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BLOW-UP OF SOLUTIONS TO SOME DIFFERENTIAL EQUATIONS AND INEQUALITIES WITH SHIFTED ARGUMENTS

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ABSTRACT. Using the test function technique, we obtain sufficient conditions for the blow-up of solutions to some differential equations and inequalities with advanced and delayed arguments, and for systems. Also we obtain upper estimates for the blow-up time.

1. Introduction

Differential equations and inequalities with shifted (advanced or delayed) argument have been considered by many authors. However most publication are devoted to obtaining sufficient conditions for existence and uniqueness of solutions. It is also known that blow-up of solutions occurs for equations and inequalities with unshifted arguments, under certain assumptions. In this article, we establish sufficient conditions for the blow-up of solutions to equations with shifted arguments. Our method is based on the test function technique suggested in [3, 4] and developed in [1, 2].

The rest of the article consists of four sections. In Section 1, we consider a single inequality with advanced arguments. In Section 2, we introduce a delayed arguments in the right-hand side, and in Section 3, in the left-hand side. Finally, Section 4 is devoted to systems of differential inequalities with advanced arguments.

2. Single inequality with advanced argument

Let $q > 1$. Consider the problem of finding a function $y(t)$, which satisfies the first-order differential inequality with advanced argument

$$\frac{dy(t)}{dt} \geq |y(t+\tau)|^q \quad t > 0, \tag{2.1}$$

and the initial condition

$$y(0) = y_0 > 0. \tag{2.2}$$

Definition 2.1. A function $y(t)$ which satisfies (2.1)–(2.2) for all $t \in (0, t^*)$, with $t^* > 0$, is called a *local solution.*

If (2.1)–(2.2) is satisfied for all $t > 0$, then $y(t)$ is called a *global solution.*

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If $y(t)$ is not a global solution, the largest possible t^* is called the *blow-up time* for problem (2.1)–(2.2).

Definition 2.2. *Let* $0 < t_* < \infty$*. A* test function *for problem* (2.1)*)–*(2.2) *is a function* $\varphi(t) \geq 0$ *that is continuous differentiable on* $[0,\infty)$ *and*

$$\varphi(0) = 1, \quad \varphi'(0) = 0, \tag{2.3}$$

$$\varphi(t_*) = \varphi'(t_*) = 0. \tag{2.4}$$

We use test functions of the form

$$\varphi(t) = \varphi_{t_*}(t) = \varphi_1(\tilde{t}), \quad \tilde{t} = \frac{t}{t_*}, \tag{2.5}$$

where

$$\varphi_1(\tilde{t}) = \begin{cases} 1 & 0 \leq \tilde{t} \leq 1/2, \\ 0 & \tilde{t} \geq 1. \end{cases} \tag{2.6}$$

Lemma 2.3. *There exists a function* $\varphi_1(\tilde{t}) \geq 0$ *continuous differentiable on* $[0,\infty)$*, which satisfies conditions* (2.6) *and*

$$\int_0^1 \frac{|\varphi_1'(\tilde{t})|^{q'}}{(\varphi_1(\tilde{t}))^{q'-1}}\, d\tilde{t} < \infty, \tag{2.7}$$

where $q' = \frac{q}{q-1}$.

Proof. Take $\varphi_1(\tilde{t})$ equal to $(1-\tilde{t})^\lambda$ with $\lambda > 0$ large enough in a left neighborhood of 1. □

Theorem 2.4. *If* $q > 1$*, then there is a blow-up for problem* (2.1)*–*(2.2)*.*

Proof. Let $0 < t_* < \infty$. Multiply (2.1) by a test function $\varphi(t) = \varphi_{t^*}(t) \geq 0$ satisfying (2.3)–(2.7). Then integrate its left-hand side by parts to obtain

$$-y_0 - \int_0^{t_*} y(t)\frac{d\varphi(t)}{dt}\, dt \geq \int_0^{t_*} |y(t+\tau)|^q \varphi(t)\, dt,$$

which, by the sign of $\frac{d\varphi}{dt}$ on the left, yields

$$-y_0 + \int_0^{t_*} y(t)\Big|\frac{d\varphi(t)}{dt}\Big|\, dt \geq \int_0^{t_*} |y(t+\tau)|^q \varphi(t)\, dt. \tag{2.8}$$

Equation (2.1) implies $y'(t) > 0$, which by (2.2) leads to

$$y(t) > y(0) = y_0 > 0 \quad \text{for all } t > 0.$$

Apply the Lagrange formula to the function $|y(t+\tau)|^q$ on the right-hand side of (2.8),

$$|y(t+\tau)|^q = y^q(t+\tau) = y^q(t) + \tau q y^{q-1}(t+\theta\tau) y'(t+\theta\tau),$$

where $\theta = \theta(t) \in (0,1)$. Hence by (2.1) we obtain

$$|y(t+\tau)|^q = y^q(t) + \tau q y^{q-1}(t+\theta\tau) y^q(t+(1+\theta)\tau) \geq y^q(t) + \tau q y_0^{2q-1}\,.$$

Further we have

$$\begin{aligned}\int_0^{t_*} |y(t+\tau)|^q \varphi(t)\,dt &\geq \int_0^{t_*} (y^q(t) + \tau q y_0^{2q-1})\varphi(t)\,dt \\ &\geq \int_0^{t_*} y^q(t)\varphi(t)\,dt + \tau q y_0^{2q-1} \int_0^{t_*/2} \varphi(t)\,dt \\ &\geq \int_0^{t_*} y^q(t)\varphi(t)\,dt + \tau q y_0^{2q-1} t_*/2.\end{aligned} \tag{2.9}$$

Combining (2.8) with (2.9), we obtain

$$\int_0^{t_*} |y(t)|^q \varphi(t)\,dt + \tau q y_0^{2q-1} t_*/2 \leq \int_0^{t_*} y(t)|\frac{d\varphi}{dt}|\,dt - y_0. \tag{2.10}$$

Hence by Young's inequality with a parameter $\epsilon > 0$

$$ab \leq \frac{\epsilon}{q} a^q + \frac{1}{q' \epsilon^{q'-1}} b^{q'}, \quad a, b \geq 0,$$

where $\frac{1}{q} + \frac{1}{q'} = 1$, we find

$$\big(1 - \frac{\epsilon}{q}\big) \int_0^{t_*} |y(t)|^q \varphi(t)\,dt \leq -y_0 - \tau q y_0^{2q-1} t_*/2 + \frac{1}{q' \epsilon^{q'-1}} \int_0^{T_1} \frac{|\varphi'(t)|^{q'}}{(\varphi(t))^{q'-1}}\,dt - y_0.$$

Thus for any $q > \epsilon > 0$, $q > 1$, we get an a priori estimate for $y(t)$. This implies

$$\int_0^{t_*} |y|^q \varphi\,dt \leq \int_0^{T_1} \frac{|\varphi'(t)|^{q'}}{(\varphi(t))^{q'-1}}\,dt - q'(y_0 + \tau q y_0^{2q-1} t_*/2),$$

since for $q > 1$

$$\min_{0<\epsilon<q} \frac{q-1}{(q-\epsilon)\epsilon^{q'-1}} = 1$$

is attained at $\epsilon = 1$.

Taking into account that the test function $\varphi(t)$ satisfies (2.6) and (2.7), we have

$$\int_0^{t_*} |y(t)|^q \varphi(t)\,dt \leq \frac{1}{t_*^{q'-1}} \int_0^1 \frac{|\varphi_1'(\tau)|^{q'}}{(\varphi_1(\tau))^{q'-1}}\,d\tau - q'(y_0 + \tau q y_0^{2q-1} t_*/2). \tag{2.11}$$

Using (2.7), we put

$$c_1 = \int_0^1 \frac{|\varphi_1'(\tau)|^{q'}}{(\varphi_1(\tau))^{q'-1}}\,d\tau.$$

Then (2.11) implies

$$\int_0^{t_*} |y|^q \varphi\,dt \leq c_1 t_*^{1-q'} - q'(y_0 + \tau q y_0^{2q-1} t_*/2). \tag{2.12}$$

From this estimate and (2.2), taking $t_* \to \infty$, we immediately obtain a contradiction implying the nonexistence of a global solution for (2.1)–(2.2) with any $q > 1$. □

Remark 2.5. From the proof of Theorem 2.4 it follows that a local solution of (2.1)–(2.2) does not exist on $[0, t_{**}]$, where t_{**} is the zero of the right-hand side of (2.12). Thus the blow-up time for problem (2.1)–(2.2) does not exceed t_{**}.

Note that

$$t_{**} < \tilde{t} := \Big(\frac{c_1}{q' y_0}\Big)^{\frac{1}{q'-1}}, \tag{2.13}$$

since for $t_* \to 0_+$ the right-hand side of (2.12) tends to $+\infty$, and for $t_* = \tilde{t}$ it is negative. As is known, $\tilde{t}$ is the blow-up time for a Cauchy problem consisting of an ODE with an unshifted argument

$$\frac{dy}{dt} = |y(t)|^q \quad t > 0, \tag{2.14}$$

and of the initial condition (2.2).

3. A single inequality with delayed argument

Consider the first-order differential inequality

$$\frac{dy(t)}{dt} \geq |y(t-\tau)|^q \quad t > 0 \tag{3.1}$$

with the initial condition

$$y(t) = f(t) \quad t \in [-\tau, 0], \tag{3.2}$$

where $f \in C[-\tau, 0]$ is a monotone growing positive function satisfying the compatibility condition

$$f'(0) \geq f^q(-\tau) \tag{3.3}$$

and the additional condition

$$f(0) + \int_{-\tau}^{0} |f(t)|^q \, dt < 0. \tag{3.4}$$

Theorem 3.1. *Let $q > 1$. Then* (3.1)–(3.2) *has a blow-up, when f satisfies* (3.3) *and* (3.4).

Proof. Similarly to Theorem 2.4, multiplying both parts of (3.1) by the test function φ and integrating by parts, we obtain

$$- y_0 + \int_0^{t_*} y(t) \Big| \frac{d\varphi(t)}{dt} \Big| \, dt \geq \int_0^{t_*} |y(t-\tau)|^q \varphi(t) \, dt. \tag{3.5}$$

Since the function $\varphi = \varphi_{t_*}$ can be assumed to be monotone decreasing and with parameter $t_* > \tau$, we can estimate the right-hand side of (3.5) from below as

$$\begin{aligned} \int_0^{\infty} |y(t-\tau)|^q \varphi(t) \, dt &= \int_{-\tau}^{\infty} |y(t)|^q \varphi(t+\tau) \, dt \\ &\geq \int_{-\tau}^{0} |f(t)|^q dt + \int_0^{\infty} |y(t)|^q \varphi(t) \, dt. \end{aligned} \tag{3.6}$$

Repeating the arguments from the proof of Theorem 2.4 concerning the left-hand side of (3.5) and combining them with (3.6), we obtain

$$\int_0^{t_*} |y|^q \varphi \, dt \leq c_1 t_*^{1-q'} - q' \Big(f(0) + \int_{-\tau}^{0} |f(t)|^q dt \Big).$$

Then we complete the proof similarly to that of Theorem 2.4. □

4. A single equation with delayed argument on the left-hand side

Now consider the first-order differential equation

$$y^p(t-\tau)\frac{dy(t)}{dt} = |y(t)|^q \quad t>0 \tag{4.1}$$

with the initial condition

$$y(t) = f(t) \quad t \in [-\tau, 0], \tag{4.2}$$

where $f \in C[-\tau, 0]$ is a monotone growing positive function satisfying the compatibility condition

$$f^p(-\tau)f'(0) = f^q(0). \tag{4.3}$$

We define local and global solutions, blow-up time, test functions for (4.1)–(4.2) as in Definitions 2.1 and 2.2.

Theorem 4.1. *Let $p>0$, $q>\max(p+1, 2p)$. Then problem (4.1)–(4.3) has a blow-up.*

Proof. Note that by equation (4.1) one has $y'(t)>0$ for all $t>0$ where $y(t)$ is defined, whence

$$y(t) > y(0) = f(0) \geq \min_{t\in[-\tau,0]} f(t) = f(-\tau), \quad \forall t>0. \tag{4.4}$$

Multiplying equation (4.1) by a test function $\varphi(t) = \varphi_{t^*}(t) \geq 0$ satisfying conditions (2.3)–(2.7), we obtain

$$\int_0^{t_*} y^p(t-\tau)y'(t)\varphi(t)\,dt = \int_0^{t_*} y^q(t)\varphi(t)\,dt. \tag{4.5}$$

We transform the left-hand side of this equality by the Lagrange formula

$$\int_0^{t_*} y^p(t-\tau)y'(t)\varphi(t)\,dt = \int_0^{t_*} [y^p(t) - \tau p y^{p-1}(t-\theta\tau)y'(t-\theta\tau)]y'(t)\varphi(t)\,dt. \tag{4.6}$$

By equation (4.1), we have

$$y'(t) = \frac{y^q(t)}{y^p(t-\tau)}, \tag{4.7}$$

$$y'(t-\theta\tau) = \frac{y^q(t-\theta\tau)}{y^p(t-(1+\theta)\tau)}. \tag{4.8}$$

Substituting (4.7) and (4.8) into (4.6), we obtain

$$\int_0^{t_*} y^p(t-\tau)y'(t)\varphi(t)\,dt = \int_0^{t_*} \Big[y^p(t) - \tau p \frac{y^{q+p-1}(t-\theta\tau)}{y^p(t-(1+\theta)\tau)}\Big]\frac{y^q(t)}{y^p(t-\tau)}\varphi(t)\,dt.$$

Then (4.4) and the monotonicity of $y(t)$ imply

$$\frac{y^{q+p-1}(t-\theta\tau)}{y^p(t-(1+\theta)\tau)}\frac{y^q(t)}{y^p(t-\tau)} \geq f^{q+p-1}(-\tau)y^{q-2p}(t) \geq f^{2q-p-1}(-\tau),$$

whence

$$\int_0^{t_*} y^p(t-\tau)y'(t)\varphi(t)\,dt \leq \int_0^{t_*} y^p(t)y'(t)\varphi(t)\,dt - \tau p f^{2q-p-1}(-\tau)\int_0^{t_*}\varphi(t)\,dt\,.$$

After integration by parts

$$\begin{aligned}&\int_0^{t_*} y^p(t-\tau)y'(t)\varphi(t)\,dt\\&\le -\frac{f^{p+1}(0)}{p+1}-\frac{1}{p+1}\int_0^{t_*} y^{p+1}(t)\varphi(t)\,dt-\tau p t_* f^{2q-p-1}(-\tau).\end{aligned}\tag{4.9}$$

From (4.5) and (4.9) we have

$$\int_0^{t_*} y^q(t)\varphi(t)\,dt\le -\frac{f^{p+1}(0)}{p+1}-\frac{1}{p+1}\int_0^{t_*} y^{p+1}(t)\varphi(t)\,dt-\tau p t_* f^{2q-p-1}(-\tau).\tag{4.10}$$

Applying the Young inequality to the integral on the right of (4.10), similarly to (2.3) we obtain

$$\int_0^{t_*} y^q(t)\varphi(t)\,dt\le -\frac{f^{p+1}(0)}{p+1}-\tau p t_* f^{2q-p-1}(-\tau)+ct_*^{-\frac{p+1}{q-p-1}},\tag{4.11}$$

which leads to a contradiction for $t_*>t_{**}$, where t_{**} is a zero of the right-hand side of (4.11). This proves the claim. □

Remark 4.2. The proof of Theorem 4.1 implies that a local solution of (4.1)–(4.3) cannot be defined on any interval $[0,t_*)$ with $t_*>t_{**}$. Thus the blow-up time for (4.1)–(4.3) does not exceed $t_{**}<\tilde{t}$, where $\tilde{t}$ is a constant from (2.13).

5. Systems of inequalities with advanced arguments

Now we consider a system of inequalities with advanced arguments

$$\begin{aligned}\frac{dy(t)}{dt}&\ge |z(t+\tau_1)|^{q_1}\quad t>0,\\ \frac{dz(t)}{dt}&\ge |y(t+\tau_2)|^{q_2}\quad t>0\end{aligned}\tag{5.1}$$

with initial conditions

$$\begin{aligned}y(0)&=y_0>0,\\ z(0)&=z_0>0\end{aligned}\tag{5.2}$$

with $\tau_1,\tau_2>0$. We define solutions of this system and test functions similarly to the previous sections.

Theorem 5.1. *Let $q_1,q_2>1$. Then a blow-up situation takes place for problem (5.1)–(5.2).*

Proof. Let $0<t_*<\infty$. Multiply inequalities (5.1) by a test function $\varphi(t)=\varphi_{t^*}(t)\ge 0$, which satisfies conditions (2.3)–(2.7). Then integrate their left-hand sides by parts to obtain

$$\begin{aligned}-y_0-\int_0^{t_*} y(t)\frac{d\varphi(t)}{dt}\,dt&\ge\int_0^{t_*}|z(t+\tau_1)|^{q_1}\varphi(t)\,dt,\\ -z_0-\int_0^{t_*} z(t)\frac{d\varphi(t)}{dt}\,dt&\ge\int_0^{t_*}|y(t+\tau_2)|^{q_2}\varphi(t)\,dt,\end{aligned}$$

which, by the sign of $\frac{d\varphi}{dt}$ on the left, yields

$$\begin{aligned} -y_0 + \int_0^{t_*} y(t)|\frac{d\varphi(t)}{dt}|\,dt &\geq \int_0^{t_*} |z(t+\tau_1)|^{q_1}\varphi(t)\,dt, \\ -z_0 + \int_0^{t_*} z(t)|\frac{d\varphi(t)}{dt}|\,dt &\geq \int_0^{t_*} |y(t+\tau_2)|^{q_2}\varphi(t)\,dt. \end{aligned} \tag{5.3}$$

Equation (5.1) implies $y'(t) > 0$. This fact and (5.2) lead to

$$y(t) > y(0) = y_0 > 0 \quad \text{for all } t > 0.$$

Apply the Lagrange formula to the functions $|z(t+\tau_1)|^{q_1}$ and $|y(t+\tau_2)|^{q_2}$ on the right-hand side of (5.3),

$$\begin{aligned} |z(t+\tau_1)|^{q_1} &= z^{q_1}(t+\tau_1) = z^{q_1}(t) + \tau_1 q_1 z^{q_1-1}(t+\theta_1\tau)z'(t+\theta_1\tau), \\ |y(t+\tau_2)|^{q_2} &= y^{q_2}(t+\tau_2) = y^{q_2}(t) + \tau_2 q_2 y^{q_2-1}(t+\theta_1\tau)y'(t+\theta_2\tau), \end{aligned}$$

where $\theta = \theta(t) \in (0,1)$. Hence by inequalities (5.1) we obtain

$$\begin{aligned} |z(t+\tau_1)|^{q_1} &= z^{q_1}(t) + \tau_1 q_1 z^{q_1-1}(t+\theta_1\tau_1)y^{q_2}(t+\theta_1\tau_1) \\ &\geq z^{q_1}(t) + \tau_1 \cdot q_1 z_0^{q_1-1} y_0^{q_2}, \\ |y(t+\tau_2)|^{q_2} &= y^{q_2}(t) + \tau_2 q_2 y^{q_2-1}(t+\theta_2\tau_2)z^{q_1}(t+(1+\theta_2)\tau_2) \\ &\geq y^{q_2}(t) + \tau_2 \cdot q_2 y_0^{q_2-1} z_0^{q_1}\,. \end{aligned}$$

Further,

$$\begin{aligned} \int_0^{t_*} |z(t+\tau_1)|^{q_1}\varphi(t)\,dt &\geq \int_0^{t_*} (z^{q_1}(t) + \tau_1 q_1 z_0^{q_1-1} y_0^{q_2})\varphi(t)\,dt \\ &\geq \int_0^{t_*} z^{q_1}(t)\varphi(t)\,dt + \tau_1 \cdot q_1 z_0^{q_1-1} y_0^{q_2} \int_0^{t_*/2} \varphi(t)\,dt \\ &\geq \int_0^{t_*} z^{q_1}(t)\varphi(t)\,dt + \tau_1 q_1 z_0^{q_1-1} y_0^{q_2} t_*/2, \\ \int_0^{t_*} |y(t+\tau_2)|^{q_2}\varphi(t)\,dt &\geq \int_0^{t_*} (y^{q_2}(t) + \tau_2 \cdot q_2 y_0^{q_2-1} z_0^{q_1})\varphi(t)\,dt \\ &\geq \int_0^{t_*} y^{q_2}(t)\varphi(t)\,dt + \tau_2 \cdot q_2 y_0^{q_2-1} z_0^{q_1} \int_0^{t_*/2} \varphi(t)\,dt \\ &\geq \int_0^{t_*} y^{q_2}(t)\varphi(t)\,dt + \tau_2 \cdot q_2 y_0^{q_2-1} z_0^{q_1} t_*/2. \end{aligned} \tag{5.4}$$

Combine (5.3) and (5.4) and use the Young inequality with parameters $\epsilon_1 > 0$ for the terms with y and ϵ_2 for those with z, (2.6) and (2.7). Similarly to Section 2 we obtain

$$\begin{aligned} \int_0^{t_*} |z|^{q_1}\varphi\,dt &\leq -y_0 - \tau_1 q_1 z_0^{q_1-1} y_0^{q_2} t_*/2 + \epsilon_1 \int_0^{t_*} |y|^{q_2}\varphi\,dt + c_1 t_*^{1-q_1'}, \\ \int_0^{t_*} |y|^{q_2}\varphi\,dt &\leq -z_0 - \tau_2 q_2 y_0^{q_2-1} z_0^{q_1} t_*/2 + \epsilon_2 \int_0^{t_*} |z|^{q_1}\varphi\,dt + c_2 t_*^{1-q_2'}, \end{aligned} \tag{5.5}$$

and, substituting the second inequality (5.5) into the first and vice versa,

$$\begin{aligned}
&(1-\epsilon_2)\int_0^{t_*}|z|^{q_1}\varphi\,dt\\
&\le -y_0-\tau_1 q_1 z_0^{q_1-1}y_0^{q_2}t_*/2-\epsilon_1(z_0+\tau_2 q_2 y_0^{q_2-1}z_0^{q_1}t_*/2)+c_3 t_*^{1-q_1'}+c_4 t_*^{1-q_2'},\\
&(1-\epsilon_1)\int_0^{t_*}|y|^{q_2}\varphi\,dt\\
&\le -z_0-\tau_2 q_2 y_0^{q_2-1}z_0^{q_1}t_*/2-\epsilon_2(y_0+\tau_1 q_1 z_0^{q_1-1}y_0^{q_2}t_*/2)+c_5 t_*^{1-q_1'}+c_6 t_*^{1-q_2'},
\end{aligned} \tag{5.6}$$

where $0<\epsilon_1<1$, $0<\epsilon_2<1$, and $c_1,\dots,c_6$ are some positive constants. From (5.6), taking $t_*\to\infty$, we immediately obtain a contradiction implying nonexistence of global solution for problem (5.1)–(5.2) for any $q_1,q_2>1$. □

Remark 5.2. From the proof of Theorem 2.4 it follows that a local solution of (2.1)–(2.2) does not exist on $[0,t_{**}]$, where t_{**} can be defined similarly to the previous section. Thus the blow-up time for problem (2.1)–(2.2) does not exceed t_{**}.

Remark 5.3. Combining the techniques from this section and the previous ones, one can easily obtain similar results for systems with delayed arguments in left-hand or the right-hand side.

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Addendum posted on March 23, 2016

In response to comments from a reader, the author wants to correct the inequality in (3.4). It should be

$$f(0)+\int_{-\tau}^{0}|f(t)|^q\,dt>0. \tag{3.4}$$

End of addendum.

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