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GROUND STATE SOLUTIONS FOR HAMILTONIAN ELLIPTIC SYSTEM WITH SIGN-CHANGING POTENTIAL

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Abstract. This article concerns the Hamiltonian elliptic system

$$\begin{gathered}-\Delta u+V(x)u=H_v(x,u,v), \quad x\in\mathbb{R}^N,\\ -\Delta v+V(x)v=H_u(x,u,v), \quad x\in\mathbb{R}^N,\\ u(x)\to 0, \quad v(x)\to 0, \quad \text{as } |x|\to\infty,\end{gathered}$$

where $z=(u,v):\mathbb{R}^N\to\mathbb{R}\times\mathbb{R}$, $N\geq 3$ and the potential $V(x)$ is allowed to be sign-changing. Under weak superquadratic assumptions for the nonlinearities, by applying the variant generalized weak linking theorem for strongly indefinite problem developed by Schechter and Zou, we obtain the existence of nontrivial and ground state solutions.

1. Introduction and statement of main results

In this article, we study the superquadratic Hamiltonian elliptic system

$$\begin{gathered}-\Delta u+V(x)u=H_v(x,u,v), \quad x\in\mathbb{R}^N,\\ -\Delta v+V(x)v=H_u(x,u,v), \quad x\in\mathbb{R}^N,\\ u(x)\to 0, \quad v(x)\to 0, \quad \text{as } |x|\to\infty,\end{gathered} \tag{1.1}$$

where $z=(u,v):\mathbb{R}^N\to\mathbb{R}\times\mathbb{R}$, $N\geq 3$, $V\in C(\mathbb{R}^N,\mathbb{R})$ and $H\in C^1(\mathbb{R}^N\times\mathbb{R}^2,\mathbb{R})$.

A number of authors have focused on the case of bounded domain, see for instance [6, 9, 10, 11, 15, 17] and the references therein. Recently, system (1.1) or systems similar to (1.1) in the whole space $\mathbb{R}^N$ was considered by some authors. See for instance [1, 2, 3, 4, 12, 13, 14, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34] and the references therein. Most of these authors focused on the case that $V\equiv 1$. The lack of compactness for Sobolev's embedding theorem is the main difficulty of this problem. A usual way to overcome this difficulty is to work on the radically symmetric function space which possesses compact embedding. In this way, De Figueiredo and Yang [12] obtained a positive radially symmetric solution which decays exponentially to 0 at infinity. Sirakov [19] generalized the results of De Figueiredo and Yang. Later, Bartsch and De Figueiredo [4] proved that the system possesses infinitely many radial solutions as well as non-radial solutions. By a

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linking argument, Li and Yang [18] proved that the system has a positive ground state solution for that case that $V = 1$ and with an asymptotically quadratic nonlinearity. Another usual way to overcome the difficulty is to avoid the indefinite character of the original functional by applying the dual variational method, see for instance [1, 2, 3].

Very recently, many authors considered system (1.1) with general periodic potential, see [25, 28, 32, 33]. By applying a generalized linking theorem for the strongly indefinite functionals developed recently by Bartsch and Ding [5] (see also [16, 8]), the authors obtained the existence of solutions (ground state) and multiple geometrically distinct solutions under different assumptions. For more detailed descriptions related to the non-periodic potential, see [26, 27, 23] for asymptotically quadratic case, in [14, 24] for superquadratic case. Moreover, for other related topics including the superquadratic singular perturbation problem and concentration phenomenon of semi-classical states, we refer the readers to [13, 29, 31, 34] and the references therein.

Motivated by these works, we continue to consider system (1.1) with non-periodic and sign-changing potential and superquadratic nonlinearities. Under some mild assumptions which are different from those studied previously, we mainly study the existence of solutions and ground states via variational methods. To state our results, we need the following assumptions:

- (H1) $V \in C(\mathbb{R}^N, \mathbb{R})$ and $\inf_{x\in\mathbb{R}^N} V(x) > -\infty$, and there exists a constant $l_0 > 0$ such that
$$\lim_{|y|\to\infty} \operatorname{meas}\big\{x \in \mathbb{R}^N : |x - y| \le l_0, V(x) \le h\big\} = 0, \quad \forall h > 0, \tag{1.2}$$
where $\operatorname{meas}(\cdot)$ denotes the Lebesgue measure in $\mathbb{R}^N$;
- (H2) $H \in C^1(\mathbb{R}^N \times \mathbb{R}^2, [0,\infty))$ and $|H_z(x, z)| \le c(1 + |z|^{p-1})$ for some $c > 0$ and $2 < p < 2^*$, where $2^* = \frac{2N}{N-2}$ is the Sobolev critical exponent;
- (H3) $|H(x, z)| \le \frac{1}{2}\gamma|z|^2$ if $|z| < \delta$ for some $0 \le \gamma < \mu$, where $\delta > 0$ and μ will be defined later in (2.14);
- (H4) $\frac{H(x,z)}{|z|^2} \to \infty$ as $|z| \to \infty$ uniformly in x;
- (H5) $H(x, z + \eta) - H(x, z) - rH_z(x, z)\eta + \frac{(r-1)^2}{2} H_z(x, z)z \ge -W_1(x)$, $r \in [0, 1]$, $W_1(x) \in L^1(\mathbb{R}^N)$ and $z, \eta \in \mathbb{R}^2$.

On the existence of solutions and ground state solutions we have the following results.

Theorem 1.1. *Let* (H1)–(H5) *be satisfied, then system* (1.1) *has at least one solution.*

Theorem 1.2. *Let $\mathcal{M}$ be the collection of solutions of system* (1.1)*. Then there is a solution that minimizes the energy functional Φ over $\mathcal{M}$, where Φ will be defined later. In addition, if $|H_z(x, z)| = o(|z|)$ uniformly in x as $|z| \to 0$, then there is a nontrivial solution that minimize the energy functional over $\mathcal{M} \setminus \{0\}$.*

Remark 1.3. Condition (H5) was first introduced by Schechter [20] in studying the scalar Schrödinger equation, it replaces the usual monotonic condition. Our main results provide general existence results for semilinear elliptic systems of Hamiltonian type with general superquadratic nonlinearities and can be viewed as extension to the main results in [20] from the scalar Schrödinger equation to the elliptic system.

Remark 1.4. It is not difficult to find the functions V satisfying (H1). For example, let $V(x)$ be a zig-zag function with respect to $|x|$ defined by

$$V(x) = \begin{cases} 2n|x| - 2n(n-1) + a_0, & n-1 \le |x| < (2n-1)/2, \\ -2n|x| + 2n^2 + a_0, & (2n-1)/2 \le |x| \le n, \end{cases} \tag{1.3}$$

where $n \in \mathbb{N}$ and $a_0 \in \mathbb{R}$.

Remark 1.5. There are functions satisfying conditions (H2)–(H5). For example,

$$H(x,z) = \frac{1}{p}|z|^p \quad \text{and} \quad H_z(x,z) = |z|^{p-2}z, \quad \text{where } p > 2. \tag{1.4}$$

Clearly, the function H satisfies the conditions (H2)–(H4). Note that $h(|z|) := |z|^{p-2}$ is strictly increasing on $[0,+\infty)$, Therefore, H satisfies the condition (H5) by the argument in [20].

The rest of this article is organized as follows. In Section 2, we establish the variational framework associated with (1.1), and we also give some preliminary lemmas, which are useful in the proofs of our main results. In Section 3, we give the detailed proofs of our main results.

2. Variational setting and preliminary lemmas

Here, by $\|\cdot\|_q$ we denote the usual L^q-norm, $(\cdot,\cdot)_2$ denote the usual L^2 inner product, c_i, C, C_i stand for different positive constants. Let X and Y be two Banach spaces with norms $\|\cdot\|_X$ and $\|\cdot\|_Y$. We always choose equivalent norm $\|(x,y)\|_{X\times Y} = (\|x\|_X^2 + \|y\|_Y^2)$ on the product space $X \times Y$. In particular, if X and Y are two Hilbert spaces with inner products $(\cdot,\cdot)_X$ and $(\cdot,\cdot)_Y$, we choose the inner product $((x,y),(w,z)) = (x,w)_X + (y,z)_Y$ on the product space $X \times Y$.

For the sake of simplicity, let $A := -\Delta + V$ and $\sigma(A)$, $\sigma_d(A)$ be the spectrum of A, the discrete spectrum of A, respectively. It is well known that under condition (H1), the operator A is a selfadjoint operator on $L^2 := L^2(\mathbb{R}^N,\mathbb{R}^2)$ with $\mathcal{D}(A) \subset H^2(\mathbb{R}^N,\mathbb{R}^2)$. To establish a variational setting for the system (1.1), we have the following result.

Lemma 2.1. *Suppose* (H1) *holds, then* $\sigma(A) = \sigma_d(A)$.

Following the ideas of [7, 35], it is easy to prove the above lemma, so we omit its proof. From Lemma 2.1, we know that the operator A has a sequence of eigenvalues

$$\lambda_1 < \lambda_2 \le \dots \le \lambda_n \le \dots \to \infty, \tag{2.1}$$

and corresponding eigenfunctions $\{e_i\}_{i\in\mathbb{N}}$, forming an orthogonal basis in L^2. Let $n^- = \sharp\{i|\lambda_i < 0\}$, $n^0 = \sharp\{i|\lambda_i = 0\}$ and $n^+ = n^- + n^0$. Moreover, we have an orthogonal decomposition

$$L^2 = L^- \oplus L^0 \oplus L^+, \quad u = u^- + u^0 + u^+,$$

such that A is negative definite on L^- and positive definite on L^+ and $L^0 = \mathrm{Ker}A$. Let $|A|$ denote the absolute of A and $|A|^{1/2}$ be the square root of $|A|$, $\{F_\lambda : \lambda \in \mathbb{R}\}$ be the spectral family of A, $A = U|A|$ is the polar decomposition of A, where $U = I - F_0 - F_{-0}$, I is the identity operator. Then U commutes with A, $|A|$ and $|A|^{1/2}$. Set $H := \mathcal{D}(|A|^{1/2})$ be the domain of the selfadjoint operator $|A|^{1/2}$ which is a Hilbert space equipped with the inner product

$$(u,v)_H = (|A|^{1/2}u, |A|^{1/2}v)_2 + (u^0, v^0)_2$$

and the norm $\|u\|_H^2 = (u,u)_H$. Let

$$H^- := \operatorname{span}\{e_1, \dots, e_{n^-}\}, \quad H^0 := \operatorname{span}\{e_{n^-+1}, \dots, e_{n^+}\},$$
$$H^+ := \overline{\operatorname{span}\{e_{n^++1}, \dots\}}.$$

Then there is an induced decomposition $H = H^- \oplus H^0 \oplus H^+$ which is orthogonal with respect to the inner products $(\cdot,\cdot)_2$ and $(\cdot,\cdot)_H$. Let $E = H \times H$ with the inner product

$$((u,v),(\varphi,\psi)) = (u,\varphi)_H + (v,\psi)_H$$

and the corresponding norm

$$\|(u,v)\| = \left[\|u\|_H^2 + \|v\|_H^2\right]^{1/2}.$$

Setting

$$E^+ = H^+ \times H^-, \quad E^- = H^- \times H^+, \quad E^0 = H^0 \times H^0.$$

Then for any $z = (u,v) \in E$, we have $z = z^- + z^0 + z^+$, where $z^+ = (u^+, v^-)$, $z^- = (u^-, v^+)$ and $z^0 = (u^0, v^0)$. Clearly, E^-, E^0, E^+ are the orthogonal with respect to the products $(\cdot,\cdot)_2$ and $(\cdot,\cdot)_H$. Hence $E = E^- \oplus E^0 \oplus E^+$.

Lemma 2.2. *$E \hookrightarrow L^q := L^q(\mathbb{R}^N, \mathbb{R}^2)$ is continuous for $q \in [2, 2^*]$ and $E \hookrightarrow L^q$ is compact for $q \in [2, 2^*)$.*

Next, on E we define the functional

$$\Phi(z) = \frac{1}{2}(\|z^+\|^2 - \|z^-\|^2) - \Psi(z), \quad z \in E, \tag{2.2}$$

where

$$\Psi(z) = \int_{\mathbb{R}^N} H(x,z)dx.$$

Clearly, Φ is strongly indefinite, and our hypotheses imply that $\Phi \in C^1(E, \mathbb{R})$, and a standard argument shows that critical points of Φ are solutions of system (1.1) (see [8, 22]).

The following abstract critical point theorem plays an important role in proving our main results. Let E be a Hilbert space with norm $\|\cdot\|$ and have an orthogonal decomposition $E = N \oplus N_\perp, N \subset E$ being a closed and separable subspace. There exists a norm $|v|_\omega \le \|v\|$ for all $v \in N$ and induces a topology equivalent to the weak topology of N on a bounded subset of N. For $z = v + w \in E = N \oplus N^\perp$ with $v \in N, w \in N^\perp$, we define $|z|_\omega^2 = |v|_\omega^2 + \|w\|^2$. Particularly, if $z_n = v_n + w_n$ is $|\cdot|_\omega$-bounded and $z_n \xrightarrow{|\cdot|_\omega} z$, then $v_n \rightharpoonup v$ weakly in N, $w_n \to w$ strongly in $N^\perp$, $z_n \rightharpoonup v + w$ weakly in E.

Let $E = E^- \oplus E^0 \oplus E^+$, $e \in E^+$ with $\|e\| = 1$. Let $N := E^- \oplus E^0 \oplus \mathbb{R}e$ and $E_1^+ := N^\perp = (E^- \oplus E^0 \oplus \mathbb{R}e)^\perp$. For $R > 0$, let

$$Q := \{z := z^- + z^0 + se : s \in \mathbb{R}^+, z^- + z^0 \in E^- \oplus E^0, \|z\| < R\}. \tag{2.3}$$

For $0 < s_0 < R$, we define

$$D := \{z := se + w^+ : s \ge 0, w^+ \in E_1^+, \|se + w^+\| = s_0\}. \tag{2.4}$$

For $\Phi \in C^1(E, \mathbb{R})$, define

$$\Gamma := \Big\{h : h : \mathbb{R} \times \bar{Q} \to E \text{ is } |\cdot|_\omega\text{-continuous, } h(0,z) = z \text{ and } \Phi(h(s,z)) \le \Phi(z)$$
$$\text{for all } z \in \bar{Q}, \text{ For any } (s_0, z_0) \in \mathbb{R} \times \bar{Q}, \text{ there is a } |\cdot|_\omega\text{-neighborhood}$$

$$U(s_0,z_0) \text{ s. t. } \{z-h(t,z):(t,z)\in U(s_0,z_0)\cap(\mathbb{R}\times\bar{Q})\}\subset E_{\rm fin}.\Big\}$$

where $E_{\rm fin}$ denotes various finite-dimensional subspaces of E; $\Gamma\neq 0$ since $id\in\Gamma$.

Now we state the following variant weak linking theorem which will be used later (see [21]).

Lemma 2.3. *The family of C^1-functionals Φ_λ has the form*

$$\Phi_\lambda(z):=\lambda K(z)-J(z),\quad \forall\lambda\in[1,\lambda_0], \tag{2.5}$$

where $\lambda_0>1$. Assume that

(a) $K(z)\geq 0$, $\forall\, z\in E$, $\Phi_1=\Phi$;
(b) $|J(z)|+K(z)\to\infty$ *as* $\|z\|\to\infty$;
(c) Φ_λ *is* $|\cdot|_\omega$*-upper semicontinuous,* Φ'_λ *is weakly sequentially continuous on* E*,* Φ_λ *maps bounded sets to bounded sets;*
(d) $\sup_{\partial Q}\Phi_\lambda<\inf_D\Phi_\lambda$, $\quad\forall\lambda\in[1,\lambda_0]$.

Then for almost all $\lambda\in[1,\lambda_0]$, there exists a sequence $\{z_n\}$ such that

$$\sup_n\|z_n\|<\infty,\ \Phi'_\lambda(z_n)\to 0,\Phi_\lambda(z_n)\to c_\lambda, \tag{2.6}$$

where

$$c_\lambda:=\inf_{h\in\Gamma}\sup_{z\in\bar{Q}}\Phi_\lambda(h(1,z))\in[\inf_D\Phi_\lambda,\sup_{\bar{Q}}\Phi_\lambda]. \tag{2.7}$$

To apply Lemma 2.3, we shall prove a few Lemmas. We pick λ_0 such that $\lambda_0>1$. For $1\leq\lambda\leq\lambda_0$, we consider

$$\Phi_\lambda(z):=\frac{\lambda}{2}\|z^+\|^2-\Big(\frac{1}{2}\|z^-\|^2+\int_{\mathbb{R}^N}H(x,z)dx\Big):=\lambda K(z)-J(z). \tag{2.8}$$

It is easy to see that Φ_λ satisfies condition (a) in Lemma 2.3. To check (c), if $z_n\xrightarrow{|\cdot|_\omega}z$, and $\Phi_\lambda(z_n)\geq c$, then $z_n^+\to z^+$ and $z_n^-\rightharpoonup z^-$ in E, $z_n\to z$ *a.e.* on $\mathbb{R}^N$, going to a subsequence if necessary. Using Fatou's lemma, we know $\Phi_\lambda(z)\geq c$, which means that Φ_λ is $|\cdot|_\omega$-upper semicontinuous; Φ'_λ is weakly sequentially continuous on E, see [22].

Lemma 2.4. *Under the assumptions of Theorem 1.1,*

$$J(z)+K(z)\to\infty\quad \textit{as } \|z\|\to\infty. \tag{2.9}$$

Proof. Suppose to the contrary that there exists $\{z_n\}$ with $\|z_n\|\to\infty$ such that $J(z_n)+K(z_n)\leq M_0$ for some $M_0>0$. Let $w_n=\frac{z_n}{\|z_n\|}=w_n^-+w_n^0+w_n^+$, then $\|w_n\|=1$ and

$$\begin{aligned}\frac{M_0}{\|z_n\|^2}&\geq\frac{K(z_n)+J(z_n)}{\|z_n\|^2}\\&=\frac{1}{2}(\|w_n^+\|^2+\|w_n^-\|^2)+\int_{\mathbb{R}^N}\frac{H(x,z_n)}{\|z_n\|^2}dx\\&=\frac{1}{2}(\|w_n\|^2-\|w_n^0\|^2)+\int_{\mathbb{R}^N}\frac{H(x,z_n)}{\|z_n\|^2}dx.\end{aligned} \tag{2.10}$$

Going to a subsequence if necessary, we may assume that $w_n^-\rightharpoonup w^-$, $w_n^+\rightharpoonup w^+$, $w_n^0\to w^0$ in E and $w_n(x)\to w(x)$ a.e. on $\mathbb{R}^N$. If $w^0=0$, by (H2) and (2.10) we have

$$\frac{1}{2}\|w_n\|^2+\int_{\mathbb{R}^N}\frac{H(x,z_n)}{\|z_n\|^2}dx\leq\frac{1}{2}\|w_n^0\|^2+\frac{M_0}{\|z_n\|^2}, \tag{2.11}$$

which implies $\|w_n\| \to 0$, this contradicts $\|w_n\| = 1$. If $w^0 \neq 0$, then $w \neq 0$. Therefore, $|z_n| = |w_n|\|z_n\| \to \infty$. By (H2), (H4) and Fatou's lemma we have

$$\int_{\mathbb{R}^N} \frac{H(x,z_n)}{|z_n|^2}|w_n|^2 dx \to \infty. \tag{2.12}$$

Hence by (2.10) again, we obtain $0 \geq +\infty$, which is a contradiction. The proof is complete. ☐

Lemma 2.4 implies condition (b) holds. To continue the discussion, we still need to verify condition (d); that is done by the following two Lemmas.

Lemma 2.5. *Under the assumptions of Theorem 1.1, there are two positive constants κ, ρ such that*

$$\Phi_\lambda(z) \geq \kappa \quad \text{for } z \in E^+,\ |z\| = \rho,\ \lambda \in [1, \lambda_0]. \tag{2.13}$$

Proof. Set

$$\mu := \min\{-\lambda_{n^-}, \lambda_{n^+ +1}\}. \tag{2.14}$$

Obviously, for any $z \in E^+$, $\|z\|^2 \geq \mu\|z\|_2^2$. Thus, for any $z \in E^+$, by (H2), (H3) and Lemma 2.2, we have

$$\begin{aligned}
\Phi_\lambda(z) &= \frac{\lambda}{2}\|z\|^2 - \int_{\mathbb{R}^N} H(x,z)dx \\
&\geq \frac{1}{2}\|z\|^2 - \int_{\{|z|<\delta\}} H(x,z)dx - \int_{\{|z|\geq\delta\}} H(x,z)dx \\
&\geq \frac{1}{2}\|z\|^2 - \frac{1}{2}\gamma\int_{\{|z|<\delta\}} |z|^2 dx - c\int_{\{|z|\geq\delta\}} (|z|^2 + |z|^p)dx \\
&\geq \frac{1}{2}\|z\|^2 - \frac{\gamma}{\mu}\frac{1}{2}\|z\|^2 - C'\|z\|^p \\
&= \frac{1}{2}\|z\|^2(1 - \frac{\gamma}{\mu} - 2C'\|z\|^{p-2}), \quad 0 \leq \gamma < \mu.
\end{aligned} \tag{2.15}$$

This implies the conclusion if we take $\|z\|$ sufficiently small. ☐

Lemma 2.6. *Under the assumptions of Theorem 1.1, there exists a constant $R > 0$ such that*

$$\Phi_\lambda(z) \leq 0, \quad \text{for } z \in \partial Q_R,\ \lambda \in [1, \lambda_0], \tag{2.16}$$

where

$$Q_R := \{z := v + se : s \geq 0, v \in E^- \oplus E^0, e \in E^+ \text{ with } \|e\| = 1, \|z\| \leq R\}. \tag{2.17}$$

Proof. By contradiction, we suppose that there exit $R_n \to \infty$, $\lambda_n \in [1, \lambda_0]$ and $z_n = v_n + s_n e = v_n^- + v_n^0 + s_n e \in \partial Q_{R_n}$ such that $\Phi_{\lambda_n}(z_n) > 0$. If $s_n = 0$, by (H2), we get

$$\Phi_{\lambda_n}(z_n) = -\frac{1}{2}\|v_n^-\|^2 - \int_{\mathbb{R}^N} H(x,z_n)dx \leq -\frac{1}{2}\|v_n^-\|^2 \leq 0. \tag{2.18}$$

Therefore,

$$s_n \neq 0, \quad \|z_n\|^2 = \|v_n\|^2 + s_n^2. \tag{2.19}$$

Let

$$\tilde{z}_n = \frac{z_n}{\|z_n\|} = \tilde{s}_n e + \tilde{v}_n. \tag{2.20}$$

Then

$$\|\tilde{z}_n\|^2 = \|\tilde{v}_n\|^2 + \tilde{s}_n^2 = 1. \tag{2.21}$$

Thus, passing to a subsequence, we may assume that

$$\begin{gathered}\tilde{s}_n \to \tilde{s}, \quad \lambda_n \to \lambda,\\ \tilde{z}_n = \frac{z_n}{\|z_n\|} = \tilde{s}_n e + \tilde{v}_n \rightharpoonup \tilde{z} \quad \text{in } E,\\ \tilde{z}_n \to \tilde{z} \quad \text{a.e. on } \mathbb{R}^N.\end{gathered}$$

It follows from $\Phi_{\lambda_n}(z_n) > 0$ and the definition of Φ that

$$\begin{aligned}0 < \frac{\Phi_{\lambda_n}(z_n)}{\|z_n\|^2} &= \frac{1}{2}(\lambda_n \tilde{s}_n^2 - \|\tilde{v}_n\|^2) - \int_{\mathbb{R}^N} \frac{H(x,z_n)}{|z_n|^2}|\tilde{z}_n|^2 dx\\ &= \frac{1}{2}[(\lambda_n+1)\tilde{s}_n^2 - 1] - \int_{\mathbb{R}^N} \frac{H(x,z_n)}{|z_n|^2}|\tilde{z}_n|^2 dx.\end{aligned} \tag{2.22}$$

From (H2) and (2.22), we know that $(\lambda+1)\tilde{s}^2 - 1 \geq 0$, that is

$$\tilde{s}^2 \geq \frac{1}{1+\lambda} \geq \frac{1}{1+\lambda_0} > 0.$$

Thus $\tilde{z} \neq 0$. It follows from (H4) and Fatou's lemma that

$$\int_{\mathbb{R}^N} \frac{H(x,z_n)}{|z_n|^2}|\tilde{z}_n|^2 dx \to \infty \quad \text{as } n \to \infty, \tag{2.23}$$

which contradicts (2.22). The proof is complete. $\square$

Hence, Lemmas 2.5 and 2.6 imply condition (d) of Lemma 2.3. Applying Lemma 2.3, we obtain the following result.

Lemma 2.7. *Under the assumptions of Theorem 1.1, for almost all $\lambda \in [1, \lambda_0]$, there exists a sequence $\{z_n\}$ such that*

$$\sup_n \|z_n\| < \infty, \quad \Phi'_\lambda(z_n) \to 0, \quad \Phi_\lambda(z_n) \to c_\lambda, \tag{2.24}$$

where c_λ is defined in Lemma 2.3.

Lemma 2.8. *Under the assumptions of Theorem 1.1, for almost all $\lambda \in [1, \lambda_0]$, there exists a $z_\lambda \in E$ such that*

$$\Phi'_\lambda(z_\lambda) = 0,\ \Phi_\lambda(z_\lambda) = c_\lambda. \tag{2.25}$$

Proof. Let $\{z_n\}$ be the sequence obtained in Lemma 2.7. Since $\{z_n\}$ is bounded, we can assume $z_n \rightharpoonup z_\lambda$ in E and $z_n \to z_\lambda$ a.e. on $\mathbb{R}^N$. By Lemma 2.7 and the fact Φ'_λ is weakly sequentially continuous, we have

$$\langle \Phi'_\lambda(z_\lambda), \varphi\rangle = \lim_{n\to\infty} \langle \Phi'_\lambda(z_n), \varphi\rangle = 0, \quad \forall \varphi \in E. \tag{2.26}$$

That is $\Phi'_\lambda(z_\lambda) = 0$. By Lemma 2.7, we have

$$\Phi_\lambda(z_n) - \frac{1}{2}\langle \Phi'_\lambda(z_n), z_n\rangle = \int_{\mathbb{R}^N} \big[\frac{1}{2}H_z(x,z_n)z_n - H(x,z_n)\big] dx \to c_\lambda. \tag{2.27}$$

On the other hand, by Lemma 2.2, it is easy to prove that

$$\int_{\mathbb{R}^N} \frac{1}{2}H_z(x,z_n)z_n dt \to \int_{\mathbb{R}^N} \frac{1}{2}H_z(x,z_\lambda)z_\lambda dx, \tag{2.28}$$

$$\int_{\mathbb{R}^N} H(x,z_n) dt \to \int_{\mathbb{R}^N} H(x,z_\lambda) dx, \tag{2.29}$$

Therefore, by (2.28), (2.29) and the fact $\Phi'_\lambda(z_\lambda)=0$, we obtain

$$\Phi_\lambda(z_\lambda)=\Phi_\lambda(z_\lambda)-\frac{1}{2}\langle\Phi'_\lambda(z_\lambda),z_\lambda\rangle=\int_{\mathbb{R}^N}\big[\frac{1}{2}H_z(x,z_\lambda)z_\lambda-H(x,z_\lambda)\big]dx=c_\lambda. \tag{2.30}$$

The proof is complete. $\square$

Applying Lemma 2.8, we obtain the following result.

Lemma 2.9. *Under the assumptions of Theorem 1.1, for almost all $\lambda\in[1,\lambda_0]$, there exists sequences $z_n\in E$ and $\lambda_n\in[1,\lambda_0]$ with $\lambda_n\to\lambda$ such that*

$$\Phi'_{\lambda_n}(z_n)=0,\quad \Phi_{\lambda_n}(z_n)=c_{\lambda_n}.$$

Lemma 2.10. *Under the assumptions of Theorem 1.1,*

$$\int_{\mathbb{R}^N}\big[H(x,z)-H(x,rw)+r^2H_z(x,z)w-\frac{1+r^2}{2}H_z(x,z)z\big]dx\le C, \tag{2.31}$$

where $z\in E$, $w\in E^+$, $0\le r\le 1$ and the constant C does not depend on z,w,r.

Proof. This follows from (H5) if we take $z=z$ and $\eta=rw-z$. $\square$

Lemma 2.11. *Under the assumptions of Theorem 1.1, the sequences $\{z_n\}$ given in Lemma 2.9 are bounded.*

Proof. Suppose to the contrary that $\{z_n\}$ is unbounded. Without loss of generality, we can assume that $\|z_n\|\to\infty$ as $n\to\infty$. Let $w_n=\frac{z_n}{\|z_n\|}=w_n^++w_n^0+w_n^-$, then $\|w_n\|=1$. Going to a subsequence if necessary, we can assume that $w_n\rightharpoonup w$ in E, $w_n\to w$ in L^p for $p\in[2,2^*)$, $w_n\to w(x)$ a.e. on $\mathbb{R}^N$. For w, we have only the following two cases: $w\neq 0$ or $w=0$.

Case 1: $w\neq 0$**.** It follows from (H4) and Fatou's Lemma that

$$\int_{\mathbb{R}^N}\frac{H(x,z_n)}{\|z_n\|^2}dx=\int_{\mathbb{R}^N}\frac{H(x,z_n)}{|z_n|^2}|w_n|^2dx\to\infty\quad\text{as } n\to\infty, \tag{2.32}$$

which, together with Lemma 2.5 and 2.9 imply

$$0\le\frac{c_{\lambda_n}}{\|z_n\|^2}=\frac{\Phi_{\lambda_n}(z_n)}{\|z_n\|^2}=\frac{\lambda_n}{2}\|w_n^+\|^2-\frac{1}{2}\|w_n^-\|^2-\int_{\mathbb{R}^N}\frac{H(x,z_n)}{\|z_n\|^2}dx\to-\infty$$

as $n\to\infty$. This is a contradiction.

Case 2: $w=0$**.** We claim that there exist a constant C independent of z_n and λ_n such that

$$\Phi_{\lambda_n}(rz_n^+)-\Phi_{\lambda_n}(z_n)\le C,\quad\forall r\in[0,1]. \tag{2.33}$$

Since

$$\frac{1}{2}\langle\Phi'_{\lambda_n}(z_n),\varphi\rangle=\frac{1}{2}\lambda_n(z_n^+,\varphi^+)-\frac{1}{2}(z_n^-,\varphi^-)-\frac{1}{2}\int_{\mathbb{R}^N}H_z(x,z_n)\varphi dx=0,$$

for all $\varphi\in E$, it follows from the definition of Φ that

$$\begin{aligned}&\Phi_{\lambda_n}(rz_n^+)-\Phi_{\lambda_n}(z_n)\\&=\frac{1}{2}\lambda_n(r^2-1)\|z_n^+\|^2+\frac{1}{2}\|z_n^-\|^2+\int_{\mathbb{R}^N}\big[H(x,z_n)-H(x,rz_n^+)\big]dx\\&\quad+\frac{1}{2}\lambda_n(z_n^+,\varphi^+)-\frac{1}{2}(z_n^-,\varphi^-)-\frac{1}{2}\int_{\mathbb{R}^N}H_z(x,z_n)\varphi dx.\end{aligned} \tag{2.34}$$

Take

$$\varphi = (r^2+1)z_n^- - (r^2-1)z_n^+ + (r^2+1)z_n^0 = (r^2+1)z_n - 2r^2 z_n^+, \tag{2.35}$$

which together with Lemma 2.10 and (2.34) imply that

$$\begin{aligned}
&\Phi_{\lambda_n}(rz_n^+) - \Phi_{\lambda_n}(z_n)\\
&= -\frac{1}{2}\|z_n^-\|^2 + \int_{\mathbb{R}^N}\Big[H(x,z_n) - H(x,rz_n^+) + r^2 H_z(x,z_n)z_n^+\\
&\quad - \frac{1+r^2}{2}H_z(x,z_n)z_n\Big]dx \le C.
\end{aligned}$$

Hence, (2.33) holds. Let C_0 be a constant and take

$$r_n := \frac{C_0}{\|z_n\|} \to 0 \quad \text{as } n\to\infty.$$

Therefore, (2.33) implies

$$\Phi_{\lambda_n}(r_n z_n^+) - \Phi_{\lambda_n}(z_n) \le C$$

for all sufficiently large n. From $w_n^+ = \frac{z_n^+}{\|z_n\|}$ and Lemma 2.9 we have

$$\Phi_{\lambda_n}(C_0 w_n^+) \le C' \tag{2.36}$$

for all sufficiently large n. Note that Lemmas 2.5 and 2.9, and (H2) imply

$$\begin{aligned}
0 \le \frac{c_{\lambda_n}}{\|z_n\|^2} &= \frac{\Phi_{\lambda_n}(z_n)}{\|z_n\|^2}\\
&= \frac{\lambda_n}{2}\|w_n^+\|^2 - \frac{1}{2}\|w_n^-\|^2 - \frac{\int_{\mathbb{R}^N} H(x,z_n)dx}{\|z_n\|^2}\\
&\le \frac{\lambda_0}{2}\|w_n^+\|^2 - \frac{1}{2}\|w_n^-\|^2;
\end{aligned}$$

therefore

$$\lambda_0\|w_n^+\|^2 \ge \|w_n^-\|^2.$$

If $w_n^+ \to 0$, then from the above inequality, we have $w_n^- \to 0$, and therefore

$$\|w_n^0\|^2 = 1 - \|w_n^+\|^2 - \|w_n^-\|^2 \to 1. \tag{2.37}$$

Hence, $w_n^0 \to w^0$ because of $\dim E^0 < \infty$. Thus, $w \neq 0$, a contradiction. Therefore, $w_n^+ \nrightarrow 0$ and $\|w_n^+\|^2 \ge c_0$ for all n and some $c_0 > 0$. By (H2) and (H3), we have

$$\begin{aligned}
&\int_{\mathbb{R}^N} H(x, C_0 w_n^+)dx\\
&\le \frac{1}{2}\gamma C_0^2 \int_{\{|C_0 w_n^+|<\delta\}} |w_n^+|^2 dx + \frac{1}{2}c\int_{\{|C_0 w_n^+|\ge\delta\}} \big(C_0^2|w_n^+|^2 + C_0^p|w_n^+|^p\big)\,dx\\
&\le \frac{1}{2}\gamma C_0^2 \int_{\{|C_0 w_n^+|<\delta\}} |w_n^+|^2 dx + C_1'\int_{\{|C_0 w_n^+|\ge\delta\}} |w_n^+|^p dx.
\end{aligned} \tag{2.38}$$

For all sufficiently large n, if follows from (2.36), (2.38) and the fact $\lambda_n \to \lambda$, $w_n^+ \to w^+ = 0$ in L^p for all $[2, 2^*)$ that

$$\begin{aligned}
\Phi_{\lambda_n}(C_0 w_n^+) &= \frac{1}{2}\lambda_n C_0^2\|w_n^+\|^2 - \int_{\mathbb{R}^N} H(x, C_0 w_n^+)dx\\
&\ge \frac{1}{2}\lambda_n C_0^2\alpha - \frac{1}{2}\gamma C_0^2\int_{\{|C_0 w_n^+|<\delta\}} |w_n^+|^2 dx - C_1'\int_{\{|C_0 w_n^+|\ge\delta\}} |w_n^+|^p dx
\end{aligned}$$

$$\to \frac{1}{2}\lambda\alpha C_0^2, \quad \text{as } n \to \infty.$$

This implies that $\Phi_{\lambda_n}(C_0 w_n^+) \to \infty$ as $C_0 \to \infty$, contrary to (2.36). Therefore, $\{z_n\}$ are bounded. The proof is complete. $\square$

3. Proofs of main results

Proof of Theorem 1.1. From Lemma 2.9, there are sequences $1 < \lambda_n \to 1$ and $\{z_n\} \subset E$ such that $\Phi'_{\lambda_n}(z_n) = 0$ and $\Phi_{\lambda_n}(z_n) = c_{\lambda_n}$. By Lemma 2.11, we know $\{z_n\}$ is bounded in E, thus we can assume $z_n \rightharpoonup z$ in E, $z_n \to z$ in L^q for $q \in [2, 2^*)$, $z_n \to z(x)$ a.e. on $\mathbb{R}^N$. Therefore,

$$\langle \Phi'_{\lambda_n}(z_n), \varphi\rangle = \lambda_n(z_n^+, \varphi) - (z_n^-, \varphi) - \int_{\mathbb{R}^N} H_z(x, z_n)\varphi dx = 0, \quad \forall \varphi \in E. \tag{3.1}$$

Hence, in the limit

$$\langle \Phi'(z), \varphi\rangle = (z^+, \varphi) - (z^-, \varphi) - \int_{\mathbb{R}^N} H_z(x, z)\varphi dx = 0, \quad \forall \varphi \in E. \tag{3.2}$$

Thus $\Phi'(z) = 0$. Note that

$$\Phi_{\lambda_n}(z_n) - \frac{1}{2}\langle \Phi'_{\lambda_n}(z_n), z_n\rangle = \int_{\mathbb{R}^N} \left[\frac{1}{2}H_z(x, z_n)z_n - H(x, z_n)\right] dx = c_{\lambda_n} \geq c_1. \tag{3.3}$$

Similar to (2.28) and (2.29), we know that

$$\int_{\mathbb{R}^N} \big[\frac{1}{2}H_z(x, z_n)z_n - H(x, z_n)\big] dx \to \int_{\mathbb{R}^N} \big[\frac{1}{2}H_z(x, z)z - H(x, z)\big] dx,$$

as $n \to \infty$. It follows from $\Phi'(z) = 0$, (3.3) and Lemma 2.5 that

$$\begin{aligned}
\Phi(z) &= \Phi(z) - \frac{1}{2}\langle \Phi'(z), z\rangle \\
&= \int_{\mathbb{R}^N} \left[\frac{1}{2}H_z(x, z)z - H(x, z)\right] dx \\
&= \lim_{n\to\infty} \int_{\mathbb{R}^N} \big[\frac{1}{2}H_z(x, z_n)z_n - H(x, z_n)\big] dx \\
&\geq c_1 \geq \kappa > 0.
\end{aligned}$$

Therefore, $z \neq 0$. $\square$

Proof of Theorem 1.2. By Theorem 1.1, $\mathcal{M} \neq \emptyset$, where $\mathcal{M}$ is the collection of solution of (1.1). Let

$$\theta := \inf_{z\in\mathcal{M}} \Phi(z). \tag{3.4}$$

If z is a solution of (1.1), by Lemma 2.10, (take $r = 0$)

$$\begin{aligned}
\Phi(z) &= \Phi(z) - \frac{1}{2}\langle \Phi'(z), z\rangle \\
&= \int_{\mathbb{R}^N} \big[\frac{1}{2}H_z(x, z)z - H(x, z)\big] dx \\
&\geq -C = -\int_{\mathbb{R}^N} |W_1(x)| dx.
\end{aligned}$$

Thus, $\theta > -\infty$. Let $\{z_n\}$ be a subsequence in $\mathcal{M}$ such that

$$\Phi(z_n) \to \theta. \tag{3.5}$$

By Lemma 2.11, the sequence $\{z_n\}$ is bounded in E. Thus, $z_n \rightharpoonup z$ in E $z_n \to z$ in L^q for $q \in [2, 2^*)$ and $z_n \to z$ a.e. on $\mathbb{R}^N$, after passing to a subsequence. Therefore

$$\langle \Phi'(z_n), \varphi \rangle = (z_n^+, \varphi) - (z_n^-, \varphi) - \int_{\mathbb{R}^N} H_z(x, z_n)\varphi dx = 0, \quad \forall \varphi \in E. \tag{3.6}$$

Hence, in the limit

$$\langle \Phi'(z), \varphi \rangle = (z^+, \varphi) - (z^-, \varphi) - \int_{\mathbb{R}^N} H_z(x, z)\varphi dx = 0, \quad \forall \varphi \in E. \tag{3.7}$$

Thus, $\Phi'(z) = 0$. Similar to (2.28) and (2.29), we have

$$\begin{aligned}
\Phi(z_n) - \frac{1}{2}\langle \Phi'(z_n), z_n \rangle &= \int_{\mathbb{R}^N} \big[\frac{1}{2} H_z(x, z_n) z_n - H(x, z_n)\big] dx \\
&\to \int_{\mathbb{R}^N} \big[\frac{1}{2} H_z(x, z) z - H(x, z)\big] dx \quad \text{as } n \to \infty.
\end{aligned}$$

If follows from $\Phi'(z) = 0$ and (3.5) that

$$\begin{aligned}
\Phi(z) = \Phi(z) - \frac{1}{2}\langle \Phi'(z), z \rangle &= \int_{\mathbb{R}^N} \big[\frac{1}{2} H_z(x, z) z - H(x, z)\big] dx \\
&= \lim_{n\to\infty} \int_{\mathbb{R}^N} \big[\frac{1}{2} H_z(x, z_n) z_n - H(x, z_n)\big] dx \\
&= \lim_{n\to\infty} \Phi(z_n) = \theta.
\end{aligned}$$

Now suppose that $|H_z(x, z)| = o(|z|)$ as $|z| \to 0$. It follows from (H2) that for any $\varepsilon > 0$, there exists a constant $C_\varepsilon > 0$ such that

$$|H_z(x, z)| \le \varepsilon |z| + C_\varepsilon |z|^{p-1}. \tag{3.8}$$

Let

$$\alpha := \inf_{z \in \mathcal{M}'} \Phi(z),$$

where $\mathcal{M}' := \mathcal{M} \setminus \{0\}$. Let $\{z_n\}$ be a sequence in $\mathcal{M} \setminus \{0\}$ such that

$$\Phi(z_n) \to \alpha. \tag{3.9}$$

Note that

$$0 = \langle \Phi'(z_n), z_n^+ \rangle = \|z_n^+\|^2 - \int_{\mathbb{R}^N} H_z(x, z_n) z_n^+ dx, \tag{3.10}$$

which together with (3.8), Hölder inequality and the Sobolev embedding theorem implies

$$\begin{aligned}
\|z_n^+\|^2 &= \int_{\mathbb{R}^N} H_z(x, z_n) z_n^+ dx \\
&\le \varepsilon \int_{\mathbb{R}^N} |z_n||z_n^+| dx + C_\varepsilon \int_{\mathbb{R}^N} |z_n|^{p-1}|z_n^+| dx \\
&\le \varepsilon \|z_n\|\|z_n^+\| + C'_\varepsilon \|z_n\|_p^{p-1} \|z_n^+\| \\
&\le \varepsilon \|z_n\|\|z_n^+\| + C''_\varepsilon \|z_n\|_p^{p-2} \|z_n\|\|z_n^+\| \\
&\le \varepsilon \|z_n\|^2 + C''_\varepsilon \|z_n\|_p^{p-2} \|z_n\|^2.
\end{aligned} \tag{3.11}$$

Similarly, we obtain

$$\|z_n^-\|^2 \le \varepsilon \|z_n\|^2 + C''_\varepsilon \|z_n\|_p^{p-2} \|z_n\|^2. \tag{3.12}$$

From (3.11) and (3.12), we have

$$\|z_n\|^2 \le 2\varepsilon \|z_n\|^2 + 2C''_\varepsilon \|z_n\|_p^{p-2} \|z_n\|^2, \tag{3.13}$$

which means $\|z_n\|_p \geq c$ for some constant $c > 0$. Since $z_n \to z$ in L^p, we know $z \neq 0$. As before, $\Phi(z_n) \to \Phi(z) = \alpha$ as $n \to \infty$. $\square$

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