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OSCILLATION OF SOLUTIONS TO SECOND-ORDER NEUTRAL DIFFERENTIAL EQUATIONS

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Abstract. We study the oscillatory behavior of solutions to second-order neutral differential equations. We show that under certain conditions, all solutions are oscillatory.

1. Introduction

In this article, we study the oscillation of solutions to the second-order nonlinear neutral delay differential equation

$$(r(t)\psi(x(t))|Z'(t)|^{\alpha-1}Z'(t))' + q(t)f(x(\sigma(t))) = 0, \tag{1.1}$$

where $t \in \mathbb{I} := [t_0, \infty)$, $Z(t) := x(t) + p(t)x(\tau(t))$, and $\alpha > 0$. Throughout, we assume that the following conditions are satisfied:

(A1) $r, p, q \in C(\mathbb{I}, \mathbb{R})$, $r(t) > 0$, $0 \le p(t) \le 1$, $q(t) \ge 0$, and q is not identically zero for large t;

(A2) $\psi \in C^1(\mathbb{R}, \mathbb{R})$, $f \in C(\mathbb{R}, \mathbb{R})$, $\psi(x) > 0$, $xf(x) > 0$ for all $x \neq 0$, and there exist two positive constants k and L such that

$$\frac{f(x)}{|x|^{\alpha-1}x} \ge k \quad \text{and} \quad \psi(x) \le L^{-1} \quad \text{for all} \quad x \neq 0;$$

(A3) $\tau \in C(\mathbb{I}, \mathbb{R})$, $\tau(t) \le t$, and $\lim_{t\to\infty} \tau(t) = \infty$;

(A4) $\sigma \in C^1(\mathbb{I}, \mathbb{R})$, $\sigma'(t) > 0$, $\sigma(t) \le t$, and $\lim_{t\to\infty} \sigma(t) = \infty$.

By a solution of equation (1.1), we mean a continuous function x defined on an interval $[t_x, \infty)$ such that $r\psi(x)|Z'|^{\alpha-1}Z'$ is continuously differentiable and x satisfies (1.1) for $t \in [t_x, \infty)$. We consider only solutions satisfying condition $\sup\{|x(t)| : t \ge T \ge t_x\} > 0$ and tacitly assume that equation (1.1) possesses such solutions. As usual, a solution of (1.1) is called oscillatory if it is neither eventually positive nor eventually negative; otherwise, we call it non-oscillatory. Equation (1.1) is termed oscillatory if all its continuable solutions oscillate.

It is known that various classes of neutral differential equations are often encountered in applied problems in natural sciences and engineering; see, e.g., Hale [8]. Recently, a great deal of interest in oscillatory properties of neutral functional differential equations has been shown, we refer the reader to [1, 2, 3, 4, 5, 6, 7, 9,

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10, 11, 12, 13, 14, 15, 16] and the references cited therein. Below, we briefly review the following related results that motivated our study. Ye and Xu [16] obtained several oscillation criteria for equation (1.1), one of which we present below. For the convenience of the reader, in what follows, we use the notation

$$\varepsilon := (\alpha/(\alpha+1))^{\alpha+1}, \quad Q(t) := q(t)(1-p(\sigma(t)))^{\alpha}, \quad \pi(t) := \int_t^{\infty} \frac{\mathrm{d}s}{r^{1/\alpha}(s)}.$$

Theorem 1.1 ([16, Theorem 2.3]). *Assume that conditions* (A1)–(A4) *are satisfied and let*

$$\pi(t_0) < \infty. \tag{1.2}$$

If

$$\int^{\infty} \big[Q(t)\pi^{\alpha}(\sigma(t)) - \frac{\varepsilon}{Lk}\frac{\sigma'(t)}{\pi(\sigma(t))r^{1/\alpha}(\sigma(t))}\big]\mathrm{d}t = \infty$$

and

$$\int^{\infty} \big[Q(t)\pi^{\alpha}(t) - \frac{\varepsilon}{Lk}\frac{r(\sigma(t))}{\pi(t)(\sigma'(t))^{\alpha}r^{(\alpha+1)/\alpha}(t)}\big]\mathrm{d}t = \infty,$$

then equation (1.1) *is oscillatory.*

Note that Theorem 1.1 is not valid for the differential equation

$$\Big(\mathrm{e}^{2t}\Big(x(t) + \frac{1}{2}x(t-2)\Big)'\Big)' + \big(1 + \frac{\mathrm{e}^2}{2}\big)\mathrm{e}^{2t}x(t) = 0, \tag{1.3}$$

where $t \geq 1$. Let $r(t) = \mathrm{e}^{2t}$, $\psi(x(t)) = 1$, $p(t) = 1/2$, $q(t) = (2+\mathrm{e}^2)\mathrm{e}^{2t}/2$, $\tau(t) = t-2$, $\sigma(t) = t$, $\alpha = 1$, $L = 1$, and $k = 1$. Then $\pi(t) = \mathrm{e}^{-2t}/2$, $\pi(t_0) < \infty$, and $Q(t) = q(t)/2 = (2+\mathrm{e}^2)\mathrm{e}^{2t}/4$. Then, we conclude that

$$\begin{aligned}
&\int^{\infty} \big[Q(t)\pi^{\alpha}(\sigma(t)) - \frac{\varepsilon}{Lk}\frac{\sigma'(t)}{\pi(\sigma(t))r^{1/\alpha}(\sigma(t))}\big]\mathrm{d}t \\
&= \int^{\infty} \big[Q(t)\pi^{\alpha}(t) - \frac{\varepsilon}{Lk}\frac{r(\sigma(t))}{\pi(t)(\sigma'(t))^{\alpha}r^{(\alpha+1)/\alpha}(t)}\big]\mathrm{d}t \\
&= \int^{\infty} \frac{\mathrm{e}^2-2}{8}\mathrm{d}t = \infty.
\end{aligned}$$

Hence, by Theorem 1.1, equation (1.3) should be oscillatory. However, it is not difficult to verify that $x(t) = \mathrm{e}^{-t}$ is a non-oscillatory solution of equation (1.3).

To amend Theorem 1.1, Han et al. [9] established some oscillation results for (1.1) under the assumptions

$$p'(t) \geq 0, \quad \sigma(t) \leq \tau(t) := t - \tau_0, \tag{1.4}$$

where τ_0 is a non-negative constant. The main goal of this article is to derive new oscillation criteria for (1.1) without requiring the restrictive conditions (1.4).

2. Main results

In what follows, all functional inequalities are tacitly assumed to hold for all t large enough.

Theorem 2.1. *Assume that* (A1)–(A4) *and* (1.2) *are satisfied and assume that* $\psi(x) \geq K > 0$. *Suppose further that there exist two functions* $\rho, m \in C^1(\mathbb{I}, (0,\infty))$ *such that*

$$\frac{m(t)}{(LK)^{1/\alpha}r^{1/\alpha}(t)\pi(t)} + m'(t) \leq 0, \quad 1 - p(t)\frac{m(\tau(t))}{m(t)} > 0, \tag{2.1}$$

$$\int^{\infty} \big[\rho(s)Q(s) - \frac{1}{Lk(\alpha+1)^{\alpha+1}} \frac{(\rho'_+(s))^{\alpha+1} r(\sigma(s))}{(\rho(s)\sigma'(s))^{\alpha}}\big] \mathrm{d}s = \infty, \tag{2.2}$$

$$\int^{\infty} \big[kq(s)\pi^{\alpha}(s)\big(1 - p(\sigma(s))\frac{m(\tau(\sigma(s)))}{m(\sigma(s))}\big)^{\alpha} - \big(\frac{\alpha}{\alpha+1}\big)^{\alpha+1} \frac{1}{L\pi(s) r^{1/\alpha}(s)}\big] \mathrm{d}s = \infty, \tag{2.3}$$

where $\rho'_+(t) := \max\{0, \rho'(t)\}$. *Then equation* (1.1) *is oscillatory.*

Proof. Let x be a non-oscillatory solution of (1.1). The proofs for eventually positive and for eventually negative solutions are similar. If y is a negative solution, then $x = -y$ may not be a solution of (1.1), but x satisfies key estimates such as (2.6) with $\psi(-x)$ instead of $\psi(x)$. Then we can use that $\psi(x)$ and $\psi(-x)$ have same bounds, $K \le \psi(\cdot) \le 1/L$.

We assume that there exists a $t_1 \ge t_0$ such that $x(t) > 0$, $x(\tau(t)) > 0$, and $x(\sigma(t)) > 0$ for all $t \ge t_1$. Then $|x(t)|^{\alpha-1}x(t) = x^{\alpha}(t)$ and $Z(t) > 0$. From (1.1) it follows that for all $t \ge t_1$,

$$(r(t)\psi(x(t))|Z'(t)|^{\alpha-1}Z'(t))' \le -kq(t)x^{\alpha}(\sigma(t)) \le 0. \tag{2.4}$$

Hence, there exists a $t_2 \ge t_1$ such that either $Z'(t) > 0$ or $Z'(t) < 0$ for all $t \ge t_2$. We consider each of two cases separately.

Case 1: $Z'(t) > 0$ for all $t \ge t_2$. As in the proof of [16, Theorem 2.1], we obtain a contradiction to (2.2).

Case 2: $Z'(t) < 0$ for all $t \ge t_2$. For $t \ge t_2$, we define a Riccati substitution

$$\omega(t) := \frac{r(t)\psi(x(t))(-Z'(t))^{\alpha-1}Z'(t)}{Z^{\alpha}(t)}. \tag{2.5}$$

Then $\omega(t) < 0$ for all $t \ge t_2$. Since $(r(t)\psi(x(t))|Z'(t)|^{\alpha-1}Z'(t))' \le 0$, the function $r\psi(x)|Z'|^{\alpha-1}Z'$ is non-increasing. Thus, for all $s \ge t \ge t_2$,

$$(r(s)\psi(x(s)))^{1/\alpha}Z'(s) \le (r(t)\psi(x(t)))^{1/\alpha}Z'(t).$$

Dividing the latter inequality by $(r(s)\psi(x(s)))^{1/\alpha}$ and integrating the resulting inequality from t to l, for all $l \ge t \ge t_2$, we have

$$Z(l) \le Z(t) + (r(t)\psi(x(t)))^{1/\alpha}Z'(t) \int_t^l \frac{\mathrm{d}s}{(r(s)\psi(x(s)))^{1/\alpha}}.$$

Since $Z'(t) < 0$ and $\psi \le 1/L$, we conclude that, for all $l \ge t \ge t_2$,

$$Z(l) \le Z(t) + (Lr(t)\psi(x(t)))^{1/\alpha}Z'(t) \int_t^l \frac{\mathrm{d}s}{r^{1/\alpha}(s)}.$$

Letting $l \to \infty$ in this inequality, and using that $Z > 0$, we have that for all $t \ge t_2$,

$$0 \le Z(t) + (Lr(t)\psi(x(t)))^{1/\alpha}Z'(t)\pi(t);$$

that is, for all $t \ge t_2$,

$$(r(t)\psi(x(t)))^{1/\alpha}\pi(t)\frac{Z'(t)}{Z(t)} \ge -\frac{1}{L^{1/\alpha}}. \tag{2.6}$$

Hence, by (2.5), we conclude that, for all $t \ge t_2$,

$$-L^{-1} \le \omega(t)\pi^{\alpha}(t) \le 0. \tag{2.7}$$

From (2.6) and $K \le \psi$, we obtain
$$\frac{Z'(t)}{Z(t)} \ge -\frac{1}{L^{1/\alpha}(r(t)\psi(x(t)))^{1/\alpha}\pi(t)} \ge -\frac{1}{(LK)^{1/\alpha}r^{1/\alpha}(t)\pi(t)}.$$
Thus, we have
$$\Big(\frac{Z(t)}{m(t)}\Big)' = \frac{Z'(t)m(t) - Z(t)m'(t)}{m^2(t)} \ge -\frac{Z(t)}{m^2(t)}\Big[\frac{m(t)}{(LK)^{1/\alpha}r^{1/\alpha}(t)\pi(t)} + m'(t)\Big] \ge 0.$$
Hence, the function Z/m is non-decreasing, and so
$$\begin{aligned} x(t) &= Z(t) - p(t)x(\tau(t)) \ge Z(t) - p(t)Z(\tau(t)) \\ &\ge Z(t) - p(t)\frac{m(\tau(t))}{m(t)}Z(t) = \big(1 - p(t)\frac{m(\tau(t))}{m(t)}\big)Z(t). \end{aligned}$$
Differentiation of (2.5) yields
$$\begin{aligned} \omega'(t) = \Big(&(r(t)\psi(x(t))(-Z'(t))^{\alpha-1}Z'(t))'Z^{\alpha}(t) \\ &- \alpha r(t)\psi(x(t))(-Z'(t))^{\alpha-1}Z'(t)Z^{\alpha-1}(t)Z'(t)\Big)/Z^{2\alpha}(t). \end{aligned}$$
It follows from the latter equality and (2.4) that
$$\begin{aligned} \omega'(t) \le &-kq(t)\Big(1 - p(\sigma(t))\frac{m(\tau(\sigma(t)))}{m(\sigma(t))}\Big)^{\alpha}\frac{Z^{\alpha}(\sigma(t))}{Z^{\alpha}(t)} \\ &- \frac{\alpha r(t)\psi(x(t))(-Z'(t))^{\alpha-1}Z'(t)Z^{\alpha-1}(t)Z'(t)}{Z^{2\alpha}(t)}. \end{aligned} \tag{2.8}$$
Thus, by (2.5) and (2.8), we have
$$\omega'(t) + kq(t)\Big(1 - p(\sigma(t))\frac{m(\tau(\sigma(t)))}{m(\sigma(t))}\Big)^{\alpha} + \frac{\alpha L^{1/\alpha}}{r^{1/\alpha}(t)}(-\omega(t))^{(\alpha+1)/\alpha} \le 0. \tag{2.9}$$
Multiplying (2.9) by $\pi^{\alpha}(t)$ and integrating the resulting inequality from t_3 ($t_3 > t_2$) to t, we deduce that
$$\begin{aligned} &\pi^{\alpha}(t)\omega(t) - \pi^{\alpha}(t_3)\omega(t_3) + \alpha\int_{t_3}^{t} r^{-1/\alpha}(s)\pi^{\alpha-1}(s)\omega(s)\mathrm{d}s \\ &+ k\int_{t_3}^{t} q(s)\left(1 - p(\sigma(s))\frac{m(\tau(\sigma(s)))}{m(\sigma(s))}\right)^{\alpha}\pi^{\alpha}(s)\mathrm{d}s \\ &+ \alpha L^{1/\alpha}\int_{t_3}^{t}\frac{\pi^{\alpha}(s)}{r^{1/\alpha}(s)}(-\omega(s))^{(\alpha+1)/\alpha}\mathrm{d}s \le 0. \end{aligned} \tag{2.10}$$
Let $p := (\alpha+1)/\alpha$, $q := \alpha + 1$,
$$\begin{aligned} a &:= L^{1/(\alpha+1)}(\alpha+1)^{\alpha/(\alpha+1)}\pi^{\alpha^2/(\alpha+1)}(t)\omega(t), \\ b &:= L^{-1/(\alpha+1)}\frac{\alpha}{(\alpha+1)^{\alpha/(\alpha+1)}}\pi^{-1/(\alpha+1)}(t). \end{aligned}$$
Using Young's inequality,
$$|ab| \le \frac{1}{p}|a|^p + \frac{1}{q}|b|^q, \quad \text{where} \quad a,\ b \in \mathbb{R},\ p > 1,\ q > 1,\ \frac{1}{p} + \frac{1}{q} = 1,$$
we have
$$-\alpha\pi^{\alpha-1}(t)\omega(t) \le \alpha L^{1/\alpha}\pi^{\alpha}(t)(-\omega(t))^{(\alpha+1)/\alpha} + \big(\frac{\alpha}{\alpha+1}\big)^{\alpha+1}\frac{1}{L\pi(t)},$$

and hence

$$-\alpha\frac{\pi^{\alpha-1}(t)\omega(t)}{r^{1/\alpha}(t)} \le \alpha L^{1/\alpha}\frac{\pi^{\alpha}(t)(-\omega(t))^{(\alpha+1)/\alpha}}{r^{1/\alpha}(t)} + \big(\frac{\alpha}{\alpha+1}\big)^{\alpha+1}\frac{1}{L\pi(t)r^{1/\alpha}(t)}.$$

Therefore, it follows from (2.7) and (2.10) that

$$\begin{aligned}
&\int_{t_3}^{t}\Big[kq(s)\pi^{\alpha}(s)\Big(1-p(\sigma(s))\frac{m(\tau(\sigma(s)))}{m(\sigma(s))}\Big)^{\alpha} - \big(\frac{\alpha}{\alpha+1}\big)^{\alpha+1}\frac{1}{L\pi(s)r^{1/\alpha}(s)}\Big]\mathrm{d}s\\
&\le \pi^{\alpha}(t_3)\omega(t_3) - \pi^{\alpha}(t)\omega(t)\\
&\le L^{-1} + \pi^{\alpha}(t_3)\omega(t_3),
\end{aligned}$$

which contradicts (2.3). This completes the proof. $\square$

Remark 2.2. A function m in Theorem 2.1 can be obtained by setting $m(t) := \pi(t)$ in the case $LK \ge 1$.

It may happen that the restriction $\psi(x) \ge K > 0$ is not satisfied and Theorem 2.1 cannot be applied. For example when

$$\psi(x) = \frac{1}{x^2+1},$$

in which case the following result proves to be useful.

Theorem 2.3. *Assume that conditions* (A1)–(A4) *and* (1.2) *hold. Let ψ be non-increasing for all $x > 0$, and non-decreasing for all $x < 0$. Suppose further that there exist two functions $\rho, m \in C^1(\mathbb{I}, (0,\infty))$ such that, for any fixed constant $l > 0$,*

$$\frac{m(t)}{(L\psi(l))^{1/\alpha}r^{1/\alpha}(t)\pi(t)} + m'(t) \le 0, \quad 1 - p(t)\frac{m(\tau(t))}{m(t)} > 0, \tag{2.11}$$

and such that conditions (2.2) *and* (2.3) *are satisfied. Then equation* (1.1) *is oscillatory.*

Proof. As in the proof of Theorem 2.1, we only need to prove the case where $Z'(t) < 0$. In this case, there exists a constant $l > 0$ such that $0 < x(t) \le Z(t) \le l$. Using the monotonicity of ψ, we deduce that $\psi(x) \ge \psi(l)$. Along the same lines as in Theorem 2.1, we conclude that

$$\frac{Z'(t)}{Z(t)} \ge -\frac{1}{L^{1/\alpha}(r(t)\psi(x(t)))^{1/\alpha}\pi(t)} \ge -\frac{1}{(L\psi(l))^{1/\alpha}r^{1/\alpha}(t)\pi(t)}.$$

Hence, we have

$$\Big(\frac{Z(t)}{m(t)}\Big)' = \frac{Z'(t)m(t) - Z(t)m'(t)}{m^2(t)} \ge -\frac{Z(t)}{m^2(t)}\Big[\frac{m(t)}{(L\psi(l))^{1/\alpha}r^{1/\alpha}(t)\pi(t)} + m'(t)\Big] \ge 0.$$

Thus, the function Z/m is non-decreasing. The remainder of the proof is similar to that of Theorem 2.1, and hence is omitted. $\square$

3. Examples and discussion

The following examples illustrate possible applications of theoretical results obtained in the previous section.

Example 3.1. For $t \geq 1$, consider the second-order neutral delay equation

$$\Big(t^2 \frac{x^2(t)+2}{x^2(t)+1}\Big(x(t)+\frac{\tau^2(t)}{4t^2}x(\tau(t))\Big)'\Big)' + tx(\sigma(t)) = 0. \tag{3.1}$$

Here $r(t) = t^2$, $\psi(x) = (x^2+2)/(x^2+1)$, $p(t) = (\tau(t)/2t)^2$, $f(x) = x$, and $q(t) = t$. Then, $1 \leq \psi(x) \leq 2$ and we can fix $k = 1$, $K = 1$, and $L = 1/2$. Let $m(t) = t^{-2}$ and $\rho(t) = 1$. It is not difficult to verify that all assumptions of Theorem 2.1 are satisfied. Hence, equation (3.1) is oscillatory.

Example 3.2. For $t \geq 1$, consider the second-order neutral delay equation

$$\Big(\frac{t^2}{x^2(t)+1}\Big(x(t)+\frac{1}{t}x\big(\frac{t}{2}\big)\Big)'\Big)' + tx\big(\frac{t}{2}\big) = 0. \tag{3.2}$$

Here $r(t) = t^2$, $\psi(x) = 1/(x^2+1)$, $p(t) = 1/t$, $f(x) = x$, $\tau(t) = \sigma(t) = t/2$, and $q(t) = t$. Then, $\psi(x) \leq 1$ and we can fix $k = 1$ and $L = 1$. Let $m(t) = t^{-1-l^2}$ and $\rho(t) = 1$. It is not hard to see that all conditions of Theorem 2.3 are satisfied. Therefore, equation (3.2) is oscillatory.

In this article, using a Riccati substitution, we have established new oscillation criteria for second-order neutral delay differential equation (1.1) assuming that (1.2) is satisfied. We stress that the study of oscillatory behavior of equation (1.1) in the case (1.2) brings additional difficulties. One of the principal difficulties one encounters lies in the fact that if x is an eventually positive solution of (1.1), then the inequality

$$x(t) \geq (1-p(t))Z(t)$$

does not hold when (1.2) is satisfied, cf., for instance, [9, 12]. Contrary to [9], we do not need in our oscillation theorems restrictive conditions (1.4); see Examples 3.1 and 3.2 which, in a certain sense, is a significant improvement compared to the results in the cited papers. However, this improvement has been achieved at the cost of imposing conditions (2.1) or (2.11). The question regarding the study of oscillatory properties of equation (1.1) with other methods that do not require assumptions (2.1) and (2.11) remains open at the moment.

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