m} \beta_j(t)x(t-\tau_j(t))e^{-\gamma_j(t)x(t-\tau_j(t))}, \tag{2.9}$$

where

$$\begin{gathered}
m = 2, \quad \alpha(t) = 18 + \frac{|\sin\sqrt{2}t + \sin\sqrt{3}t|}{2}, \\
\beta_1(t) = e^{e-1}(10 + 0.005|\sin\sqrt{3}t + \sin\sqrt{2}t|), \\
\beta_2(t) = e^{e-1}(10 + 0.005|\sin\sqrt{5}t + \sin\sqrt{3}t|), \\
\tau_1(t) = e^{|\cos\sqrt{2}t+\cos t|},\ \tau_2(t) = e^{|\cos\sqrt{2}t+\cos\sqrt{3}t|}, \\
\gamma_1(t) = \gamma_2(t) = 0.25 + 0.025|\sin t + \sin\sqrt{3}t|.
\end{gathered}$$

By a direct calculation, we can obtain

$$\begin{gathered}
\alpha^- = 18, \quad \alpha^+ = 19, \quad \beta_1^- = \beta_2^- = 10e^{e-1}, \\
\beta_1^+ = \beta_2^+ = 10.01e^{e-1}, \quad \gamma_1^- = \gamma_2^- = 0.25, \quad \gamma_1^+ = \gamma_2^+ = 0.3, \\
n_2 = \sum_{j=1}^{2} \frac{\beta_j^+}{\alpha^-\gamma_j^- e} = \frac{80.08e^{e-2}}{18} > 9, \\
3.4 < n_1 = \sum_{j=1}^{2} \frac{n_2\beta_j^- e^{-\gamma_j^+ n_2}}{\alpha^+} < 3.5.
\end{gathered}$$

It is easy to see that

$$n_2 \ge n_1 \ge \frac{1}{\min_{1\le j\le 2}\{\gamma_j^+\}} = \frac{10}{3}.$$

So (H1) holds. In addition, we have

$$\sum_{j=1}^{2} \frac{\beta_j^+}{\alpha^-} = \frac{20.02e^{e-1}}{18} < e^2,$$
$$\frac{\exp(0.25n_1)}{1-0.25n_1} > e^2.$$

Thus, (H2) holds. By Theorem 2.1 and Theorem 2.5, Equation (2.9) has a unique almost periodic solution $x(t)$ in Ω, and every solution $y(t)$ satisfying (2.6) converges exponentially to $x(t)$ as $t \to +\infty$.

Remark 2.7. In Example 2.9, $\gamma_1^- = \gamma_2^- < 1$. Thus, the results in [4] cannot be applied to Example 2.9. In addition, here we have

$$n_1 < \frac{1}{\min_{1\le j\le m}\{\gamma_j^-\}} = 4.$$

So the results in [9] can not be applied to Example 2.6.

Example 2.8. Let $m = 1$, $\alpha(t) \equiv 1$, $\gamma_1(t) \equiv 1$, and $\tau_1(t) = 1 + |\sin t + \sin \pi t|$ for all $t \in \mathbb{R}$. Moreover, β_1 is a continuous $\frac{1}{2}$-periodic function, which is defined on $[0, \frac{1}{2}]$ by

$$\beta_1(t) = \begin{cases} e^2, & t \in [\frac{1}{4}-\varepsilon, \frac{1}{4}+\varepsilon], \\ e^{e-1}, & t \in [0, \frac{1}{4}-2\varepsilon] \cup [\frac{1}{4}+2\varepsilon, \frac{1}{2}], \\ \text{line segments} & t \in [\frac{1}{4}-2\varepsilon, \frac{1}{4}-\varepsilon] \cup [\frac{1}{4}+\varepsilon, \frac{1}{4}+2\varepsilon], \end{cases}$$

where $\varepsilon \in (0, \frac{1}{8})$ is a fixed constant. It is easy to check that

$$\alpha^+ = \alpha_- = \gamma^+ = \gamma_- = 1, \quad n_2 = e, \quad n_1 = 1,$$

and

$$\|\beta_1\|_{S^{1/2}} = \int_0^{1/2} \beta_1(s)ds \le 4\varepsilon e^2 + (\frac{1}{2} - 4\varepsilon)e^{e-1}.$$

Then we conclude that (H1) holds and (H3) holds for sufficiently small ε and $\lambda = \frac{1}{2}$ since

$$\lim_{\varepsilon\to 0} 4\varepsilon e^2 + (\frac{1}{2} - 4\varepsilon)e^{e-1} = \frac{1}{2}e^{e-1} < e^2(1 - e^{-1/2}).$$

Then, by Corollary 2.3, we know that

$$x'(t) = -x(t) + \beta_1(t)x(t-\tau(t))e^{-x(t-\tau(t))}$$

has an almost periodic solution provided that ε is sufficiently small.

Remark 2.9. In Example 2.8, we have

$$\sum_{j=1}^{m} \frac{\beta_j^+}{\alpha^-} = \beta_1^+ = e^2.$$

However, in [4, 9],

$$\sum_{j=1}^{m} \frac{\beta_j^+}{\alpha^-} < e^2$$

is a key assumption. So the results in [4, 9] can not be applied to Example 2.8.

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