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SINGULAR PERTURBATION METHOD FOR GLOBAL STABILITY OF RATIO-DEPENDENT PREDATOR-PREY MODELS WITH STAGE STRUCTURE FOR THE PREY

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Abstract. In this article, a singular perturbation is introduced to analyze the global asymptotic stability of positive equilibria of ratio-dependent predator-prey models with stage structure for the prey. We prove theoretical results and show numerically that the proposed approach is feasible and efficient.

1. Introduction

One of the most important and interesting topics in both ecology and mathematical ecology is the analysis between predators and their preys. This has long been and will continue to be one of the dominant themes due to its universal importance. There are many mathematical models for predator-prey behavior. The ratio-dependent type systems are very basic and important in the models of multi-species population dynamics. This can be roughly stated as that the per capita predator growth rate should be a function of the ratio of prey to predator abundance, and so should be the so-called predator functional responses. This is strongly supported by numerous field and laboratory experiments and observations; see for example Arditi and Ginzburg [1], Arditi et al. [2], Hanski [10]. Generally, a ratio-dependent predator-prey model takes the form

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= xf(x) - yp(\frac{x}{y}), \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= cyq(\frac{x}{y}) - dy. \end{aligned} \tag{1.1}$$

In previous decades, the dynamics of the ratio-dependent predator-prey system (1.1) has been systematically studied by Kuang and Beretta [12], Hsu el at. [11], Berezovskaya el at. [5], Xiao and Ruan [24], Li and Kuang [13] and Ginzburg el at. [8]. These authors have shown that system (1.1) has very rich dynamics.

In the natural world, there are many species whose individual members have a life history that take them through two stages: immature and mature. Stage-structured models have been received much attention in recent years; see for example [6, 22, 7, 21]. Recently, Wang and Chen [23], Magnusson [16], Zhang el at. [26] proposed

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and investigated predator-prey models with stage structure for prey or predator to analyze the influence of a stage structure for the prey or the predator on the dynamics of predator-prey models. In particular, Xu el at. [25] studied a ratio-dependent predator-prey model with stage structure for the prey. Their model appears as

$$\begin{aligned}
\frac{\mathrm{d}x_1}{\mathrm{d}t} &= ax_2 - r_1x_1 - bx_1,\\
\frac{\mathrm{d}x_2}{\mathrm{d}t} &= bx_1 - b_1x_2^2 - \frac{a_1x_2x_3}{mx_3 + x_2},\\
\frac{\mathrm{d}x_3}{\mathrm{d}t} &= x_3\big(-r + \frac{a_2x_2}{mx_3 + x_2}\big),
\end{aligned} \tag{1.2}$$

where x_1 represents the density of immature individual preys at time t, and X_2 denotes the density of mature individual preys at time t, y represents the density of the predator at time t. By constructing Lyapunov functions, sufficient conditions are derived for the global asymptotic stability of nonnegative equilibria of the model.

On the other hand, in a wide class of large-scale interconnected systems such as in power systems, large economies or even in networks one encounters dynamics with different speeds or multiple time scales. Singular perturbation technique is an adequate tool to describe such systems. Singular perturbation problems are of common occurrence in many branches of applied mathematics such as fluid dynamics, elasticity, chemical reactor theory, neural networks, etc.. In particular, by singular perturbation methods, [17, 18, 15, 19] analyzed the exponential stability of the competitive neural networks, [20, 27] discussed the dynamic behavior of the epidemic models, [3] considered a general linear population model with both a continuous age structure and a finite spatial structure.

Motivated by the literature survey, in this paper, we use singular perturbation theory to simplify the study of system (1.2) and analysis the global asymptotic stability of positive equilibria of system (1.2).

The paper is organized as follows. In the next section, a singular perturbed system is introduced. We state and prove a general criterion for the global asymptotically stability of positive equilibrium of system (1.2) in Section 3. In Section 4, specific examples are given to illustrate our results.

2. Model description

Obviously, system (1.2) always has equilibria $E_0(0,0,0)$, $E_1(\widetilde{x}_1,\widetilde{x}_2,0)$, where

$$\widetilde{x}_1 = \frac{a^2b}{b_1(r_1+b)^2}, \quad \widetilde{x}_2 = \frac{ab}{b_1(r_1+b)}$$

and has a positive equilibrium $E_2(x_1^*,x_2^*,x_3^*)$ if and only if $ab/(r_1+b) > a_1(a_2 - r)/(ma_2) > 0$, where

$$x_1^* = \frac{ax_2^*}{r_1+b}, \quad x_2^* = \frac{ab}{b_1(r_1+b)} - \frac{a_1(a_2-r)}{ma_2b_1}, \quad x_3^* = \frac{x_2^*(a_2-r)}{mr}. \tag{2.1}$$

On the global asymptotic stability of equilibria E_0, E_1 and E_2 of system (1.2), we have the following result.

Theorem 2.1 ([25]). *If $a_2 < r$, E_1 is locally asymptotically stable, if $a_2 > r$, which is locally unstable; the positive equilibrium E_2 is global asymptotically stable if*

$$ab/(r_1+b) > a_1(a_2-r)/(ma_2) > 0, \quad ab/(r_1+b) > 2a_1/m. \tag{2.2}$$

According to above discussion, to obtain the global asymptotically stability of positive equilibrium E_2^* of system (1.2), it is necessary to construct the Lyapunov functions. However, it is usually difficult for nonlinear systems. In the present paper, by choosing a reasonable transformation, we transform system (1.2) into the standard singular perturbation system.

It is well know that the survival of individual eggs may be very low, so millions of eggs must be produced in order for the species to successfully survive the larval stage and then to persist for a long time. The fish species provides an exact example of this phenomenon. So, we suppose r/a is small enough and re-scale time by $rt = \tau$. Further, let $x = x_1 - x_1^*$, $y = x_2 - x_2^*$, $z = x_3 - x_3^*$, then the equilibria $E_2(x_1^*, x_2^*, x_3^*)$ of system (1.2) has been shift to the origin $O(0,0,0)$. Thus, we note that system (1.2) can be rewritten as the following singular perturbation form

$$\begin{aligned} \frac{\mathrm{d}\theta}{\mathrm{d}\tau} &= f(x,\theta), \\ \varepsilon\frac{\mathrm{d}x}{\mathrm{d}\tau} &= g(x,\theta), \end{aligned} \tag{2.3}$$

where $\varepsilon = r/a$, $\theta = (y,z) \in D_\theta = \{(y,z) : y > -x_2^*, z > -x_3^*\}$ and $x \in D_x = \{x : x > -x_1^*\}$ and

$$g(x,\theta) = y - \frac{r_1}{a}x - \frac{b}{a}x,$$
$$f(x,\theta) = \begin{pmatrix} \frac{1}{r}\big[b(x+x_1^*) - b_1(y+x_2^*)^2 - \frac{a_1(y+x_2^*)(z+x_3^*)}{m(z+x_3^*)+(y+x_2^*)}\big] \\ (z+x_3^*)\big[-1 + \frac{a_2 y(y+x_2^*)}{r(m(z+x_3^*)+(y+x_2^*))}\big] \end{pmatrix}.$$

3. Main results

In this section, we are concerned with the global asymptotically stable of non-negative equilibria of system (1.2) by using the singular perturbation.

Now, we proceed to the discussion on the stability of the origin $O(0,0,0)$ by examining the reduced and boundary-layer models. Let ε tend to zero in system (2.3), we can get the first equation of system (2.3) has a unique real function root

$$x = h(\theta) = \frac{ay}{r_1 + b}. \tag{3.1}$$

It is more convenient to work in the (ϑ, y, z) coordinates, where

$$\vartheta = x - h(\theta)$$

because this change of variables shifts the equilibrium of the boundary layer model to the origin. In the new coordinates, the singularly perturbed system (2.3) can be rewritten as

$$\begin{aligned} \frac{\mathrm{d}\theta}{\mathrm{d}\tau} &= f(\vartheta + h(\theta),\theta), \\ \varepsilon\frac{\mathrm{d}\vartheta}{\mathrm{d}\tau} &= g(\vartheta + h(\theta),\theta) - \varepsilon\frac{\partial h}{\partial\theta}f(\vartheta + h(\theta),\theta). \end{aligned} \tag{3.2}$$

Then, the reduced system

$$\frac{\mathrm{d}\theta}{\mathrm{d}\tau} = f(h(\theta),\theta) \tag{3.3}$$

has equilibrium at $(0,0)$ and boundary-layer system

$$\frac{\mathrm{d}\vartheta}{\mathrm{d}s} = g(\vartheta + h(\theta),\theta) = -\frac{r_1 + b}{a}\vartheta, \tag{3.4}$$

where $s = \tau/\varepsilon$, has equilibrium at $\vartheta = 0$.

To discuss the globally asymptotically stable of equilibrium $O(0,0,0)$ of system (2.3), we first derive certain upper bound and lower bound estimates for solution of reduced system (3.3).

Theorem 3.1. *Let $(y(\tau), z(\tau))$ denote any solutions of system (3.3) corresponding to initial conditions $y(0) > 0$ and $z(0) > 0$. If $a_2 > r$ and $mab > a_1(r_1 + b)$, then there is a constant $T > 0$ such that if $t \geq T$,*

$$m_1 - x_2^* \leq y(\tau) \leq M_1 + x_2^*, \quad m_2 < z(\tau) \leq M_2,$$

where

$$m_1 = \frac{mab - a_1(r_1 + b)}{mb_1(r_1 + b)}, \quad M_1 = \frac{ab}{b_1(r_1 + b)}, \quad m_2 = 0, \quad M_2 = \frac{a_2 M_1}{mr}. \tag{3.5}$$

The proof of the above theorem is similar to that of [25, Theorem 2.1]; therefore we omit it here. Now, we state and prove our result on the globally asymptotically stable of system (1.2).

Theorem 3.2. *Let ε^* be defined by (3.24). If*

$$\frac{ab}{r_1 + b} + \frac{ra_1}{ma_2} - \frac{2a_1}{ma_2} > 0, \quad \frac{ab}{r_1 + b} - \frac{a_1}{m} > 0 \tag{3.6}$$

hold, then the equilibrium $O(0,0,0)$ of system (2.3) is globally asymptotically stable for all $\varepsilon \in (0, \varepsilon^*)$, that is, the equilibrium $E_2(x_1^*, x_2^*, x_3^*)$ of system (1.2) is globally asymptotically stable for all $\varepsilon \in (0, \varepsilon^*)$.

Proof. Let $(y(t), z(t))$ be any positive solution of system (3.3) with initial conditions $y(0) > 0$ and $z(0) > 0$. In view of the $E_2(x_1^*, x_2^*, x_3^*)$ is positive equilibrium of system (1.2), we note that system (3.3) can be rewritten as

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}\tau} &= \frac{y + x_2^*}{r}\big[-b_1 y + \frac{a_1 x_3^* y - a_1 x_2^* z}{(mx_3^* + x_2^*)[m(z + x_3^*) + y + x_2^*]}\big], \\ \frac{\mathrm{d}z}{\mathrm{d}\tau} &= \frac{ma_2(z + x_3^*)}{r}\big[\frac{x_3^* y - x_2^* z}{(mx_3^* + x_2^*)[m(z + x_3^*) + y + x_2^*]}\big]. \end{aligned} \tag{3.7}$$

Define a Lyapunov function candidate

$$V_1(\theta) = c_1\big[y - x_2^* \ln(y + x_2^*) - x_2^* \ln x_2^*\big] + c_2\big[z - x_3^* \ln(z + x_3^*) - x_3^* \ln x_3^*\big] \tag{3.8}$$

and calculating the derivative of $V_1(\theta)$ along solutions of system (3.7), it follows that

$$\begin{aligned} \frac{\mathrm{d}V_1}{\mathrm{d}\tau} &= \frac{c_1 y}{y + x_2^*}\frac{\mathrm{d}y}{\mathrm{d}\tau} + \frac{c_2 z}{z + x_3^*}\frac{\mathrm{d}z}{\mathrm{d}\tau} \\ &= -c_1\big[-\frac{b_1}{r}y^2 + \frac{a_1 x_3^* y^2 - a_1 x_2^* yz}{r(mx_3^* + x_2^*)[m(z + x_3^*) + y + x_2^*]}\big] \\ &\quad + c_2\big[\frac{-ma_2 x_2^* z^2 - ma_2 x_2^* yz}{r(mx_3^* + x_2^*)[m(z + x_3^*) + y + x_2^*]}\big]. \end{aligned} \tag{3.9}$$

Let $c_2 = 1$ and $c_1 = ma_2 x_3^*/a_1 x_2^*$. We derive from (3.9) that

$$\begin{aligned} \frac{\mathrm{d}V_1}{\mathrm{d}\tau} &= -\frac{c_1}{r}\big[b_1 - \frac{a_1 x_3^*}{(mx_3^* + x_2^*)[m(z + x_3^*) + y + x_2^*]}\big]y^2 \\ &\quad - \frac{ma_2 c_2 x_2^*}{r(mx_3^* + x_2^*)[m(z + x_3^*) + y + x_2^*]}z^2. \end{aligned} \tag{3.10}$$

From (3.6), we can choose a positive constant ϵ such that

$$\frac{ab}{r_1+b}+\frac{ra_1}{ma_2}-\frac{2a_1}{ma_2}-\epsilon>0.$$

Further, from Theorem (3.1), there is a $\tau_1 \geq 0$ such that $y(\tau) > m_1-\epsilon$ for all $\tau \geq \tau_1$. Therefore, from Theorem (3.1) and (3.10), we obtain

$$\begin{aligned}\frac{\partial V_1}{\partial \theta}f(h(\theta),\theta) &= -\big[b_1-\frac{a_1(a_2-r)}{ma_2(m_1-\epsilon)}\big]y^2-\frac{m}{mM_2+M_1}z^2\\ &\leq -\alpha_1\phi_1^2(\theta)\end{aligned}\tag{3.11}$$

for all $\tau \geq \tau_1$, where $\phi_1(\theta)=\sqrt{y^2+z^2}$ and

$$\alpha_1=\min\big\{b_1-\frac{a_1(a_2-r)}{ma_2(m_1-\epsilon)},\frac{m}{mM_2+M_1}\big\}.\tag{3.12}$$

On the other hand, Let $\vartheta(s)$ be any positive solution of the boundary-layer system (3.4) with initial condition $\vartheta(0)>0$. We define a Lyapunov function candidate

$$V_2(\vartheta)=\frac{1}{2}\vartheta^2\tag{3.13}$$

and calculating the derivative of $V_2(\vartheta)$ along solutions of system (3.4), it follows that

$$\frac{\partial V_2}{\partial \vartheta}g(\vartheta+h(\theta),\theta)=-\alpha_2\phi_2^2(\vartheta),\tag{3.14}$$

where $\phi_2(\vartheta)=|\vartheta|$ and

$$\alpha_2=\frac{r_1+b}{a}.\tag{3.15}$$

Now, for the singularly perturbed system (3.2), we consider the composite Lyapunov function candidate

$$V(\theta,\vartheta)=(1-\delta)V_1(\theta)+\delta V_2(\vartheta),\tag{3.16}$$

where $0<\theta<1$ is to be chosen. Calculating the derivative of $V(\theta,\vartheta)$ along the solutions of the full system (3.2), we obtain

$$\begin{aligned}\frac{\mathrm{d}V}{\mathrm{d}\tau} &= (1-\delta)\frac{\partial V_1}{\partial \theta}f(\vartheta+h(\theta),\theta)+\frac{\delta}{\varepsilon}\frac{\partial V_2}{\partial \vartheta}g(\vartheta+h(\theta),\theta)-\delta\frac{\partial V_2}{\partial \vartheta}\frac{\partial h}{\partial \theta}f(\vartheta+h(\theta),\theta)\\ &= (1-\delta)\frac{\partial V_1}{\partial \theta}f(h(\theta),\theta)+\frac{\delta}{\varepsilon}\frac{\partial V_2}{\partial \vartheta}g(\vartheta+h(\theta),\theta)\\ &\quad+(1-\delta)\frac{\partial V_1}{\partial \theta}[f(\vartheta+h(\theta),\theta)-f(h(\theta),\theta)]+\delta\big[\frac{\partial V_2}{\partial \theta}-\frac{\partial V_2}{\partial \vartheta}\frac{\partial h}{\partial \theta}\big]f(\vartheta+h(\theta),\theta).\end{aligned}\tag{3.17}$$

Further, from (3.8), systems (3.2) and (3.3) we have

$$\frac{\partial V_1}{\partial \theta}[f(\vartheta+h(\theta),\theta)-f(h(\theta),\theta)]=(\frac{y}{y+x_2^*},\frac{z}{z+x_3^*})\begin{pmatrix}\frac{b\vartheta}{r}\\ 0\end{pmatrix}\leq \beta_1\phi_1(\theta)\phi_2(\vartheta)\tag{3.18}$$

for all $\tau \geq \tau_1$, where

$$\beta_1=\frac{b_1}{m_1 r}.\tag{3.19}$$

By (3.1), (3.13) and system (3.2), we obtain

$$\begin{aligned} &\Big[\frac{\partial V_2}{\partial \theta} - \frac{\partial V_2}{\partial \vartheta}\frac{\partial h}{\partial \theta}\Big] f(\vartheta + h(\theta), \theta) \\ &= \frac{-a\vartheta}{r(r_1+b)}\Big\{b\vartheta - (y+x_2^*)\Big[-b_1 y + \frac{a_1x_3^*y - a_1x_2^*z}{(mx_3^*+x_2^*)[m(z+x_3^*)+y+x_2^*]}\Big]\Big\} \\ &\le -\alpha_3\phi_2^2(\vartheta) + \beta_2\phi_1(\theta)\phi_2(\vartheta) \end{aligned} \tag{3.20}$$

for all $\tau \ge \tau_1$, where

$$\alpha_3 = \frac{ab}{r(r_1+b)}, \quad \beta_2 = \frac{a}{r(r_1+b)}\Big[\frac{a_1(a_2-r+mr)}{ma_2} + \frac{ab}{r_1+b}\Big] \tag{3.21}$$

Using (3.14), (3.17), inequalities (3.18) and (3.20), we obtain

$$\begin{aligned} \frac{\mathrm{d}V}{\mathrm{d}\tau} &\le -(1-\delta)\alpha_1\phi_1^2(\theta) - \delta\Big[\frac{\alpha_2}{\varepsilon}+\alpha_3\Big]\phi_2^2(\vartheta) + (1-\delta)\beta_1\phi_1(\theta)\phi_2(\vartheta) + \delta\beta_2\phi_1(\theta)\phi_2(\vartheta) \\ &= -\phi^T(\theta,\vartheta)\Lambda\phi(\theta,\vartheta) \end{aligned} \tag{3.22}$$

for all $\tau \ge \tau_1$, where

$$\phi^T(\theta,\vartheta) = (\phi_1(\theta), \phi_2(\theta))$$

and

$$\Lambda = \begin{bmatrix} (1-\delta)\alpha_1 & -\frac{(1-\delta)\beta_1+\delta\beta_2}{2} \\ -\frac{(1-\delta)\beta_1+\delta\beta_2}{2} & \delta[\frac{\alpha_2}{\varepsilon}+\alpha_3] \end{bmatrix}.$$

The right-hand side of inequality (3.22) is a quadratic form in ϕ. The quadratic form is negative definite when

$$\alpha_1\Big[\frac{\alpha_2}{\varepsilon}+\alpha_3\Big] > \frac{[(1-\delta)\beta_1+\delta\beta_2]^2}{4\delta(1-\delta)}. \tag{3.23}$$

It can be easily seen that the minimum value of inequality (3.23) at $\delta^* = \beta_1/(\beta_1+\beta_2)$ and is given by $\beta_1\beta_2$. So, the inequality (3.23) is equivalent to

$$\alpha_1\Big[\frac{\alpha_2}{\varepsilon}+\alpha_3\Big] > \beta_1\beta_1.$$

Therefore, The quadratic form is negative definite for all $\varepsilon < \varepsilon^*$, where

$$\varepsilon^* = \begin{cases} +\infty, & \text{if } \alpha_1\alpha_3 \ge \beta_1\beta_2; \\ \frac{\alpha_1\alpha_2}{\beta_1\beta_2-\alpha_1\alpha_3}, & \text{if } \alpha_1\alpha_3 < \beta_1\beta_2, \end{cases} \tag{3.24}$$

and α_1, α_2, α_3, β_1 and β_2 be defined by (3.12), (3.15), (3.19) and (3.21), respectively. It follows that the origin of system (3.2) is global asymptotically stable for all $\varepsilon < \varepsilon^*$. That is the equilibrium $E_2(x_1^*, x_2^*, x_3^*)$ of system (1.2) is globally asymptotically stable for all $\varepsilon < \varepsilon^*$. This completes the proof of this theorem. $\square$

Remark 3.3. Xu et al [25] studied the globally asymptotically stable of the positive equilibrium of system (1.2) by using the technique of directly constructing Lyapunov function. Obviously, their method is different from our method, and our result improve theirs, in Theorem 2.1 for $\varepsilon = a/r$ small enough. So our results are more general.

From the proof of Theorem (3.2), we have the following corollary.

Corollary 3.4. *Suppose that*

$$\frac{ab}{r_1+b}+\frac{ra_1}{ma_2}-\frac{2a_1}{ma_2}>0,\qquad \frac{ab}{r_1+b}-\frac{a_1}{m}>0.$$

If $\alpha_1\alpha_3 \geq \beta_1\beta_2$ *holds, then the positive equilibrium* $E_2(x_1^*,x_2^*,x_3^*)$ *of system* (1.2) *is globally asymptotically stable, where* α_1, α_2, α_3, β_1 *and* β_2 *be defined by* (3.12), (3.15), (3.19) *and* (3.21), *respectively.*

4. Example and numerical simulation

To check the validity of our results we consider the ratio-dependent predator-prey model with stage structure for the prey,

$$\begin{aligned}
\frac{\mathrm{d}x_1}{\mathrm{d}t}&=25x_2-22x_1-1.9x_1,\\
\frac{\mathrm{d}x_2}{\mathrm{d}t}&=1.9x_1-2.8x_2^2-\frac{2.2x_2x_3}{2.2x_3+x_2},\\
\frac{\mathrm{d}x_3}{\mathrm{d}t}&=x_3\big(-1+\frac{1.5x_2}{2.2x_3+x_2}\big).
\end{aligned}\tag{4.1}$$

It is easy to compute that

$$\frac{ab}{r_1+b}-\frac{2a_1}{m}=\frac{25\times 1.9}{22+1.9}-\frac{2\times 2.2}{2.2}\approx -0.0126<0.$$

So, conditions (2.2) of Theorem 2.1 do not hold. Thus, we cannot guarantee the global asymptotically stability of positive equilibrium of system (4.1) from Theorem 2.1. However, it is also easy to verify that

$$\frac{ab}{r_1+b}+\frac{ra_1}{ma_2}-\frac{2a_1}{ma_2}=\frac{25\times 1.9}{22+1.9}+\frac{1\times 2.2}{2.2\times 1.5}-\frac{2\times 2.2}{2.2}\approx 0.6541>0$$

and

$$\varepsilon=\frac{r}{a}=0.0400<\varepsilon^*=\frac{\alpha_1\alpha_2}{\beta_1\beta_2-\alpha_1\alpha_3}\approx 0.0409.$$

Therefore, from Theorem 3.2, the positive equilibrium E_2 of system (1.2) is globally asymptotically stable. Which is shown in Figure 1.

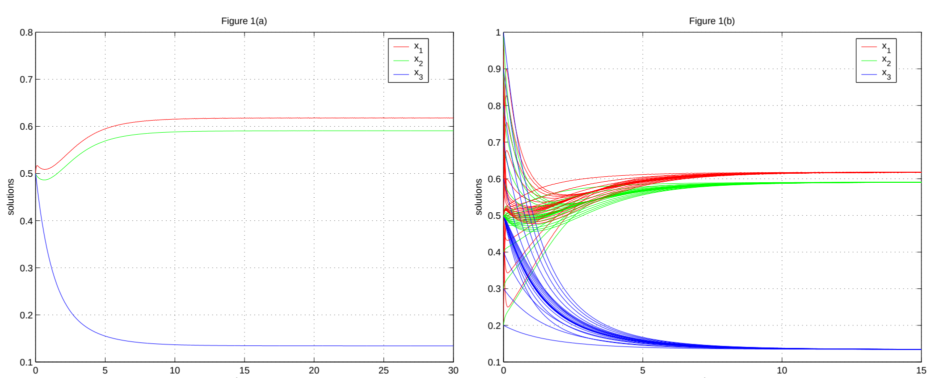

Figure 1. Trajectory of system (4.1) with $a=25$, $r_1=22$, $b=1.9$, $b_1=2.8$, $a_1=m=2.2$, $a_2=1.5$, $r=1$ and $\varepsilon=0.04$

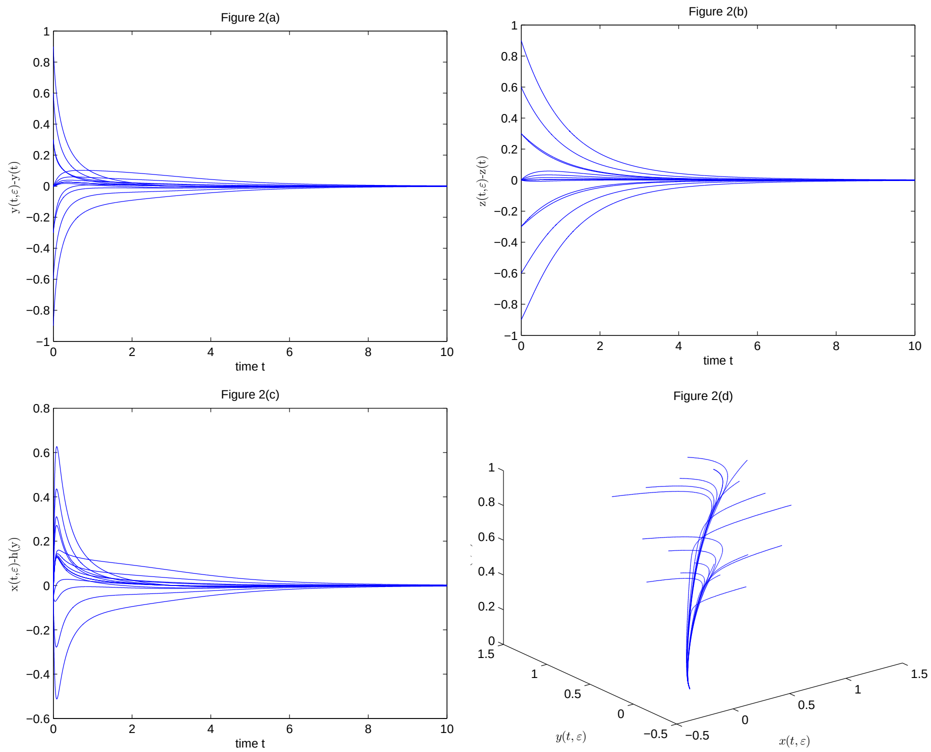

Figure 2. Trajectory of system (4.1) and its reduced system with $a = 25$, $r_1 = 22$, $b = 1.9$, $b_1 = 2.8$, $a_1 = m = 2.2$, $a_2 = 1.5$, $r = 1$ and $\varepsilon = 0.04$

Further, to show how the reduced system (3.3) approximates to the full system (1.2) and how the small parameter ε affects the stability of zero solution of system (1.2). By the equivalence of systems (1.2) and (2.3), we only focus on the numerical analysis of system (2.3) and its reduced system (3.3). Let $(x(t,\varepsilon), y(t,\varepsilon), z(t,\varepsilon))$ be solution of system (2.3), $(y(t), z(t))$ be the solution of system (3.3). If ε is small enough, the solutions of the reduced system (3.3) closely approximate to the solutions of the full system (2.3) and the errors (i.e. $x(t,\varepsilon) - h(y)$, $y(t,\varepsilon) - y(t)$, $z(t,\varepsilon) - z(t)$) quickly converge to zero after oscillation, and all solutions of system (2.3) approach to zero solution. Which are shown in Figure 2(a)-(d).

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