Electronic Journal of Differential Equations, Vol. 2015 (2015), No. 247, pp. 1–14.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu

PERSISTENCE AND EXTINCTION FOR STOCHASTIC LOGISTIC MODEL WITH LÉVY NOISE AND IMPULSIVE PERTURBATION

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Abstract. This article investigates a stochastic logistic model with Lévy noise and impulsive perturbation. In the model, the impulsive perturbation and Lévy noise are taken into account simultaneously. This model is new and more feasible and more accordance with the actual. The definition of solution to a stochastic differential equation with Lévy noise and impulsive perturbation is established. Based on this definition, we show that our model has a unique global positive solution and obtains its explicit expression. Sufficient conditions for extinction are established as well as nonpersistence in the mean, weak persistence and stochastic permanence. The threshold between weak persistence and extinction is obtained.

1. Introduction

Persistence and extinction of logistic model is one of the important topics in mathematical biology. Many scholars have investigated the topic for the classical stochastic logistic model with Lévy noise (see [6, 5, 16, 36]):

$$dx(t) = x(t)(r(t) - a(t)x(t))dt + \sigma(t)x(t)dB(t) + x(t^-)\int_{\mathbb{Y}} \gamma(u)\tilde{N}(dt, du), \quad (1.1)$$

where $x(t)$ is the population size, $B(t)$ is a standard Brownian motion, $x(t^-) = \lim_{s\uparrow t} x(s)$, $N(dt, du)$ is a real-valued Poisson counting measure with characteristic measure λ on a measurable subset $\mathbb{Y}$ of $\mathbb{R}_+ = [0, \infty)$ with $\lambda(\mathbb{Y}) < \infty, \tilde{N}(dt, du) = N(dt, du) - \lambda(du)dt$ and $\gamma(u) > -1$. There is an important and interesting literature about stochastic differential equation with jumps (see [1, 2, 3, 8, 32]). To simulate the phenomena well in reality, e.g., epidemics, earthquakes, hurricanes, ocean red tide and so on, Lots of authors have introduced the Lévy noise into biological model (see [15, 24, 25, 26, 28, 33]).

However, in the real world, owing to some natural and man-made factors, such as fire, drought, crop-dusting, deforestation, hunting, harvesting, etc., the growth of species often undergoes some discrete changes of relatively short time interval at some fixed times. These phenomena cannot be considered continually, so in this case, system (1.1) cannot describe these phenomena. Introducing the impulsive

2010 *Mathematics Subject Classification.* 64H10, 60J75, 35R12.
Key words and phrases. Logistic equation; Lévy noise; impulsive perturbation; stochastic permanence.

Submitted September 5, 2014. Published September 23, 2015.

effects, which can not boil down to Lévy noise from its definition, into the model may describe such phenomena well, see [4, 10].

Recently, several authors have incorporated the impulsive perturbation into the stochastic population dynamics and some results on dynamical behavior for such systems have been reported (see [17, 18, 19, 27, 34]) and the references therein. However, so far as we know, there are no papers published which consider the impulsive perturbation in stochastic population model with Lévy noise. Motivated by these arguments presented above, we will consider the following stochastic logistic model with Lévy noise and impulsive perturbation:

$$
\begin{gathered}
dx(t) = x(t)(r(t) - a(t)x(t))dt + \sigma(t)x(t)dB(t) + x(t^-)\int_{\mathbb{Y}} \gamma(u)\tilde{N}(dt, du), \\
t \neq t_k, \quad K \in \mathbb{N} \\
x(t_k^+) - x(t_k) = h_k x(t_k), \quad k \in \mathbb{N}
\end{gathered}
\tag{1.2}
$$

where N denotes the set of positive integers, $0 < t_1 < t_2 \dots, \lim_{k\to\infty} t_k = \infty$, $r(t), a(t)$ and $\sigma(t)$ are continuous and boundeded function on $\mathbb{R}_+$ and $\inf_{t\in\mathbb{R}_+} a(t) > 0$. Here, we assume that $B(t)$ is independent of $N(dt, du)$. Other parameters are defined and required as before.

The main contributions of this paper are listed as follows:

(1) The model includes two types of environmental noise and impulsive perturbation which is more grounded in the real world. We establish the definition of solution to a stochastic differential equation with Lévy noise and impulsive perturbation. The explicit solution for the model is given in Theorem 2.3;

(2) We give sufficient conditions for extinction, nonpersistence in the mean, weak persistence and stochastic permanence of the solution. In addition, the threshold between weak persistence and extinction is obtained.

(3) The effects of the impulsive perturbation on the population are investigated in detail, see Remark 3.10, examples and figures. Our results imply that the impulsive perturbation has great impacts on the model.

For model (1.2) we assume the following conditions:

- (A1) As far as biological meanings is concerned, we consider $1 + h_k > 0,\ k \in \mathbb{N}$. When $h_k > 0$, is satisfied, the perturbation turn to be the description process of planting of species and harvesting if not $h_k < 0$.
- (A2) For each $m > 0$ there exists L_m such that $\int_{\mathbb{Y}} |H(x, u) - H(y, u)|^2 \lambda(du) \leq L_m|x - y|^2$ where $H(x, u) = \gamma(u)x(t^-)$ with $|x| \vee |y| \leq m$.
- (A3) There exists a constant $c > 0$ such that $\int_{\mathbb{Y}} (\ln(1 + \gamma(u))^2 \lambda(du) \leq c$.

For simplicity, we define the notation:

$$
\langle f(t)\rangle = \frac{1}{t}\int_0^t f(s)ds, \quad f_* = \liminf_{t\to\infty} f(t), \quad f^* = \limsup_{t\to\infty} f(t).
$$

If $\nu(t)$ is a continuous bounded function on $\mathbb{R}_+$, define $\hat{\nu} = \sup_{t\in\mathbb{R}_+} \nu(t)$ and $\check{\nu} = \inf_{t\in\mathbb{R}_+} \nu(t)$. The following definitions are commonly used and we list them here.

1. The population $x(t)$ is said to be extinct if $\lim_{t\to\infty} x(t) = 0$.
2. The population $x(t)$ is said to be nonpersistence in the mean [29] if $\limsup_{t\to\infty}\langle x(t)\rangle = 0$.
3. The population $x(t)$ is said to be weak persistence [7] if $\limsup_{t\to\infty} x(t) > 0$.

4. The population $x(t)$ is said to be stochastic permanence [16] if for an arbitrary $\varepsilon > 0$, there are constants $\beta > 0, \alpha > 0$ such that

$$\liminf_{t\to\infty} \mathcal{P}\{x(t) \geq \beta\} \geq 1-\varepsilon \quad \text{and} \quad \liminf_{t\to\infty} \mathcal{P}\{x(t) \leq \alpha\} \geq 1-\varepsilon.$$

2. Positive and global solutions

Throughout this paper, let $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, \mathcal{P})$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions and $B(t)$ denotes a standard Brownian motion defined on this probability space.

Definition 2.1. Consider the stochastic differential equation with Lévy noise and impulsive perturbation:

$$\begin{gathered} dx(t) = f(t,x(t),\omega)dt + g(t,x(t),\omega)dB(t) + \int_{\mathbb{Y}} \gamma(t,x(t^-),u,\omega)\tilde{N}(dt,du), \\ t \neq t_k, k \in \mathbb{N} \\ x(t_k^+) - x(t_k) = h_k x(t_k), \quad k \in \mathbb{N} \end{gathered} \tag{2.1}$$

with initial condition $x(0) = x_0$. Here, $x(t^-) = \lim_{s\uparrow t} x(s)$, $N(dt,du)$ is a real-valued Poisson counting measure with characteristic measure λ on a measurable subset $\mathbb{Y}$ of $\mathbb{R}_+$ with $\lambda(\mathbb{Y}) < \infty, \tilde{N}(dt,du) = N(dt,du) - \lambda(du)dt$ and $B(t)$ is independent of N. A stochastic process $x(t)$, $t \in \mathbb{R}_+$, is said to be a solution of (2.1) if

(i) $x(t)$ is $\mathscr{F}_t$-adapted on $(0,t_1)$ and each interval $(t_k,t_{k+1}) \in \mathbb{R}_+, k \in \mathbb{N}$; $f(t,x) : \mathbb{R}_+ \times R \times \Omega \to R$, $g(t,x) : \mathbb{R}_+ \times R \times \Omega \to R$ and $\gamma : \mathbb{R}_+ \times R \times \mathbb{Y} \times \Omega \to R$ are jointly measurable and $\mathscr{F}_t$-adapted where, furthermore, γ is $\mathscr{F}_t$-predictable;

(ii) For each $t_k, k \in \mathbb{N}, x(t_k^+) = \lim_{t\to t_k^+} x(t)$ and $x(t_k^-) = \lim_{t\to t_k^-} x(t)$ exist and $x(t_k) = x(t_k^-)$ with probability one;

(iii) For almost all $t \in [0,t_1]$ and $k \in \mathbb{N}, x(t)$, $x(t)$ satisfies the integral equation

$$\begin{aligned} x(t) = x(0) &+ \int_0^t f(s,x(s),\omega) + \int_0^t g(s,x(s),\omega)dB(s) \\ &+ \int_0^t \int_{\mathbb{Y}} \gamma(s,x(s^-),u,\omega)\tilde{N}(ds,du). \end{aligned} \tag{2.2}$$

And for almost all $t \in (t_k,t_{k+1}], k \in \mathbb{N}, x(t)$ satisfies

$$\begin{aligned} x(t) = x(t_k^+) &+ \int_{t_k}^t f(s,x(s),\omega) + \int_{t_k}^t g(s,x(s),\omega)dB(s) \\ &+ \int_{t_k}^t \int_{\mathbb{Y}} \gamma(s,x(s^-),u,\omega)\tilde{N}(ds,du). \end{aligned} \tag{2.3}$$

Moreover, $x(t)$ satisfies the impulsive conditions at each $t = t_k, k \in \mathbb{N}$ with probability one.

Remark 2.2. Now let us clarify the derivation procedure of Definition 1. Firstly, noticing that the stochastic differential equation with jumps and impulsive perturbation (2.1) becomes the following stochastic differential equation with jumps:

$$dx(t) = f(t,x(t),\omega)dt + g(t,x(t),\omega)dB(t) + \int_{\mathbb{Y}} \gamma(t,x(t^-),u,\omega)\tilde{N}(dt,du)$$

on interval $[0, t_1]$ and each interval $(t_k, t_{k+1}] \in \mathbb{R}_+, k \in \mathbb{N}$. In the light of the classical definition of a solution of stochastic differential equation with jumps (see [32, page 76]), condition (i), Equations (2.2) and (2.3) should be satisfied. Second, since there exists impulsive perturbation in (2.1), then the condition (ii) and (iii) should be satisfied. According to the two facts above, the Definition 1 is proposed.

Theorem 2.3. *Under assumptions* (A1)–(A2)*, for any initial value $x(0) = x_0 > 0$, there is a unique solution $x(t)$ to* (1.2) *a.s., which is global and represented by*

$$x(t) = \frac{\prod_{0<t_k<t}(1+h_k)\phi(t)}{\frac{1}{x_0} + \int_0^t \prod_{0<t_k<s}(1+h_k)a(s)\phi(s)ds},$$

where

$$\begin{aligned}\phi(t) = \exp\Big(&\int_0^t \Big[r(\zeta) - \frac{1}{2}\sigma^2(\zeta) + \int_{\mathbb{Y}} (\ln(1+\gamma(u)) - \gamma(u))\lambda(du)\Big]d\zeta \\ &+ \int_0^t \sigma(\zeta)dB(\zeta) + \int_0^t \int_{\mathbb{Y}} \ln(1+\gamma(u))\tilde{N}(d\zeta, du)\Big).\end{aligned}$$

Proof. Consider the stochastic differential equation with jumps

$$\begin{aligned}dy(t) = y(t)\Big[r(t) - \prod_{0<t_k<t}(1+h_k)a(t)y(t)\Big] + \sigma(t)y(t)dB(t) \\ + y(t^-)\int_{\mathbb{Y}} \gamma(u)\tilde{N}(dt, du)\end{aligned} \tag{2.4}$$

with initial value $y(0) = x_0$. Then (2.4) has the explicit solution [6, Lemma 4.2]

$$y(t) = \frac{\phi(t)}{\frac{1}{x_0} + \int_0^t \prod_{0<t_k<s}(1+h_k)a(s)\phi(s)ds},$$

where

$$\begin{aligned}\phi(t) = \exp\Big(&\int_0^t \Big[r(\zeta) - \frac{1}{2}\sigma^2(\zeta) + \int_{\mathbb{Y}} (\ln(1+\gamma(u)) - \gamma(u))\lambda(du)\Big]d\zeta \\ &+ \int_0^t \sigma(\zeta)dB(\zeta) + \int_0^t \int_{\mathbb{Y}} \ln(1+\gamma(u))\tilde{N}(d\zeta, du)\Big).\end{aligned}$$

Now let

$$x(t) = \prod_{0<t_k<t}(1+h_k)y(t).$$

We show that $x(t)$ is the solution (1.2). On the interval $[0, t_1)$ and each interval $(t_k, t_{k+1}) \in \mathbb{R}_+, k \in \mathbb{N}$, we obtain

$$\begin{aligned}dx(t) &= d\Big[\prod_{0<t_k<t}(1+h_k)y(t)\Big] \\ &= \prod_{0<t_k<t}(1+h_k)dy(t) \\ &= \prod_{0<t_k<t}(1+h_k)y(t)\Big[r(t) - \prod_{0<t_k<t}(1+h_k)a(t)y(t)\Big] \\ &\quad + \prod_{0<t_k<t}(1+h_k)\sigma(t)y(t)dB(t) + \prod_{0<t_k<t}(1+h_k)y(t^-)\int_{\mathbb{Y}} \gamma(u)\tilde{N}(dt, du)\end{aligned}$$

$$
\begin{aligned}
&= \prod_{0<t_k<t}(1+h_k)y(t)\Big[r(t)-\prod_{0<t_k<t}(1+h_k)a(t)y(t)\Big] \\
&\quad + \prod_{0<t_k<t}(1+h_k)\sigma(t)y(t)dB(t) + \prod_{0<t_k<t^-}(1+h_k)y(t^-)\int_{\mathbb{Y}}\gamma(u)\tilde{N}(dt,du) \\
&= x(t)(r(t)-a(t)x(t))dt + \sigma(t)x(t)dB(t) + x(t^-)\int_{\mathbb{Y}}\gamma(u)\tilde{N}(dt,du).
\end{aligned}
$$

Moreover, for every $k \in \mathbb{N}$ and $t_k \in [0,\infty)$,

$$
\begin{aligned}
x(t_k^+) &= \lim_{t\to t_k^+}\prod_{0<t_j<t}(1+h_j)y(t) = \prod_{0<t_j\le t_k}(1+h_j)y(t_k^+) \\
&= (1+h_k)\prod_{0<t_j<t_k}(1+h_j)y(t_k) = (1+h_k)x(t_k).
\end{aligned}
$$

In addition,

$$
\begin{aligned}
x(t_k^-) &= \lim_{t\to t_k^-}\prod_{0<t_j<t}(1+h_j)y(t) = \prod_{0<t_j<t_k}(1+h_j)y(t_k^-) \\
&= \prod_{0<t_j<t_k}(1+h_j)y(t_k) = x(t_k).
\end{aligned}
$$

Now let us prove the uniqueness of the solution. For $t \in [0, t_1]$, equation (1.2) becomes the classical equation

$$
dx(t) = x(t)(r(t)-a(t)x(t))dt + \sigma(t)x(t)dB(t) + x(t^-)\int_{\mathbb{Y}}\gamma(u)\tilde{N}(dt,du), \quad (2.5)
$$

for $t \in (0, t_1)$. Since the coefficients of (2.5) are locally Lipschitz continuous, by the theory of stochastic differential equation with jumps [2, 8], the solution of (2.5) is unique. For $t \in (t_k, t_{k+1}], k \in \mathbb{N}$, equation (1.2) becomes

$$
dx(t) = x(t)(r(t)-a(t)x(t))dt + \sigma(t)x(t)dB(t) + x(t^-)\int_{\mathbb{Y}}\gamma(u)\tilde{N}(dt,du) \quad (2.6)
$$

for $t \in (t_k, t_{k+1}]$. Note that the coefficients of (2.6) are also locally Lipschitz continuous; then the solution of (2.6) is also unique. Therefore, the solution of model (1.2) is unique. $\square$

3. Persistence and extinction for model (1.2)

For later applications, we introduce the following lemmas.

Lemma 3.1. *Let $M(t)$, $t \ge 0$ be a local martingale at time 0 and define*

$$
\rho_M(t) = \int_0^t \frac{d\langle M\rangle(s)}{(1+s)^2}, \quad t \ge 0,
$$

where $\langle M\rangle(t) = \langle M, M\rangle(t)$ is Meyer's angle bracket process. Then $\lim_{t\to\infty}\frac{M(t)}{t} = 0$ a.s. provided that $\lim_{t\to\infty}\rho_M(t) < \infty$ a.s. [13].

Theorem 3.2. *Let assumptions* (A1)–(A3) *hold. Suppose that $x(t)$ is a solution of* (1.2)*, then*

$$
\limsup_{t\to\infty} t^{-1}\ln x(t) \le \limsup_{t\to\infty} t^{-1}\Big[\sum_{0<t_k<t}\ln(1+h_k) + \int_0^t (r(s)-0.5\sigma^2(s))ds\Big]
$$

$$-\int_{\mathbb{Y}}(\gamma(u)-\ln(1+\gamma(u)))\lambda(du)=g^*, \quad a.s.,$$

where

$$g(t)=t^{-1}\Big[\sum_{0<t_k<t}\ln(1+h_k)+\int_0^t(r(s)-0.5\sigma^2(s))ds\Big]-\int_{\mathbb{Y}}(\gamma(u)-\ln(1+\gamma(u)))\lambda(du).$$

In particular, if $g^*<0$, then $\lim_{t\to\infty}x(t)=0$ a.s.

Proof. By Itô's formula [9], we derive from (2.4) that

$$\begin{aligned}
d\ln y(t)&=\Big[r(t)-\frac{\sigma^2(t)}{2}-\prod_{0<t_k<t}(1+h_k)a(t)y(t)+\int_{\mathbb{Y}}(\ln(1+\gamma(u))\\
&\quad-\gamma(u))\lambda(du)\Big]dt+\sigma(t)dB(t)+\int_{\mathbb{Y}}\ln(1+\gamma(u))\tilde N(dt,du)\\
&=\Big[r(t)-\frac{\sigma^2(t)}{2}-a(t)x(t)+\int_{\mathbb{Y}}(\ln(1+\gamma(u))-\gamma(u))\lambda(du)\Big]dt\\
&\quad+\sigma(t)dB(t)+\int_{\mathbb{Y}}\ln(1+\gamma(u))\tilde N(dt,du).
\end{aligned}$$

Integrating both sides from 0 to t, we have

$$\begin{aligned}
\ln y(t)-\ln y(0)&=\int_0^t\Big[r(s)-0.5\sigma^2(s)-a(s)x(s)\Big]ds+t\int_{\mathbb{Y}}(\ln(1+\gamma(u))\\
&\quad-\gamma(u))\lambda(du)+M_1(t)+M_2(t)
\end{aligned}\tag{3.1}$$

where

$$M_1(t)=\int_0^t\sigma(s)dB(s),\quad M_2(t)=\int_0^t\int_{\mathbb{Y}}\ln(1+\gamma(u))\tilde N(ds,du).$$

The quadratic variation of $M_1(t)$ is $\langle M_1(t),M_1(t)\rangle=\int_0^t\sigma^2(s)ds\le\hat{\sigma^2}t$. By the strong law of large numbers for martingales leads to

$$\lim_{t\to\infty}\frac{M_1(t)}{t}=0,\quad a.s.\tag{3.2}$$

Under assumption (A2), $\langle M_2\rangle(t)=\int_0^t\int_{\mathbb{Y}}(\ln(1+\gamma(u)))^2\lambda(du)ds\le ct$. We derive

$$\int_0^t\frac{1}{(1+s)^2}ds=\frac{t}{t+1}<\infty.$$

By Lemma 3.1, we then obtain

$$\lim_{t\to\infty}\frac{M_2(t)}{t}=0,\quad \text{a.s.}\tag{3.3}$$

On the other hand, it follows from (3.1) that

$$\begin{aligned}
&\sum_{0<t_k<t}\ln(1+h_k)+\ln y(t)-\ln y(0)\\
&=\sum_{0<t_k<t}\ln(1+h_k)+\int_0^t\Big[r(s)-0.5\sigma^2(s)-a(s)x(s)\Big]ds\\
&\quad+t\int_{\mathbb{Y}}(\ln(1+\gamma(u))-\gamma(u))\lambda(du)+M_1(t)+M_2(t).
\end{aligned}$$

Thus

$$\begin{aligned}\ln x(t)-\ln x(0)=\sum_{0<t_k<t}\ln(1+h_k)+\int_0^t\Big[r(s)-0.5\sigma^2(s)-a(s)x(s)\Big]ds\\ -t\int_{\mathbb{Y}}(\gamma(u)-\ln(1+\gamma(u)))\lambda(du)+M_1(t)+M_2(t).\end{aligned}\tag{3.4}$$

Using (3.2) and (3.3), we immediately obtain the desired assertion. □

Theorem 3.3. *Under assumptions* (A1)–(A3)*, if* $g^*=0$*, then the population model* (1.2) *is nonpersistence in the mean a.s.*

Proof. In view of $g^*=0$, there exists a constant T such that for all $\varepsilon>0$,

$$\begin{aligned}&t^{-1}\Big[\sum_{0<t_k<t}\ln(1+b_k)+\int_0^t b(s)ds\Big]-\int_{\mathbb{Y}}[\gamma(u)-\ln(1+\gamma(u))]\lambda(du)\\ &+\frac{M_1(t)}{t}+\frac{M_2(t)}{t}<\varepsilon,\quad t>T.\end{aligned}$$

Substituting this inequality into (3.4) yields

$$\begin{aligned}\ln x(t)&\le\ln x(0)+\sum_{0<t_k<t}\ln(1+h_k)+\int_0^t(b(s)-a(s)x(s))ds\\ &\quad-\int_0^t\int_{\mathbb{Y}}[\gamma(u)-\ln(1+\gamma(u))]\lambda(du)ds+M_1(t)+M_2(t)\\ &<\varepsilon t-\int_0^t a(s)x(s)ds\end{aligned}$$

for all $t>T$. The rest of proof is similar to [20, Theorem 3] and hence is omitted. □

Theorem 3.4. *Under the assumptions* (A1)–(A3)*, if* $g^*>0$*, then the population* $x(t)$ *modeled by* (1.2) *is weak persistence a.s.*

Proof. If this assertion is not true, let $F=\{\limsup_{t\to\infty}x(t)=0\}$ and suppose $\mathcal{P}(F)>0$. In the light of (3.4),

$$\begin{aligned}&t^{-1}\Big[\ln x(t)-\ln x(0)\Big]\\ &=t^{-1}\Big[\sum_{0<t_k<t}\ln(1+b_k)+\int_0^t(b(s)-a(s)x(s))ds\\ &\quad-\int_0^t\int_{\mathbb{Y}}[\gamma(u)-\ln(1+\gamma(u))]\lambda(du)ds\Big]+\frac{M_1(t)}{t}+\frac{M_2(t)}{t}.\end{aligned}\tag{3.5}$$

On the other hand, for for all $\omega\in F$, we have $\lim_{t\to\infty}x(t,\omega)=0$. Therefore, substituting (3.2) and (3.3) into (3.5), one can deduce the contradiction

$$0\ge\limsup_{t\to\infty}[t^{-1}\ln x(t,\omega)]=g^*>0.$$

□

Remark 3.5. Theorems 3.2–3.4 have a direct biological explanation. It is obvious to see that the extinction and persistence of population $x(t)$ modeled by (1.2) largely rely on g^*. Under the assumption (A1)–(A3), if $g^*>0$, the population $x(t)$ will be weakly persistent; Under the assumption (A1)–(A3), if $g^* < 0$, the population $x(t)$ will go to extinction. That is to say, under the assumption (A1)–(A3), g^* is the threshold between weak persistence and extinction for the population $x(t)$.

When it comes to the study of population system, the role of stochastic permanence indicating the eternal existence of the population, can never be ignorant with its theoretical and practical significance. And its importance has catched the eyes of scientists all over the world. So now let us show that $x(t)$ modeled by (1.2) is stochastic permanent in some cases. We define the assumption

- (A4) There are two positive constants m and M such that $m \le \prod_{0<t_k<t}(1+h_k) \le M$ for all $t > 0$.

Remark 3.6. Assumption (A4) is easy to be satisfied. For example, if $h_k = e^{\frac{(-1)^{k+1}}{k}} - 1$, then $e^{0.5} < \prod_{0<t_k<t}(1+h_k) < e$ for all $t > 0$. Thus $1 \le \prod_{0<t_k<t}(1+h_k) \le e$ for all $t > 0$.

Theorem 3.7. *Under assumptions* (A1), (A2), (A4). *If*

$$\big(r(t) - 0.5\sigma^2(t)\big)_* - \int_{\mathbb{Y}} \gamma(u)\lambda(du) > 0$$

and $\gamma(u) \ge 0$*, then the population* $x(t)$ *represented by* (1.2) *will be stochastic permanence.*

Proof. First, we claim that for arbitrary $\varepsilon > 0$, there is constant $\beta > 0$ such that $\liminf_{t\to\infty} \mathcal{P}\{x(t) \ge \beta\} \ge 1 - \varepsilon$.

Define $V_1(y) = 1/y$ for $y > 0$. Applying Itô's formula to (2.4) we can obtain that

$$\begin{aligned} dV_1(y) &= -V_1(y)\Big[r(t) - \prod_{0<t_k<t}(1+h_k)a(t)y(t)\Big]dt \\ &\quad + V_1(y)\int_{\mathbb{Y}}\Big(\frac{1}{1+\gamma(u)} - 1 + \gamma(u)\Big)\lambda(du)dt + V_1(y)\sigma^2(t)dt \\ &\quad - V_1(y)\sigma(t)dw(t) + V_1(y)\int_{\mathbb{Y}}\Big(\frac{1}{1+\gamma(u)} - 1\Big)\tilde{N}(dt,du). \end{aligned}$$

Since $\big(r(t)-0.5\sigma^2(t)\big)_* - \int_{\mathbb{Y}}\gamma(u)\lambda(du) > 0$, we can choose a sufficient small constant $0 < \kappa < 1$ such that $r(t) - 0.5\sigma^2(t) - \int_{\mathbb{Y}}\gamma(u)\lambda(du) - 0.5\kappa\sigma^2(t) > 0$.

Define $V_2(y) = (1 + V_1(y))^\kappa$. Using Itô's formula again leads to

$$\begin{aligned} &dV_2 \\ &= \kappa(1+V_1(y))^{\kappa-1}dV_1 + 0.5\kappa(\kappa-1)(1+V_1(y))^{\kappa-2}V_1^2(y)\sigma^2(t)dt \\ &\quad + \int_{\mathbb{Y}}\Big[\Big(1 + V_1(y) + V_1(y)\Big(\frac{1}{(1+\gamma(u))} - 1\Big)\Big)^\kappa - (1+V_1(y))^\kappa - \kappa(1+V_1(y))^{\kappa-1} \\ &\quad \times V_1(y)\Big(\frac{1}{1+\gamma(u)} - 1\Big)\Big]\lambda(du)dt - \kappa(1+V_1(y))^{\kappa-1}V_1(y)\sigma(t)dw(t) \\ &\quad + \int_{\mathbb{Y}}\Big[\Big(1 + V_1(y) + V_1(y)\Big(\frac{1}{1+\gamma(u)} - 1\Big)\Big)^\kappa - (1+V_1(y))^\kappa\Big]\tilde{N}(dt,du) \\ &\le \kappa(1+V_1(y))^{\kappa-2}\Big\{ - (1+V_1(y))V_1(y)\Big[r(t) - Ma(t)y(t)\Big] + (1+V_1(y))V_1(y) \\ &\times \int_{\mathbb{Y}}\Big(\frac{1}{1+\gamma(u)} - 1 + \gamma(u)\Big)\lambda(du) + (1+V_1(y))V_1(y)\sigma^2(t) + 0.5(\kappa-1)V_1^2(y)\sigma^2(t) \end{aligned}$$

$$
\begin{aligned}
&- (1+V_1(y))V_1(y)\int_{\mathbb{Y}}\Big(\frac{1}{1+\gamma(u)}-1\Big)\lambda(du)\Big\}dt-\kappa(1+V_1(y))^{\kappa-1}V_1(y)\sigma(t)dw(t)\\
&+\int_{\mathbb{Y}}\Big[\Big(1+V_1(y)+V_1(y)\Big(\frac{1}{1+\gamma(u)}-1\Big)\Big)^{\kappa}-(1+V_1(y))^{\kappa}\Big]\tilde N(dt,du)\\
&=\kappa(1+V_1(y))^{\kappa-2}\Big\{-V_1^2(y)\Big[r(t)-0.5\sigma^2(t)-\int_{\mathbb{Y}}\gamma(u)\lambda(du)-0.5\kappa\sigma^2(t)\Big]\\
&\quad+V_1(y)\Big[Ma(t)-r(t)+\sigma^2(t)+\int_{\mathbb{Y}}\gamma(u)\lambda(du)\Big]+Ma(t)\Big\}dt\\
&+\int_{\mathbb{Y}}\Big[\Big(1+V_1(y)+V_1(y)\Big(\frac{1}{1+\gamma(u)}-1\Big)\Big)^{\kappa}-(1+V_1(y))^{\kappa}\Big]\tilde N(dt,du)\\
&\quad-\kappa(1+V_1(y))^{\kappa-1}V_1(y)\sigma(t)dw(t)
\end{aligned}
$$

for sufficiently large $t \ge T$. The first inequity follows from $\int_{\mathbb{Y}}\Big[\Big(1+V_1(x)+V_1(x)\Big(\frac{1}{(1+\gamma(u))^2}-1\Big)\Big)^{\kappa}-(1+V_1(x))^{\kappa}\Big]\lambda(du)\le 0$ for $\gamma(u)\ge 0$. Now, let $\eta>0$ be sufficiently small satisfy

$$
0<\eta/\kappa<r(t)-0.5\sigma^2(t)-\int_{\mathbb{Y}}\gamma(u)\lambda(du)-0.5\kappa\sigma^2(t).
$$

Define $V_3(y)=e^{\eta t}V_2(y)$. By Itô's formula

$$
\begin{aligned}
&dV_3(y(t))\\
&=\eta e^{\eta t}V_2(y)+e^{\eta t}dV_2(y)\\
&\le\kappa e^{\eta t}(1+V_1(y(t)))^{\kappa-2}\Big\{\eta(1+V_1(y))^2/\kappa-V_1^2(y)\Big[r(t)-0.5\sigma^2(t)\\
&\quad-\int_{\mathbb{Y}}\gamma(u)\lambda(du)-0.5\kappa\sigma^2(t)\Big]+V_1(y)\Big[Ma(t)-r(t)+\sigma^2(t)\\
&\quad+\int_{\mathbb{Y}}\gamma(u)\lambda(du)\Big]+Ma(t)\Big\}dt-e^{\eta t}\kappa(1+V_1(y))^{\kappa-1}V_1(y)\sigma(t)dw(t)\\
&\quad+e^{\eta t}\int_{\mathbb{Y}}\Big[\Big(1+V_1(y)+V_1(y)\Big(\frac{1}{1+\gamma(u)}-1\Big)\Big)^{\kappa}-(1+V_1(y))^{\kappa}\Big]\tilde N(dt,du)\\
&\le\kappa e^{\eta t}(1+V_1(y(t)))^{\kappa-2}\Big\{-V_1^2(y)\Big[\Big(r(t)-0.5\sigma^2(t)\Big)_*\\
&\quad-\int_{\mathbb{Y}}\gamma(u)\lambda(du)-\frac{\eta}{\kappa}-0.5\kappa\sigma^2(t)\Big]+V_1(y)\Big[\frac{2\eta}{\kappa}+M\hat a-\check r+\hat{\sigma^2}\\
&\quad+\int_{\mathbb{Y}}\gamma(u)\lambda(du)\Big]+M\hat a+\frac{\eta}{\kappa}\Big\}dt+e^{\eta t}\int_{\mathbb{Y}}\Big[\Big(1+V_1(y)\\
&\quad+V_1(y)\Big(\frac{1}{1+\gamma(u)}-1\Big)\Big)^{\kappa}-(1+V_1(y))^{\kappa}\Big]\tilde N(dt,du)\\
&=e^{\eta t}H(y)dt-e^{\eta t}\kappa(1+V_1(y))^{\kappa-1}V_1(y)\sigma(t)dw(t)+e^{\eta t}\int_{\mathbb{Y}}\Big[\Big(1+V_1(y)\\
&\quad+V_1(y)\Big(\frac{1}{1+\gamma(u)}-1\Big)\Big)^{\kappa}-(1+V_1(y))^{\kappa}\Big]\tilde N(dt,du)
\end{aligned}
$$

for $t\ge T$. Note that $H(y)$ is upper bounded in $\mathbb{R}_+$, namely $H=\sup_{y\in\mathbb{R}_+}H(y)<\infty$. Consequently,

$$
dV_3(y(t))
$$

$$
\begin{aligned}
&= He^{\eta t}dt - e^{\eta t}\kappa(1+V_1(y))^{\kappa-1}V_1(y)\sigma(t)dw(t) \\
&\quad + e^{\eta t}\int_{\mathbb{Y}}\Big[\Big(1+V_1(y)+V_1(y)\Big(\frac{1}{1+\gamma(u)}-1\Big)\Big)^{\kappa}-(1+V_1(y))^{\kappa}\Big]\tilde{N}(dt,du)
\end{aligned}
$$

for sufficiently large t. Integrating both sides of the above inequality and then taking eypectations gives

$$
E[V_3(y(t))] = E[e^{\eta t}(1+V_1(y(t)))^{\kappa}] \le e^{\eta T}(1+V_1(y(T)))^{\kappa} + \frac{H}{\eta}\Big(e^{\eta t}-e^{\eta T}\Big).
$$

That is to say

$$
\limsup_{t\to\infty} E[V_1^{\kappa}(y(t))] \le \limsup_{t\to\infty} E[(1+V_1(y(t)))^{\kappa}] < \frac{H}{\eta}.
$$

In other words, we have already shown that

$$
\limsup_{t\to\infty} E\Big[\frac{1}{y^{\kappa}(t)}\Big] \le \frac{H}{\eta}.
$$

Then

$$
\limsup_{t\to\infty} E[1/x^{\kappa}(t)] = \limsup_{t\to\infty}\Big[\prod_{0<t_k<t}(1+h_k)\Big]^{-\kappa}E[1/y^{\kappa}(t)] \le m^{-\kappa}\frac{H}{\eta} = H_1.
$$

So for $\varepsilon>0$, we set $\beta=\varepsilon^{1/\kappa}/H_1^{1/\kappa}$, by Chebyshev's inequality, one can derive that

$$
\mathcal{P}\{x(t)<\beta\} = \mathcal{P}\Big\{\frac{1}{x^{\kappa}(t)} > \frac{1}{\beta^{\kappa}}\Big\} \le \frac{E[1/x^{\kappa}(t)]}{1/\beta^{\kappa}}.
$$

This is to say

$$
\limsup_{t\to\infty}\{x(t)<\beta\} \le \beta^{\kappa}H_1 = \varepsilon.
$$

Consequently

$$
\liminf_{t\to\infty}\{x(t)\ge\beta\} \ge 1-\varepsilon.
$$

Next, we prove that for arbitrary $\varepsilon>0$, there are constants $\alpha>0$ such that $\liminf_{t\to\infty}\mathcal{P}\{x(t)\le\alpha\}\ge 1-\varepsilon$.

Let $0<p<1$, and compute

$$
\begin{aligned}
&dy^p(t) \\
&= py^{p-1}(t)\Big[y(t)\Big(r(t)-\prod_{0<t_k<t}(1+h_k)a(t)y(t)\Big)dt+\sigma(t)y(t)dB(t)\Big] \\
&\quad + \frac{1}{2}p(p-1)\sigma^2(t)y^p(t)dt + \int_{\mathbb{Y}}[(1+\gamma(u))^p-1-p\gamma(u)]x^p(t)\lambda(du)dt \\
&\quad + \int_{\mathbb{Y}}[(1+\gamma(u))^p-1]\tilde{N}(dt,du)x^p(t) \\
&\le \Big(-pma(t)y^{p+1}(t)+pr(t)y^p(t)+\frac{1}{2}p(p-1)\sigma^2(t)y^p(t)\Big)dt + p\sigma(t)y^p(t)dB(t) \\
&\quad + \int_{\mathbb{Y}}[(1+\gamma(u))^p-1-p\gamma(u)]x^p(t)\lambda(du)dt + \int_{\mathbb{Y}}[(1+\gamma(u))^p-1]\tilde{N}(dt,du)x^p(t).
\end{aligned}
$$

Using the inequality $x^q\le 1+q(x-1)$, $x\ge0$, $0\le q\le1$, we have $\int_{\mathbb{Y}}[(1+\gamma(u))^p-1-p\gamma(u)]\lambda(du)<0$. And there exists $K>0$ such that $-pma(t)y^{p+1}(t)+\Big(pr(t)+$

$\frac{1}{2}p(p-1)\sigma^2(t)\Big)y^p(t) \le K$. Then,

$$dy^p(t) \le Kdt + p\sigma(t)y^p(t)dB(t) + \int_{\mathbb{Y}}[(1+\gamma(u))^p - 1]\tilde{N}(dt,du)x^p(t).$$

Therefore

$$E(e^t x^p(t)) \le x_0^p + \int_0^t e^s ds = x_0^p + K(e^t - 1).$$

This immediately implies that $\limsup_{t\to\infty} E(y^p(t)) \le K$. Consequently,

$$\limsup_{t\to\infty} E(x^p(t)) = \limsup_{t\to\infty} \Big[\prod_{0<t_k<t}(1+h_k)\Big]^p E(x^p(t)) \le M^p K = \alpha.$$

Then the desired assertion follows from the Chebyshev inequality. This completes the proof. □

Remark 3.8. Generally speaking, as the biology has implied, Theorem 3.2 reveals that the population probably will go to an end in the worst cases, while Theorem 3.3 shows that the living chances are considerably rare. From Theorem 3.4 we can easily find that the population size is limited to zero with the time permitted, however, the opportunity of the survival of it still exists. Theorem 3.7 means that if the time is sufficiently large, the population size will be neither too small nor too large with large probability. That is to say, the population will stably exist, which is the best result. In other words, the survival conditions of Theorem 3.7 are better than Theorems 3.2–3.4. This can well explain why the conditions are gradually stronger from Theorem 3.2 to Theorem 3.7.

Remark 3.9. When the jump coefficient $\gamma(u)$ degenerates to zero, our results in Theorems 2.3 and 3.2 coincide with [17, Theorems 1 and 2]. Therefore, our results generalize the work of [17].

Remark 3.10. In view of

$$\begin{aligned} g^* = \limsup_{t\to\infty} t^{-1}\Big[\sum_{0<t_k<t}\ln(1+h_k) + \int_0^t (r(s) - 0.5\sigma^2(s))ds\Big] \\ - \int_{\mathbb{Y}}(\gamma(u) - \ln(1+\gamma(u)))\lambda(du) \end{aligned}$$

in Theorems 3.2–3.7, we can find that the impulse does not affect the persistence and extinction if the impulsive perturbations satisfy assumption (A4). If the impulsive perturbations do not satisfy assumption (A4), it can effect the population: the positive impulses h_k are advantageous for the population and the negative impulses h_k not favorable to the population.

4. Examples and numerical simulations

In this section, we shall use the Euler scheme [31] to illustrate the analytical findings.

In Figure 1 (a), (b) (c), we choose $r(t) = 0.28 + 0.05\sin t$, $a(t) = 0.2 + 0.01\cos t$, $\sigma^2(t) = 0.3$, $\mathbb{Y} = (0,\infty)$, $\lambda(\mathbb{Y}) = 1$, $\gamma(u) = 0.63$, $x_0 = 0.3$ and step size $\Delta t = 0.001$ and $t_k = 100k$ for $k \in \mathbb{N}$. The only difference in Figure 1 (a) (b) and (c) is that the representations of h_k are different. In Figure 1(a), we choose $h_k = 0$, then $g^* = -0.01 < 0$. From Theorem 3.2, the population $x(t)$ will go to extinction.

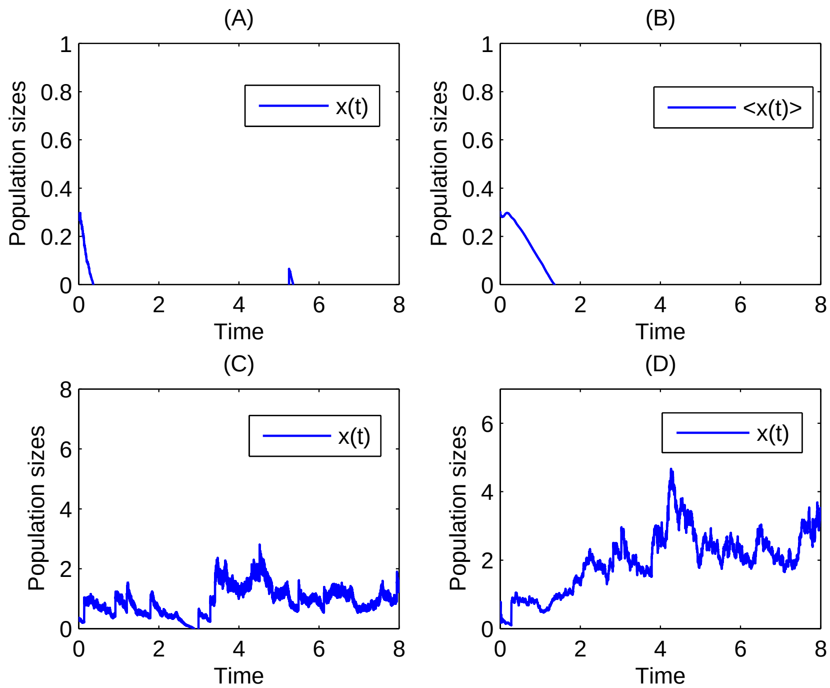

FIGURE 1. The horizontal axis and the vertical axis in this and following figures represent the time t and the populations size $x(t)$ (step size $\Delta t = 0.001$).

In Figure 1(b), we choose $t_k = 10k$ and $h_k = e^{0.1} - 1$, then $g^* = 0$. In view of Theorem 3.3, population $x(t)$ will be nonpersistence in the mean. In Figure 1(c), we consider $t_k = 10k$ and $h_k = e^{0.2} - 1$, then $g^* = 0.01 > 0$. By Theorem 3.4, population $x(t)$ will be weak persistence. By the numerical simulations above, we can find that the impulsive perturbation can change the properties of the population models significantly.

In Figure 1(d), we consider $r(t) = 0.43 + 0.06 \sin t$, $a(t) = 0.2 + 0.01 \cos t$, $\gamma(u) = 0.24$, $\sigma^2(t) = 0.3$, $\mathbb{Y} = (0, \infty)$, $\lambda(\mathbb{Y}) = 1$, $x_0 = 0.3$, step size $\Delta t = 0.001$, $t_k = 10k$ and $h_k = e^{\frac{(-1)^{k+1}}{k}} - 1$, then $g^* = 0.04 > 0$. Using Theorem 3.7, the population $x(t)$ will be stochastic permanence.

Conclusions and future directions. In this article, we considered a stochastic logistic model with Lévy noise and impulsive perturbation. From the conclusions we know that the impulsive perturbation can have an impact on the population in some degree. Generally speaking, the impulsive perturbation is small when compared with Lévy jumps. Yet it may represent human factor to protect the population even if it suffer sudden environmental shocks that can be modelled by Lévy noise. When the population will be extinct, we should take measure, i.e. positive impulsive perturbation, to avoid the case as far as possible. In contrast, we may go into action, i.e. negative impulsive perturbation, to take precautions against population explosion ahead of schedule.

Some interesting and significant topics deserve our further engagement. One may put forward a more realistic and sophisticated model to integrate the colored noise into the model [11, 12, 30]. Another significant problem is that one should incorporate Lévy noise and impulsive perturbation into multidimensional stochastic model with time delay or without time delay [14, 21, 22, 23, 35], and such investigations are to be done in future.

Acknowledgments. The authors would like to thank the editor and the referee for their important and valuable comments. This work was supported by the National Natural Science Foundation of China (Nos. 11501150 and 11271101), the NNSF of Shandong Province in China (ZR2010AQ021), the Key Project of Science and Technology of Weihai (No. 2014DXGJ14), the Natural Scientific Research Innovation Foundation in Harbin Institute of Technology (No. HIT. NSRIF. 2016081), the Disciplinary Construction Guide Foundation of Harbin Institute of Technology at Weihai (No. WH20140206) and the Scientific Research Foundation of Harbin Institute of Technology at Weihai (No. HIT(WH)201319).

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