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PERIODIC SOLUTIONS FOR A FOOD CHAIN SYSTEM WITH MONOD-HALDANE FUNCTIONAL RESPONSE ON TIME SCALES

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Abstract. In this article, we study a three species food chain model on time scales, with Monod-Haldane functional response and time delay. With the help of coincidence degree theory, we establish the existence of periodic solutions.

1. Introduction

Research on food chain system has been a hot spot in population dynamics. Dynamical behavior of these models governed by differential equations and difference equations has been extensively studied in [5, 10, 12, 13, 15]. Hsu, Hwang and Kuang [9] considered the ratio-dependent food chain model

$$\begin{aligned}\dot{x}(t) &= rx(1-\frac{x}{K}) - \frac{1}{\eta_1}\frac{m_1xy}{a_1y+x},\\ \dot{y}(t) &= \frac{m_1xy}{a_1y+x} - d_1y - \frac{1}{\eta_2}\frac{m_2yz}{a_2z+y},\\ \dot{z}(t) &= \frac{m_2yz}{a_2z+y} - d_2z,\end{aligned}$$

where x, y and z sand for the population densities of prey, predator and top predator, respectively. The boundness, extinction and periodicity were studied.

Xu, Chaplain and Davidson [14] studied the delayed three-species Lotka-Volterra food chain system

$$\begin{aligned}\dot{x}_1(t) &= x_1(t)[r_1(t) - a_{11}(t)x_1(t-\tau_{11}) - a_{12}(t)x_2(t)],\\ \dot{x}_2(t) &= x_2(t)[-r_2(t) + a_{21}(t)x_1(t-\tau_{21}) - a_{22}(t)x_2(t-\tau_{22}) - a_{23}(t)x_3(t)],\\ \dot{x}_3(t) &= x_3(t)[-r_3(t) + a_{32}(t)x_2(t-\tau_{32}) - a_{33}x_3(t-\tau_{33})].\end{aligned}$$

The existence, uniqueness and global stability of positive periodic solutions of the system were studied. In population dynamics, the relationship between predator and prey can be represented as the functional response which refers to the change in the density of prey attached per unit time per predator as the prey density changes. Holling [8] gave three different kinds of functional responses, which are monotonic in the first quadrant. But some experiments and observations indicate

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that nonmonotonic response is more realistic [1]. To model such an inhibitory effect, Andrews suggested the so-called Monod-Haldane function proposed in [1]. From above all, we can get the following non-autonomous food chain model with time delays,

$$\begin{aligned} \dot{u}_1(t) &= u_1(t)[r_1(t) - d_1(t)u_1(t) - \frac{m_{12}(t)u_2(t)}{a_1(t) + b_1(t)u_1(t) + u_1^2(t)}], \\ \dot{u}_2(t) &= u_2(t)[-r_2(t) + \frac{m_{21}u_1(t-\tau)}{a_1(t) + b_1(t)u_1(t-\tau) + u_1^2(t-\tau)} - d_2(t)u_2(t) \\ &\quad - \frac{m_{23}(t)u_3(t)}{a_2(t) + b_2(t)u_2(t) + u_2^2(t)}], \\ \dot{u}_3(t) &= u_3(t)[-r_3(t) + \frac{m_{32}(t)u_2(t-\sigma)}{a_2(t) + b_2(t)u_2(t-\sigma) + u_2^2(t-\sigma)} - d_3(t)u_3(t)], \end{aligned} \tag{1.1}$$

where $u_1(t)$, $u_2(t)$ and $u_3(t)$ stand for the population density of prey, predator and top–predator at time t, respectively. All coefficients are positive continuous functions. $m_{i,i+1}(t)$ is the capture rate of the predator, $m_{i+1,i}(t)$ is a measure of the food quality that the prey provided for conversion into predator birth, where $i = 1, 2$.

On the other hand, if the populations have non-overlapping generations, the discrete model governed difference equations is more appropriate

$$\begin{aligned} u_1(n+1) &= u_1(n)\exp[r_1(n) - d_1(n)u_1(n) - \frac{m_{12}(n)u_2(n)}{a_1(n) + b_1(n)u_1(n) + u_1^2(n)}], \\ u_2(n+1) &= u_2(n)\exp[\frac{m_{21}u_1(n-\tau)}{a_1(n) + b_1(n)u_1(n-\tau) + u_1^2(n-\tau)} - r_2(n) - d_2(n)u_2(n) \\ &\quad - \frac{m_{23}(n)u_3(n)}{a_2(n) + b_2(n)u_2(n) + u_2^2(n)}], \\ u_3(n+1) &= u_3(n)\exp[\frac{m_{32}(n)u_2(n-\sigma)}{a_2(n) + b_2(n)u_2(n-\sigma) + u_2^2(n-\sigma)} - r_3(n) - d_3(n)u_3(n)], \end{aligned} \tag{1.2}$$

where all the coefficients are positive periodic sequences.

To explore the periodic solutions of differential equation and difference equation models, coincidence degree theory is a common method. However, for these two types of systems, the methods and results are significantly similar. Enlightened by the idea of Stefan Hilger [7], to unify the continuous and discrete dynamic systems, we consider the following dynamic system on time scales,

$$\begin{aligned} x^{\Delta}(t) &= r_1(t) - d_1(t)e^{x(t)} - \frac{m_{12}(t)e^{y(t)}}{a_1(t) + b_1(t)e^{x(t)} + e^{2x(t)}}, \\ y^{\Delta}(t) &= -r_2(t) + \frac{m_{21}(t)e^{x(t-\tau)}}{a_1(t) + b_1e^{x(t-\tau)} + e^{2x(t-\tau)}} - d_2(t)e^{y(t)} \\ &\quad - \frac{m_{23}(t)e^{z(t)}}{a_2(t) + b_2(t)e^{y(t)} + e^{2y(t)}}, \\ z^{\Delta}(t) &= -r_3(t) + \frac{m_{32}(t)e^{y(t-\sigma)}}{a_2(t) + b_2(t)e^{y(t-\sigma)} + e^{2y(t-\sigma)}} - d_3(t)e^{z(t)}, \end{aligned} \tag{1.3}$$

where $t \in \mathbb{T}$ and $\mathbb{T}$ is a time scale that is unbounded above. $x^{\Delta}(t)$ is the delta derivative of x at t, which is defined in [4]. All the coefficients are positive $\omega-$periodic functions. Set $u_1(t) = e^{x(t)}$, $u_2(t) = e^{y(t)}$, $u_3(t) = e^{z(t)}$, then (1.3) can be reduced to (1.1) and (1.2) when $\mathbb{T} = \mathbb{R}$ and $\mathbb{T} = \mathbb{Z}$, respectively.

The purpose of this paper is to study the periodicity of three-species food chain system on time scales and this model has not been investigated before. We would like to mention that there are several papers on periodicity in dynamic systems on time scales by using the coincidence degree theory, see [2, 3, 11, 17]. The remainder of the paper is organized as follows. In the following section, some preliminary results about calculus on time scales and the continuation theorem are stated. Next, the sufficient conditions for the existence of periodic solutions are explored.

2. Preliminaries

For convenience, we first present the useful lemma about time scales and the continuation theorem of the coincidence degree theory; more details can be found in [16, 6].

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of real numbers $\mathbb{R}$. Throughout this paper, we assume that the time scale $\mathbb{T}$ is unbounded above and below, such as $\mathbb{R}$, $\mathbb{Z}$ and $\cup_{k\in\mathbb{Z}}[2k, 2k+1]$. The following definitions and lemmas about time scales are from [16, 6].

Lemma 2.1 ([16]). *Let t_1, $t_2 \in I_\omega$ and $t \in \mathbb{T}$. If $g : \mathbb{T} \to \mathbb{R} \in C_{rd}(\mathbb{T})$ is ω-periodic, then*

$$g(t) \le g(t_1) + \frac{1}{2}\int_k^{k+\omega} |g^{\Delta}(s)|\Delta s,$$
$$g(t) \ge g(t_2) - \frac{1}{2}\int_k^{k+\omega} |g^{\Delta}(s)|\Delta s,$$

where the constant factor $1/2$ is the best possible.

For simplicity, we use the following notation throughout this paper. Let $\mathbb{T}$ be ω-periodic; that is, $t \in \mathbb{T}$ implies $t + \omega \in \mathbb{T}$,

$$k = \min\{\mathbb{R}^+ \cap \mathbb{T}\}, \quad I_\omega = [k, k+\omega] \cap \mathbb{T}, \quad g^L = \inf_{t\in\mathbb{T}} g(t),$$
$$g^M = \sup_{t\in\mathbb{T}} g(t), \quad \bar{g} = \frac{1}{\omega}\int_{I_\omega} g(s)\Delta s = \frac{1}{\omega}\int_k^{k+\omega} g(s)\Delta s,$$

where $g \in C_{rd}(\mathbb{T})$ is an ω-periodic real function; i.e., $g(t+\omega) = g(t)$ for all $t \in \mathbb{T}$.

Next, we state the Mawhin's continuation theorem, which is a main tool in the proof of our theorem.

Lemma 2.2 ([6]). *Let L be a Fredholm mapping of index zero and N be L-compact on $\bar{\Omega}$. Suppose*

(a) *for each $\lambda \in (0, 1)$, every solution u of $Lu = \lambda Nu$ is such that $u \notin \partial\Omega$;*
(b) *$QNu \ne 0$ for each $u \in \partial\Omega \cap \ker L$ and the Brouwer degree $\deg\{JQN, \Omega \cap \ker L, 0\} \ne 0$.*

Then the operator equation $Lu = Nu$ has at least one solution lying in $\operatorname{Dom} L \cap \bar{\Omega}$.

3. Main Results

Theorem 3.1. *If* $m_{32}^L e^{L_2} > r_3^M(a_2^M + b_2^M e^{M_2} + e^{2M_2})$ *holds, where*

$$L_2 = \ln \frac{a_2^L r_3^L}{m_{32}^M} - \frac{m_{21}^M \omega}{b_1^L}$$

and

$$M_2 = \ln \frac{r_1^M(a_1^M + \frac{b_1^M r_1^M}{d_1^L} + \frac{2r_1^M}{d_1^L})}{m_{12}^L} + \frac{m_{21}^M \omega}{b_1^L},$$

then (1.3) *has at least one* ω*-periodic solution.*

Proof. Let

$$\begin{gathered}
X = Z = \Big\{(x,y,z)^T \in C(\mathbb{T}, \mathbb{R}^3) : x(t+\omega) = x(t), y(t+\omega) = y(t), \\
z(t+\omega) = z(t), \forall t \in \mathbb{T}\Big\}, \\
\|(x,y,z)^T\| = \max_{t\in I_\omega} |x(t)| + \max_{t\in I_\omega} |y(t)| + \max_{t\in I_\omega} |z(t)|, \quad (x,y,z)^T \in X \quad \text{(or in } Z).
\end{gathered}$$

Then X and Z are both Banach spaces when they are endowed with the above norm $\|\cdot\|$. Let

$$N \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix},$$

where

$$\begin{aligned}
N_1 &= r_1(t) - d_1(t)e^{x(t)} - \frac{m_{12}(t)e^{y(t)}}{a_1(t) + b_1(t)e^{x(t)} + e^{2x(t)}}, \\
N_2 &= -r_2(t) + \frac{m_{21}(t)e^{x(t-\tau)}}{a_1(t) + b_1 e^{x(t-\tau)} + e^{2x(t-\tau)}} - d_2(t)e^{y(t)} \\
&\quad - \frac{m_{23}(t)e^{z(t)}}{a_2(t) + b_2(t)e^{y(t)} + e^{2y(t)}}, \\
N_3 &= -r_3(t) + \frac{m_{32}(t)e^{y(t-\sigma)}}{a_2(t) + b_2(t)e^{y(t-\sigma)} + e^{2y(t-\sigma)}} - d_3(t)e^{z(t)}.
\end{aligned}$$

$$L \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x^\Delta \\ y^\Delta \\ z^\Delta \end{bmatrix}, \quad P \begin{bmatrix} x \\ y \\ z \end{bmatrix} = Q \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{\omega}\int_k^{k+\omega} x(t)\Delta t \\ \frac{1}{\omega}\int_k^{k+\omega} t(t)\Delta t \\ \frac{1}{\omega}\int_k^{k+\omega} z(t)\Delta t \end{bmatrix}.$$

Obviously, $\ker L = \mathbb{R}^3$, $\operatorname{Im} L = \big\{(x,y,z)^T \in Z : \bar{x} = \bar{y} = \bar{z} = 0, t \in \mathbb{T}\big\}$, $\dim\ker L = 3 = \operatorname{codim} \operatorname{Im} L$. Since $\operatorname{Im} L$ is closed in Z, then L is a Fredholm mapping of index zero. It is easy to show that P and Q are continuous projections such that $\operatorname{Im} P = \ker L$ and $\operatorname{Im} L = \ker Q = \operatorname{Im}(I - Q)$. Furthermore, the generalized inverse (of L) $K_P : \operatorname{Im} L \to \ker P \cap \operatorname{Dom} L$ exists and is given by

$$K_P \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \int_k^t x(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t x(s)\Delta s \Delta t \\ \int_k^t y(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t y(s)\Delta s \Delta t \\ \int_k^t z(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t z(s)\Delta s \Delta t \end{bmatrix}.$$

Thus

$$QN\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}\frac{1}{\omega}\int_k^{k+\omega}(r_1(t)-d_1(t)e^{x(t)}-\frac{m_{12}(t)e^{y(t)}}{a_1(t)+b_1(t)e^{x(t)}+e^{2x(t)}})\Delta t\\ \frac{1}{\omega}\int_k^{k+\omega}(-r_2(t)+\frac{m_{21}(t)e^{x(t-\tau)}}{a_1(t)+b_1e^{x(t-\tau)}+e^{2x(t-\tau)}}\\ -d_2(t)e^{y(t)}-\frac{m_{23}(t)e^{z(t)}}{a_2(t)+b_2(t)e^{y(t)}+e^{2y(t)}})\Delta t\\ \frac{1}{\omega}\int_k^{k+\omega}(-r_3(t)+\frac{m_{32}(t)e^{y(t-\sigma)}}{a_2(t)+b_2(t)e^{y(t-\sigma)}+e^{2y(t-\sigma)}}\\ -d_3(t)e^{z(t)})\Delta t\end{bmatrix},$$

and

$$K_P(I-Q)N\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}\int_k^t x(s)\Delta s-\frac{1}{\omega}\int_k^{k+\omega}\int_k^t x(s)\Delta s\Delta t\\ -\Big(t-k-\frac{1}{\omega}\int_k^{k+\omega}(t-k)\Delta t\Big)\bar{x}\\ \int_k^t y(s)\Delta s-\frac{1}{\omega}\int_k^{k+\omega}\int_k^t y(s)\Delta s\Delta t\\ -\Big(t-k-\frac{1}{\omega}\int_k^{k+\omega}(t-k)\Delta t\Big)\bar{y}\\ \int_k^t z(s)\Delta s-\frac{1}{\omega}\int_k^{k+\omega}\int_k^t z(s)\Delta s\Delta t-\\ \Big(t-k-\frac{1}{\omega}\int_k^{k+\omega}(t-k)\Delta t\Big)\bar{z}\end{bmatrix}.$$

Clearly, QN and $K_P(I-Q)N$ are continuous. According to the Arzela-Ascoli theorem, it is not difficulty to show that $K_P(I-Q)N(\bar{\Omega})$ is compact for any open bounded set $\Omega\subset X$ and $QN(\bar{\Omega})$ is bounded. Thus, N is L-compact on $\bar{\Omega}$.

Now, we shall search an appropriate open bounded subset Ω for the application of the continuation theorem, Lemma 2.2. For the operator equation $Lu=\lambda Nu$, where $\lambda\in(0,1)$, we have

$$\begin{aligned}u_1^{\Delta}(t)&=\lambda(r_1(t)-d_1(t)e^{x(t)}-\frac{m_{12}(t)e^{y(t)}}{a_1(t)+b_1(t)e^{x(t)}+e^{2x(t)}}),\\ u_2^{\Delta}(t)&=\lambda(-r_2(t)+\frac{m_{21}(t)e^{x(t-\tau)}}{a_1(t)+b_1e^{x(t-\tau)}+e^{2x(t-\tau)}}-d_2(t)e^{y(t)}\\ &\quad-\frac{m_{23}(t)e^{z(t)}}{a_2(t)+b_2(t)e^{y(t)}+e^{2y(t)}}),\\ u_3^{\Delta}(t)&=\lambda(-r_3(t)+\frac{m_{32}(t)e^{y(t-\sigma)}}{a_2(t)+b_2(t)e^{y(t-\sigma)}+e^{2y(t-\sigma)}}-d_3(t)e^{z(t)}).\end{aligned}\tag{3.1}$$

Assume that $(u_1,u_2,u_3)^T\in X$ is a solution of system (3.1) for a certain $\lambda\in(0,1)$. Integrating (3.1) on both sides from k to $k+\omega$, we obtain

$$\begin{gathered}\int_k^{k+\omega}[d_1(t)e^{x(t)}+\frac{m_{12}(t)e^{y(t)}}{a_1(t)+b_1(t)e^{x(t)}+e^{2x(t)}}]\Delta t=\bar{r}_1\omega,\\ \int_k^{k+\omega}[r_2(t)+d_2(t)e^{y(t)}+\frac{m_{23}(t)e^{z(t)}}{a_2(t)+b_2(t)e^{y(t)}+e^{2y(t)}}]\Delta t\\ =\int_k^{k+\omega}\frac{m_{21}(t)e^{x(t-\tau)}}{a_1(t)+b_1e^{x(t-\tau)}+e^{2x(t-\tau)}}\Delta t,\\ \int_k^{k+\omega}[r_3(t)+d_3(t)e^{z(t)}]\Delta t=\int_k^{k+\omega}\frac{m_{32}(t)e^{y(t-\sigma)}}{a_2(t)+b_2(t)e^{y(t-\sigma)}+e^{2y(t-\sigma)}}\Delta t.\end{gathered}\tag{3.2}$$

Since $(x,y,z)^T \in X$, there exist $\xi_i, \eta_i \in I_\omega$, $i=1,2,3$, such that

$$\begin{aligned} x(\xi_1) &= \min_{t\in I_\omega}\{x(t)\}, \quad x(\eta_1) = \max_{t\in I_\omega}\{x(t)\}, \\ y(\xi_2) &= \min_{t\in I_\omega}\{y(t)\}, \quad y(\eta_2) = \max_{t\in I_\omega}\{y(t)\}, \\ z(\xi_3) &= \min_{t\in I_\omega}\{z(t)\}, \quad z(\eta_3) = \max_{t\in I_\omega}\{z(t)\}. \end{aligned} \tag{3.3}$$

From (3.1) and (3.2), we have

$$\begin{gathered} \int_k^{k+\omega} |x^\Delta(t)|\Delta t \le 2\bar{r}_1\omega, \\ \int_k^{k+\omega} |y^\Delta(t)|\Delta t \le 2\frac{m_{21}^M\omega}{b_1^L}, \\ \int_k^{k+\omega} |z^\Delta(t)|\Delta t \le 2\frac{m_{32}^M\omega}{b_2^L}. \end{gathered}$$

By the first equation of (3.2) and (3.3),

$$d_1(\xi_1)e^{x(\xi_1)} < r_1(\xi_1);$$

that is,

$$x(\xi_1) < \ln\frac{r_1^M}{d_1^L}.$$

From the second equation of (3.2), we have

$$r_2(\eta_2) < \frac{m_{21}(\eta_2)e^{x(\eta_2-\tau)}}{a_1(\eta_2)}$$

and

$$x(\eta_1) \ge x(\eta_2-\tau) > \ln\frac{r_2^La_1^L}{m_{21}^M}.$$

According to Lemma 2.1, we have

$$\begin{gathered} x(t) \le x(\xi_1) + \frac{1}{2}\int_k^{k+\omega} |x^\Delta(t)|\Delta t \le \ln\frac{r_1^M}{d_1^L} + \bar{r}_1\omega := M_1, \\ x(t) \ge x(\eta_1) - \frac{1}{2}\int_k^{k+\omega} |x^\Delta(t)|\Delta t \ge \ln\frac{r_2^La_1^L}{m_{21}^M} - \bar{r}_1\omega := L_1. \end{gathered}$$

From the first equation of (3.2) and (3.3), we obtain

$$\begin{gathered} \frac{m_{12}(\xi_1)e^{y(\xi_1)}}{a(\xi_1)+b_1(\xi_1)e^{x(\xi_1)}+e^{2x(\xi_1)}} < r_1(\xi_1), \\ y(\xi_2) < y(\xi_1) < \ln\frac{r_1^M(a_1^M + \frac{b_1^Mr_1^M}{d_1^L} + \frac{2r_1^M}{d_1^L})}{m_{12}^L}. \end{gathered}$$

Then

$$y(t) \le y(\xi_2) + \frac{1}{2}\int_k^{k+\omega} |y^\Delta(t)|\Delta t < \ln\frac{r_1^M(a_1^M + \frac{b_1^Mr_1^M}{d_1^L} + \frac{2r_1^M}{d_1^L})}{m_{12}^L} + \frac{m_{21}^M\omega}{b_1^L} := M_2.$$

From the third equation of (3.2), we have

$$\frac{m_{32}(\xi_3)}{a_2(\xi_3)}e^{y(\xi_3-\sigma)} > r_3(t),$$

this reduces to

$$y(\eta_2) \geq y(\xi_3 - \sigma) > \ln \frac{a_2^L r_3^L}{m_{32}^M}.$$

Then

$$y(t) \geq y(\eta_2) - \frac{1}{2}\int_k^{k+\omega} \left|y^{\Delta}(t)\right| \Delta t \geq \ln \frac{a_2^L r_3^L}{m_{32}^M} - \frac{m_{21}^M \omega}{b_1^L} := L_2.$$

According to the first equation of (3.2), we have

$$\frac{m_{32}(\xi_3)}{b_2(\xi_3)} > d_3(\xi_3) e^{z(\xi_3)}.$$

Then

$$z(t) \leq z(\xi_3) + \frac{1}{2}\int_k^{k+\omega} |z^{\Delta}(t)| \Delta t \leq \ln \frac{m_{32}^M}{b_2^L d_3^L} + \frac{m_{32}^M \omega}{b_2^L} := M_3.$$

Also we have

$$d_3^M e^{z(\eta_3)} > \frac{m_{32}^L e^{L_2}}{a_2^M + b_2^M e^{M_2} + e^{2M_2}} - r_3^M.$$

Then

$$z(t) \geq z(\eta_3) - \frac{1}{2}\int_k^{k+\omega} |z^{\Delta}(t)| \Delta t \geq \ln \frac{\frac{m_{32}^L e^{L_2}}{a_2^M + b_2^M e^{M_2} + e^{2M_2}} - r_3^M}{d_3^M} - \frac{m_{32}^M \omega}{b_2^L} := L_3.$$

Therefore, we have

$$\begin{aligned}
\max_{t \in [k, k+\omega]} |x(t)| &\leq \max\{|M_1|, |L_1|\} := R_1, \\
\max_{t \in [k, k+\omega]} |y(t)| &\leq \max\{|M_2|, |L_2|\} := R_2, \\
\max_{t \in [k, k+\omega]} |z(t)| &\leq \max\{|M_3|, |L_3|\} := R_3.
\end{aligned}$$

Clearly, R_1, R_2 and R_3 are independent of λ. Let $R = R_1 + R_2 + R_3 + R_0$, where R_0 is taken sufficiently large such that for the algebraic equations

$$\begin{gathered}
\bar{r}_1 - \bar{d}_1 e^x - \frac{1}{\omega}\int_\kappa^{\kappa+\omega} \frac{m_{12}(t) e^y}{a_1(t) + b_1(t) e^x + e^{2x}} \Delta t = 0, \\
-\bar{r}_2 + \frac{1}{\omega}\int_\kappa^{\kappa+\omega} \frac{m_{21}(t) e^x}{a_1(t) + b_1(t) e^x + e^{2x}} \Delta t - \bar{d}_2 e^y \\
- \frac{1}{\omega}\int_\kappa^{\kappa+\omega} \frac{\bar{m}_{23} e^z}{a_2(t) + b_2(t) e^y + e^{2y}} \Delta t = 0, \\
-\bar{r}_3 + \frac{1}{\omega}\int_\kappa^{\kappa+\omega} \frac{m_{32}(t) e^y}{a_2(t) + b_2(t) e^y + e^{2y}} \Delta t - \bar{d}_3 e^z = 0,
\end{gathered} \tag{3.4}$$

every solution $(x^*, y^*, z^*)^T$ of (3.4) satisfies $\|(x^*, y^*, z^*)^T\| < R$. Now, we define $\Omega = \{(x, y, z)^T \in X : \|(x, y, z)^T\| < R\}$. Then it is clear that Ω satisfies the requirement (a) of Lemma 2.2. If $(u_1, u_2, u_3)^T \in \partial\Omega \cap \ker L = \partial\Omega \cap \mathbb{R}^3$, then $(x, y, z)^T$ is a constant vector in $\mathbb{R}^3$ with $\|(x, y, z)^T\| = |x| + |y| + |z| = R$, so we have

$$QN \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

By the assumption in Theorem 3.1 and the definition of topological degree, a direct calculation yields $\deg(JQN, \Omega \cap \ker L, 0) \neq 0$. We have verified that Ω

satisfies all requirements of Lemma 2.2; therefore, system (1.3) has at least one ω-periodic solution in $\operatorname{Dom} L \cap \bar{\Omega}$. This completes the proof. $\square$

Conclusion. In this article, a three species food chain model on time scales is proposed. This model not only unifies the food chain system with Monod-Haldane functional response and time delay governed by differential equations and their discrete analogues in form of difference equations, but also extends the results to more general time scales. By using the Mawhin's continuation theorem of coincidence degree theory, the existence of periodic solutions is established, which means that we do not have to investigate the same problem in systems (1.1) and (1.2) repeatedly. Moreover, based on the sharp inequalities in [16], the priori estimates of periodic solutions are better than previous work.

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References

[1] Andrews, J. F.; A mathematical model for the continuous culture of microorganisms utilizing inhabitory substrates, *Biotechnol Bioengrg*, Vol. 10(1968), 707–723.
[2] Bohner, M.; Fan, M.; Zhang, J. M.; Existence of periodic solutions in predator–prey and competition dynamic systems, *Nonl. Anal: RWA*, Vol. 7(2006), 1193–1204.
[3] Bohner, M.; Fan, M.; Zhang, J. M.; Periodicity of scalar dynamic equations and applications to population models, *J. Math. Anal. Appl.* Vol. 330(2007), 1–9.
[4] Bohner, M.; Peterson, A.; *Dynamic Equations on Time Scales: An Introduction with Applications*, Birkhäuser, Boston, 2001.
[5] Francoise, J. P.; Llibre, J.; Analytical study of a triple Hopf bifurcation in a tritrophic food chain model, *Applied Mathematics and Computation*, Vol. 217(2011), 7146–7154.
[6] Gaines, R. E.; Mawhin, J. L.; *Coincidence Degree and Nonlinear Differential Equations*, Lecture Notes in Mathematics, Springer-Verlag, Berlin, 1977.
[7] Hilger, S.; Analysis on measure chains-a unified approach to continuous and discrete calculus, *Results Math.*, Vol. 18(1990), 18–56.
[8] Holling C. S.; The functional response of predator to prey density and its role in mimicry and population regulations, *Mem. Ent. Sec. Can. Vol.*, 45(1965), 1–60.
[9] Hsu, S. B.; Hwang, T.W.; Kuang, Y.; A ratio-dependent food chain model and its applications to biological control, *Math. Biosci.*, Vol. 181(2003), 55–83.
[10] Mukhopadhyay B., Bhattacharyya R.; A stage-structured food chain model with stage dependent predation: Existence of codimension one and codimension two bifurcations, *Nonlinear Analysis: RWA*, Vol. 12(2011), 3056–3072.
[11] Tong, Y.; Liu, Z. J.; Gao, Z. Y.; Wang, Y. H.; Existence of periodic solutions for a predator-prey system with sparse effect and functional response on time scales, *Commun. Nonlinear Sci. Numer. Simulat.*, Vol. 17(2012), 3360–3366.
[12] Wang, W. M.; Wang, H. L.; Li, Z. Q.; The dynamic complexity of a three–species Beddington-type food chain with impulsive control strategy, *Chaos, Solitons and Fractals*, Vol. 32(2007), 1772–1785.
[13] Xu, C. Y.; Wang, M. J.; Permanence for a delayed discrete three-level food-chain model with Beddington-DeAngelis functional response, *Applied Mathematics and Computation*, Vol. 187(2007), 1109–1119.
[14] Xu, R.; Chaplain, M. A. J.; Davidson, F. A.; Periodic solution for a three-species Lotka-Volterra food-chain model with time delays, *Mathematical and Computer Modelling*, Vol. 40(2004), 823–837.

[15] Xu, R.; Chen, L. S.; Hao, F. L.; Periodic solutions of a discrete time Lotka-Volterra type food-chain model with delays, *Applied Mathematics and Computation*, Vol. 171(2005), 91–103.
[16] Zhang, B. B., Fan, M.; A remark on the application of coincidence degree to periodicity of dynamic equtions on time scales, *J. Northeast Normal University(Natural Science Edition)*, Vol. 39(2007), 1–3. (in Chinese)
[17] Zhuang, K. J.; Periodic solutions for a three species predator–prey system on time scales, *Int. J. Dynamical Systems and Differential Equations*, Vol. 3(2011), 3-11.

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