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ASYMPTOTIC BEHAVIOR OF STOCHASTIC THREE-SPECIES PREDATOR-PREY SYSTEMS WITH WHITE AND LÉVY NOISE

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ABSTRACT. In this article, we propose a three-species prey-predator system with Holling II functional response and stochastic perturbations involving white noise and Lévy noise. Firstly, we study the existence and uniqueness of a global positive solution and stochastic ultimate boundedness. Then, we obtain sufficient conditions for stability, extinction, strongly persistence in the mean and stochastic permanence in the sense of probability for the stochastic system. The results show that both white noise and Lévy noise may change the asymptotic properties of the population system. Finally, some examples that chaotic dynamics can be influenced by stochastic noises.

1. Introduction

The predator-prey models have attracted great attention because of their rich and complicated dynamical behaviors, in which functional response plays an important role to determine dynamical behaviors such as stability, oscillation, bifurcation and even chaos (see [4]-[18]). In the past few decades, food chain models with Holling-type functional response have been widely studied by many researchers; see [3, 4, 5, 7, 18]. For instance, the famous Hastings and Powell's model depicted a three-species food chain with the Holling II functional response [3]

$$\begin{aligned} dx_1(t) &= x_1(t)[1 - x_1(t) - \frac{a_1x_2(t)}{1+b_1x_1(t)}],\\ dx_2(t) &= x_2(t)[-r_2 + \frac{a_1x_1(t)}{1+b_1x_1(t)} - \frac{a_2x_3(t)}{1+b_2x_2(t)}],\\ dx_3(t) &= x_3(t)[-r_3 + \frac{a_2x_2(t)}{1+b_2x_2(t)}], \end{aligned} \tag{1.1}$$

where x_i are the population densities of prey, middle predator and top predator [3, 7] respectively. A more general three-special predator-prey model with the Holling II

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functional response has the form

$$\begin{aligned}\frac{dx_1(t)}{dt} &= x_1(t)[r_1 - a_{11}x_1(t) - \frac{a_{12}x_2(t)}{1+b_1x_1(t)}],\\ \frac{dx_2(t)}{dt} &= x_2(t)[-r_2 + \frac{a_{21}x_1(t)}{1+b_1x_1(t)} - a_{22}x_2(t) - \frac{a_{23}x_3(t)}{1+b_2x_2(t)}],\\ \frac{dx_3(t)}{dt} &= x_3(t)[-r_3 + \frac{a_{32}x_2(t)}{1+b_2x_2(t)} - a_{33}x_3(t)],\end{aligned} \tag{1.2}$$

where $x_i(t)$, $i = 1, 2, 3$ denote the population densities of prey, meso-predator and super-predator at time t respectively, r_1 is intrinsic growth rate and $r_i > 0$ $(i = 2, 3)$ are the death rates, $a_{ii} > 0$ $(i = 1, 2, 3)$ represent the intraspecies competition coefficients, $a_{12} > 0$ and $a_{23} > 0$ stand for the capture rates, $a_{21} > 0$ and $a_{32} > 0$ represent the efficiency of food conversion, and $1/b_i$ $(i = 1, 2)$ denote the half-saturation constant of meso-predator and super-predator respectively.

As we known, in the real world, ecosystems are unavoidably subject to stochastic perturbations because of random fluctuation of the birth rates, death rates, carrying capacity and so on. In recent years, white noise driven by Brownian motion has been taken into consideration in the process of modeling [11], and the study of dynamical behaviors for stochastic population systems with white noise has become fascinating [6, 8, 10, 13]. Except for white noise, ecosystems may suffer sudden environmental perturbations such as earthquakes, hurricanes, floods and so on, which may cause jumps of population number and great influences for dynamical properties of the systems. So it is reasonable to introduce Lévy noise described by Lévy random processes into the systems. Stochastic population systems with Lévy noise have been extensively studied by some scholars in the last few years [17]. In most of ecosystems, the functional responses are linear, but linear ones have some limitations and may be unable to accurately describe various natural phenomena [12]. In fact, nonlinear function response has much richer dynamical behaviors than linear function response, and lots of critical properties are demonstrated only via nonlinear function response [3]. Therefore, it is interesting to further study the population systems with nonlinear functional response and stochastic perturbations such as white noise and Lévy noise.

Motivated by the above discussions, we take white noise and colored Lévy noise into the model (1.2), and formulate the following hybrid stochastic three-species predator-prey system with the Holling II functional response

$$\begin{aligned}dx_1(t) &= x_1(t)[r_1 - a_{11}x_1(t) - \frac{a_{12}x_2(t)}{1+b_1x_1(t)}]dt\\ &\quad + \sigma_1 x_1(t)dB_1(t) + x_1(t^-)\int_{\mathbb{Y}} \gamma_1(u)\tilde{N}(dt,du),\\ dx_2(t) &= x_2(t)[-r_2 + \frac{a_{21}x_1(t)}{1+b_1x_1(t)} - a_{22}x_2(t) - \frac{a_{23}x_3(t)}{1+b_2x_2(t)}]dt\\ &\quad + \sigma_2 x_2(t)dB_2(t) + x_2(t^-)\int_{\mathbb{Y}} \gamma_2(u)\tilde{N}(dt,du),\\ dx_3(t) &= x_3(t)[-r_3 + \frac{a_{32}x_2(t)}{1+b_2x_2(t)} - a_{33}x_3(t)]dt\\ &\quad + \sigma_3 x_3(t)dB_3(t) + x_3(t^-)\int_{\mathbb{Y}} \gamma_3(u)\tilde{N}(dt,du),\end{aligned} \tag{1.3}$$

where $x_i(t^-)$ is the left limit of $x_i(t)$, $B_i(t)$ is the standard Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions, σ_i^2 is the intensity of white noise, N is a Poisson counting measure with characteristic measure λ on a measurable subset $\mathbb{Y}$ of $(0, +\infty)$ with $\lambda(\mathbb{Y}) < +\infty$, $\widetilde{N}(dt, du) = N(dt, du) - \lambda(du)dt$ is the compensated random measure, $\gamma_i(u) > -1 (u \in \mathbb{Y})$ are bounded functions $(i = 1, 2, 3)$, and the meaning of other parameters are same with model (1.2).

We shall investigate the dynamical behaviors such as well-posedness, boundedness, stability, extinction and persistence for the above stochastic system. The main contributions of this paper are listed as follows. Firstly, we formulate a three-species predator-prey system with the Holling II functional response and hybrid stochastic perturbations involving white noise and Lévy noise. Secondly, we discuss the extinction and persistence in the mean and in the stochastic trajectory path. Lastly, we show that both white noise and Lévy noise have significant impacts on dynamical properties of the system.

The remaining part of this paper is organized as follows. In Section 2, we give some preliminary results on system(1.3). In Section 3, we analyze the asymptotic behaviors of system(1.3). In Section 4, the theoretical results are illustrated by some examples.

2. Preliminaries

Throughout this paper, we denote $R_+^3 = \{x = (x_1, x_2, x_3)^T \in R^3 : x_i > 0,\ i = 1, 2, 3\}$ with the norm $|x| = \sqrt{x_1^2 + x_2^2 + x_3^2}$, and assume $B_i(t) (i = 1, 2, 3)$ and N are independent. For convenience, we define the following notations

$$\beta_i = \frac{\sigma_i^2}{2} - \int_{\mathbb{Y}} \ln(1 + \gamma_i(u))\lambda(du), \quad i = 1, 2, 3;$$
$$Q_i(t) = \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_i(u))\tilde{N}(ds, du), \quad i = 1, 2, 3;$$
$$\overline{f(t)} = \frac{1}{t}\int_0^t f(s)ds, \quad f^* = \limsup_{t\to\infty} f(t), \quad f_* = \liminf_{t\to\infty} f(t).$$

To obtain the main results, we introduce the following assumptions.

(A1) There is a positive constant c such that $\int_{\mathbb{Y}} [\ln(1 + \gamma_i(u))]^2 \lambda(du) < c$, $i = 1, 2, 3$;

(A2) For any $t \geq 0$, $\sup_{t\geq 0} \int_0^t \int_{\mathbb{Y}} e^{s-t}[\gamma_i(u) - \ln(1 + \gamma_i(u))]\lambda(du)ds < \infty$, $i = 1, 2, 3$;

(A3) $B = \min\{r_1 - \beta_1, -r_2 - \beta_2, -r_3 - \beta_3\} > 0$.

Definition 2.1 ([8])**.** The solutions $x(t)$ of system (1.3) are called stochastically ultimately bounded if for each $\epsilon \in (0, 1)$, there is a positive constant $H := H(\epsilon)$ such that $x(t)$ with any initial value $x(0) \in R_+^3$ has the property that

$$\limsup_{t\to\infty} P(|x(t)| > H) < \epsilon.$$

Definition 2.2 ([1])**.** The system (1.3) is said to be stochastically permanent if for any $\epsilon \in (0, 1)$, there exist constants $\delta_1 = \delta_1(\epsilon) > 0$ and $\delta_2 = \delta_2(\epsilon) > 0$ such that

$$\liminf_{t\to\infty} P\{|x(t)| \geq \delta_1\} \geq 1 - \epsilon, \quad \liminf_{t\to\infty} P\{|x(t)| \leq \delta_2\} \geq 1 - \epsilon.$$

Definition 2.3 ([20]). Let $x(t) = (x_1(t), x_2(t), x_3(t))^T \in R^3_+$ be a solution to system (1.3), then for $i = 1, 2, 3$,

(1) the population $x_i(t)$ becomes extinct if $\lim_{t\to\infty} x_i(t) = 0$ a.s.;
(2) the population $x_i(t)$ becomes strongly persistent in the mean if $\liminf_{t\to\infty} \frac{1}{t}\int_0^t x_i(s)ds > 0$ a.s.;
(3) the population $x_i(t)$ is said to be stable in the mean if $\lim_{t\to\infty} \frac{1}{t}\int_0^t x_i(s)ds = c > 0$ a.s.

From the above definitions we can find that the stability in the mean must be strongly persistence in the mean, stochastic permanence implies stochastically ultimate boundedness, and stochastically ultimate boundedness means the solution will be ultimately bounded with large probability, stochastic permanence is the strongest property, indicating the eternal existence of the population.

To ensure that the system (1.3) has biological significance, we give well-posedness for the solution of system (1.3).

Lemma 2.4. *For any given initial value $x(0) \in R^3_+$, the system* (1.3) *has a unique global solution $x(t) \in R^3_+$ for all $t \geq 0$ almost surely.*

Proof. First, we prove that (1.3) has a unique positive local solution. For $t \geq 0$, we consider the system

$$\begin{aligned} du_1(t) &= (r_1 - \beta_1 - a_{11}e^{u_1(t)} - \frac{a_{12}e^{u_2(t)}}{1 + b_1e^{u_1(t)}})dt + \sigma_1 dB_1(t) \\ &\quad + \int_{\mathbb{Y}} \ln(1 + \gamma_1(u))\widetilde{N}(dt, du), \\ du_2(t) &= (-r_2 - \beta_2 + \frac{a_{21}e^{u_1(t)}}{1 + b_1e^{u_1(t)}} - a_{22}e^{u_2(t)} - \frac{a_{23}e^{u_3(t)}}{1 + b_2e^{u_2(t)}})dt + \sigma_2 dB_2(t) \\ &\quad + \int_{\mathbb{Y}} \ln(1 + \gamma_2(u))\widetilde{N}(dt, du), \\ du_3(t) &= (-r_3 - \beta_3 + \frac{a_{32}e^{u_2(t)}}{1 + b_2e^{u_2(t)}} - a_{33}e^{u_3(t)})dt + \sigma_3 dB_3(t) \\ &\quad + \int_{\mathbb{Y}} \ln(1 + \gamma_3(u))\widetilde{N}(dt, du), \end{aligned} \tag{2.1}$$

with initial value $(u_1(0), u_2(0), u_3(0))^{\mathrm{T}} = (\ln x_1(0), \ln x_2(0), \ln x_3(0))^{\mathrm{T}}$.

Clearly, (2.1) satisfies local Lipschitz condition, there is a unique local solution $(u_1(t), u_2(t), u_3(t))^{\mathrm{T}}$ on $[0, \tau_e)$, where τ_e is the explosion time. By Itô's formula, $(x_1(t), x_2(t), x_3(t))^T = (e^{u_1(t)}, e^{u_2(t)}, e^{u_3(t)})^T$ is the unique positive local solution to the system (1.3) with initial value $x_i(0) > 0$. Then, we will use the comparison theorem to prove $x(t)$ is global, i.e., $\tau_e = +\infty$. Considering the following stochastic system

$$
\begin{aligned}
dy_1(t) &= y_1(t)[r_1 - a_{11}y_1(t)]dt + \sigma_1 y_1(t)dB_1(t)\\
&\quad + y_1(t^-)\int_{\mathbb{Y}} \gamma_1(u)\tilde{N}(dt,du),\\
dy_2(t) &= y_2(t)[-r_2 + \frac{a_{21}}{b_1} - a_{22}y_2(t)]dt + \sigma_2 y_2(t)dB_2(t)\\
&\quad + y_2(t^-)\int_{\mathbb{Y}} \gamma_2(u)\tilde{N}(dt,du),\\
dy_3(t) &= y_3(t)[-r_3 + \frac{a_{32}}{b_2} - a_{33}y_3(t)]dt + \sigma_3 y_3(t)dB_3(t)\\
&\quad + y_3(t^-)\int_{\mathbb{Y}} \gamma_3(u)\tilde{N}(dt,du),
\end{aligned} \tag{2.2}
$$

with initial value $y_i(0) = x_i(0) > 0$, $i = 1,2,3$. By the comparison theorem for stochastic differential equation, we obtain for $t \in [0,\tau_e)$, $x_i(t) \le y_i(t)$, a.s., $i = 1,2,3$. According to [1, Theorem 2.1], the system (2.2) has a unique global solution $y_1(t)$, $y_2(t)$ and $y_3(t)$ for $t \ge 0$. Hence we have $\tau_e = +\infty$. □

The following lemma gives ultimate boundedness for the system (1.3).

Lemma 2.5. *For any initial value $x(0) \in R^3_+$ and $p > 0$, there is a constant K such that the solution $x(t)$ of system* (1.3) *satisfies* $\limsup_{t\to\infty} E|x(t)|^p \le K$, *and is stochastically ultimately bounded.*

Proof. Define a Lyapunov function $V(x) = x_1^p + x_2^p + x_3^p$, $p > 0$. Applying the generalized Itô's formula, we obtain

$$
E(e^t V(x)) = V(x(0)) + E\int_0^t e^s[V(x(s)) + \mathcal{L}V(x(s))]ds,
$$

where

$$
\begin{aligned}
\mathcal{L}V(x) =& - a_{11}px_1^{p+1} + x_1^p(pr_1 + \frac{p(p-1)}{2}\sigma_1^2\\
&+ \int_{\mathbb{Y}}[(1+\gamma_1(u))^p - 1]\lambda(du)) - \frac{a_{12}px_1^px_2}{1+b_1x_1}\\
&- a_{22}px_2^{p+1} + x_2^p(-pr_2 + \frac{p(p-1)}{2}\sigma_2^2\\
&+ \int_{\mathbb{Y}}[(1+\gamma_2(u))^p - 1]\lambda(du)) + \frac{a_{21}px_2^px_1}{1+b_1x_1} - \frac{a_{23}px_2^px_3}{1+b_2x_2}\\
&- a_{33}px_3^{p+1} + x_3^p(-pr_3 + \frac{p(p-1)}{2}\sigma_3^2\\
&+ \int_{\mathbb{Y}}[(1+\gamma_3(u))^p - 1]\lambda(du)) + \frac{a_{32}px_3^px_2}{1+b_2x_2}.
\end{aligned}
$$

From $a_{ij} > 0$, we can deduce that there exists a constant $K(p) > 0$ such that

$$
\begin{aligned}
V(x) + \mathcal{L}V(x) \le& - a_{11}px_1^{p+1} + x_1^p(1 + pr_1 + \frac{p(p-1)}{2}\sigma_1^2\\
&+ \int_{\mathbb{Y}}[(1+\gamma_1(u))^p - 1]\lambda(du)) - a_{22}px_2^{p+1}\\
&+ x_2^p(1 - pr_2 + \frac{a_{21}p}{b_1} + \frac{p(p-1)}{2}\sigma_2^2
\end{aligned}
$$

$$
\begin{aligned}
&+\int_{\mathbb{Y}}[(1+\gamma_2(u))^p-1]\lambda(du))-a_{33}px_3^{p+1}\\
&+x_3^p(1-pr_3+\frac{a_{32}p}{b_2}+\frac{p(p-1)}{2}\sigma_3^2\\
&+\int_{\mathbb{Y}}[(1+\gamma_3(u))^p-1]\lambda(du))\\
&\le K(p).
\end{aligned}
$$

Hence,

$$
E(e^tV(x_1(t),x_2(t),x_3(t)))\le V(x_1(0),x_2(0),x_3(0))+K(p)(e^t-1).
$$

Then

$$
\limsup_{t\to\infty}E(x_1^p(t)+x_2^p(t)+x_3^p(t))\le K(p).
$$

Since $n^{(1-\frac{p}{2})\wedge 0}|x|^p\le\sum_{i=1}^n x_i^p\le n^{(1-\frac{p}{2})\vee 0}|x|^p$, for all $p>0$, $x\in R_+^n$, we can find a constant $K=\frac{K(p)}{3^{(1-\frac{p}{2})\wedge 0}}>0$, this yields that $\limsup_{t\to\infty}E|x(t)|^p\le K$. And combining with Chebyshev inequality, we can derive that the solution of (1.3) is stochastically ultimately bounded. The proof is complete. □

The following lemma gives the pathwise estimation of system state.

Lemma 2.6. *Let* (A2) *hold, for any initial value* $x(0)\in R_+^3$, *the solution* $x(t)$ *of* (1.3) *has the property that* $\limsup_{t\to\infty}\frac{\ln x_i(t)}{t}\le 0$ *a.s.,* $i=1,2,3$.

Proof. Using the same method as in [1, Lemma 4.4] with (A2), we obtain that the solution $(y_1(t),y_2(t),y_3(t))$ of (2.2) satisfies $\limsup_{t\to\infty}\frac{\ln y_i(t)}{\ln t}\le 1$ a.s., $i=1,2,3$. Combining this and the limit $\lim_{t\to\infty}\frac{\ln t}{t}=0$, we have $\limsup_{t\to\infty}\frac{\ln y_i(t)}{t}\le 0$ a.s., $i=1,2,3$. Then by the inequality $x_i(t)\le y_i(t)$, $t\ge 0$, $i=1,2,3$, we can gain the desired result. □

Lastly, we also introduce the following basic lemma given in [9].

Lemma 2.7. *Let* (A1) *hold and* $Z(t)\in C(\Omega\times[0,+\infty),R_+)$.

(1) *If there exist two positive constants* T *and* λ_0 *such that for all* $t\ge T$,

$$
\ln Z(t)\le\lambda t-\lambda_0\int_0^t Z(s)ds+\sum_{i=1}^n\sigma_iB_i(t)+\sum_{i=1}^n\lambda_iQ_i(t),
$$

where λ,σ_i,λ_i *are constants,then*

$$
\overline{Z}^*\le\frac{\lambda}{\lambda_0}
$$

$$
quad a.s.,\ if \lambda\ge 0;
$$

$$
\lim_{t\to\infty}Z(t)=0\quad a.s.,\ if\ \lambda<0.
$$

(2) *If there exist there positive constants* T, λ *and* λ_0 *such that for all* $t\ge T$,

$$
\ln Z(t)\ge\lambda t-\lambda_0\int_0^t Z(s)ds+\sum_{i=1}^n\sigma_iB_i(t)+\sum_{i=1}^n\lambda_iQ_i(t),
$$

then $\overline{Z}_*\ge\frac{\lambda}{\lambda_0}$ *a.s.*

3. Asymptotic behavior of system (1.3)

In this section, we shall investigate the asymptotic behaviors such as extinction, persistence and stability for system (1.3). Firstly, we give main results on extinction, strongly persistence in the mean and stability in the mean.

Theorem 3.1. *Let* (A1) *and* (A2) *hold. We have the following statements for system* (1.3).

(i) *If* $r_1 - \beta_1 < 0$ *and* $-r_i - \beta_i < 0$, $i = 2, 3$, *then all populations become extinct.*

(ii) *If* $r_1 - \beta_1 > 0$, $-r_2 - \beta_2 + \frac{a_{21}}{b_1} < 0$ *and* $-r_3 - \beta_3 < 0$, *then the populations* $x_2(t), x_3(t)$ *become extinct and* $x_1(t)$ *is stable in the mean, namely,*

$$\lim_{t\to\infty} \overline{x_1(t)} = \frac{r_1 - \beta_1}{a_{11}} \quad a.s.$$

(iii) *If* $-r_3 - \beta_3 + \frac{a_{32}}{b_2} < 0$, *then population* $x_3(t)$ *becomes extinct. Moreover, if* $r_1 - \beta_1 > \max\{0, a_{12}\frac{-r_2-\beta_2+\frac{a_{21}}{b_1}}{a_{22}}\}$ *and* $-r_2 - \beta_2 > 0$, *then the populations* $x_1(t), x_2(t)$ *are strongly persistent in the mean, that is,*

$$\frac{r_1 - \beta_1 - a_{12}\frac{-r_2-\beta_2+\frac{a_{21}}{b_1}}{a_{22}}}{a_{11}} \le \overline{x_1(t)}_* \le \overline{x_1(t)}^* \le \frac{r_1 - \beta_1}{a_{11}} \quad a.s.,$$

$$\frac{-r_2 - \beta_2}{a_{22}} \le \overline{x_2(t)}_* \le \overline{x_2(t)}^* \le \frac{-r_2 - \beta_2 + a_{21}}{a_{22}} \quad a.s.$$

(iv) *If* $-r_3 - \beta_3 > 0$, *then the population variable* $x_3(t)$ *satisfies*

$$\frac{-r_3 - \beta_3}{a_{33}} \le \overline{x_3(t)}_* \le \overline{x_3(t)}^* \le \frac{-r_3 - \beta_3 + a_{32}}{a_{33}} \quad a.s.$$

Furthermore, if $r_1 - \beta_1 > \max\big\{0, a_{12}\frac{-r_2-\beta_2+\frac{a_{21}}{b_1}}{a_{22}}\big\}$ *and* $-r_2 - \beta_2 > \max\big\{0, a_{23}\frac{-r_3-\beta_3+\frac{a_{32}}{b_2}}{a_{33}}\big\}$, *then*

$$\frac{r_1 - \beta_1 - a_{12}\frac{-r_2-\beta_2+\frac{a_{21}}{b_1}}{a_{22}}}{a_{11}} \le \overline{x_1(t)}_* \le \overline{x_1(t)}^* \le \frac{r_1 - \beta_1}{a_{11}} \quad a.s.,$$

$$\frac{-r_2 - \beta_2 - a_{23}\frac{-r_3-\beta_3+\frac{a_{32}}{b_2}}{a_{33}}}{a_{22}} \le \overline{x_2(t)}_* \le \overline{x_2(t)}^* \le \frac{-r_2 - \beta_2 + a_{21}}{a_{22}} \quad a.s.$$

That is, all populations are strongly persistent in the mean.

Proof. Applying generalized Itô's formula to $\ln x_1(t)$ leads to

$$\begin{aligned} d\ln x_1(t) &= (r_1 - \beta_1 - a_{11}x_1(t) - \frac{a_{12}x_2(t)}{1 + b_1x_1(t)})dt + \sigma_1 dB_1(t) \\ &\quad + \int_{\mathbb{Y}} \ln(1 + \gamma_1(u))\widetilde{N}(dt, du). \end{aligned}$$

Integrating from 0 to t and then dividing it by t yields

$$\begin{aligned} &\frac{\ln(x_1(t)/x_1(0))}{t} \\ &= r_1 - \beta_1 - a_{11}\overline{x_1(t)} - a_{12}\overline{\frac{x_2(t)}{1 + b_1x_1(t)}} + \frac{\sigma_1 B_1(t)}{t} + \frac{Q_1(t)}{t}. \end{aligned} \tag{3.1}$$

Similarly,

$$\begin{aligned} \frac{\ln(x_2(t)/x_2(0))}{t} &= -r_2 - \beta_2 + a_{21}\overline{\frac{x_1(t)}{1+b_1x_1(t)}} - a_{22}\overline{x_2(t)} - a_{23}\overline{\frac{x_3(t)}{1+b_2x_2(t)}} \\ &\quad + \frac{\sigma_2 B_2(t)}{t} + \frac{Q_2(t)}{t}, \end{aligned} \tag{3.2}$$

$$\begin{aligned} \frac{\ln(x_3(t)/x_3(0))}{t} &= -r_3 - \beta_3 + a_{32}\overline{\frac{x_2(t)}{1+b_2x_2(t)}} - a_{33}\overline{x_3(t)} + \frac{\sigma_3 B_3(t)}{t} \\ &\quad + \frac{Q_3(t)}{t}. \end{aligned} \tag{3.3}$$

Firstly, we shall prove the conclusion in case (i). By (3.1),

$$\frac{\ln(x_1(t)/x_1(0))}{t} \le r_1 - \beta_1 - a_{11}\overline{x_1(t)} + \frac{\sigma_1 B_1(t)}{t} + \frac{Q_1(t)}{t}.$$

Note that $r_1 - \beta_1 < 0$, hence by case (1) in Lemma 2.7,

$$\lim_{t\to\infty} x_1(t) = 0 \quad \text{a.s.}$$

Thus we have that

$$|\overline{\frac{x_1(t)}{1+b_1x_1(t)}}| \le |\overline{x_1(t)}| < \epsilon,$$

for sufficiently large t, where $0 < \epsilon < \frac{\beta_2+r_2}{a_{21}}$. Then for(3.2), we obtain

$$\frac{\ln(x_2(t)/x_2(0))}{t} \le -r_2 - \beta_2 + a_{21}\epsilon - a_{22}\overline{x_2(t)} + \frac{\sigma_2 B_2(t)}{t} + \frac{Q_2(t)}{t}.$$

Note that $-r_2 - \beta_2 < 0$ and $0 < \epsilon < \frac{\beta_2+r_2}{a_{21}}$, hence by case (1) in Lemma 2.7,

$$\lim_{t\to\infty} x_2(t) = 0 \quad \text{a.s.}$$

Similarly, applying this to (3.3), we have

$$\lim_{t\to\infty} x_3(t) = 0 \; a.s.$$

Secondly, we will give the proof of case (ii). From (3.2),

$$\frac{\ln(x_2(t)/x_2(0))}{t} \le -r_2 - \beta_2 + \frac{a_{21}}{b_1} - a_{22}\overline{x_2(t)} + \frac{\sigma_2 B_2(t)}{t} + \frac{Q_2(t)}{t}.$$

Since $-r_2 - \beta_2 + \frac{a_{21}}{b_1} < 0$, by case (1) in Lemma 2.7,

$$\lim_{t\to\infty} x_2(t) = 0 \; a.s.$$

The proof of $\lim_{t\to\infty} x_3(t) = 0$ a.s. is the same with that in (i), hence the details are omitted. For (3.1), we obtain

$$\frac{\ln(x_1(t)/x_1(0))}{t} \le r_1 - \beta_1 - a_{11}\overline{x_1(t)} + \frac{\sigma_1 B_1(t)}{t} + \frac{Q_1(t)}{t}.$$

By case (1) in Lemma 2.7, we deduce that

$$\overline{x_1(t)}^* \le \frac{r_1 - \beta_1}{a_{11}} \quad \text{a.s.}$$

From $\lim_{t\to\infty} x_2(t)=0$, we have $|\overline{\frac{x_2(t)}{1+b_1x_1(t)}}| \le |\overline{x_2(t)}| < \epsilon$, for sufficiently large t, where $0<\epsilon<\frac{r_1-\beta_1}{a_{12}}$. Then for (3.1), we obtain

$$\frac{\ln(x_1(t)/x_1(0))}{t} \ge r_1-\beta_1-a_{12}\epsilon-a_{11}\overline{x_1(t)}+\frac{\sigma_1B_1(t)}{t}+\frac{Q_1(t)}{t}.$$

From case (2) in Lemma 2.7, we deduce that

$$\overline{x_1(t)}_* \ge \frac{r_1-\beta_1-a_{12}\epsilon}{a_{11}} \quad \text{a.s.}$$

In view of the arbitrariness of ϵ, we obtain

$$\lim_{t\to\infty} \overline{x_1(t)} = \frac{r_1-\beta_1}{a_{11}} \quad \text{a.s.}$$

Thirdly, we shall prove the conclusion in case (iii). By (3.3),

$$\frac{\ln(x_3(t)/x_3(0))}{t} \le -r_3-\beta_3+\frac{a_{32}}{b_2}-a_{33}\overline{x_3(t)}+\frac{\sigma_3B_3(t)}{t}+\frac{Q_3(t)}{t}.$$

Note that $-r_3-\beta_3+\frac{a_{32}}{b_2}<0$, hence by case (1) in Lemma 2.7,

$$\lim_{t\to\infty} x_3(t)=0 \quad \text{a.s.}$$

According to (3.2),

$$\frac{\ln(x_2(t)/x_2(0))}{t} \le -r_2-\beta_2+\frac{a_{21}}{b_1}-a_{22}\overline{x_2(t)}+\frac{\sigma_2B_2(t)}{t}+\frac{Q_2(t)}{t}.$$

By case (1) in Lemma 2.7, we deduce that

$$\overline{x_2(t)}^* \le \frac{-r_2-\beta_2+\frac{a_{21}}{b_1}}{a_{22}} \quad \text{a.s.}$$

From $\lim_{t\to\infty} x_3(t)=0$, we have that $|\overline{\frac{x_3(t)}{1+b_2x_2(t)}}| \le |\overline{x_3(t)}| < \epsilon$, for sufficiently large t, where $0<\epsilon<\frac{-r_2-\beta_2}{a_{23}}$. Then for (3.2), we obtain

$$\frac{\ln(x_2(t)/x_2(0))}{t} \ge -r_2-\beta_2-a_{22}\overline{x_2(t)}-a_{23}\epsilon+\frac{\sigma_2B_2(t)}{t}+\frac{Q_2(t)}{t}.$$

Using case (2) in Lemma 2.7, we deduce that

$$\overline{x_2(t)}_* \ge \frac{-r_2-\beta_2-a_{23}\epsilon}{a_{22}} \quad \text{a.s.}$$

Therefore, in view of the arbitrariness of ϵ, we obtain

$$\frac{-r_2-\beta_2}{a_{22}} \le \overline{x_2(t)}_* \le \overline{x_2(t)}^* \le \frac{-r_2-\beta_2+\frac{a_{21}}{b_1}}{a_{22}} \quad \text{a.s.} \tag{3.4}$$

Through (3.1),

$$\frac{\ln(x_1(t)/x_1(0))}{t} \le r_1-\beta_1-a_{11}\overline{x_1(t)}+\frac{\sigma_1B_1(t)}{t}+\frac{Q_1(t)}{t}.$$

It follows from case (1) in Lemma 2.7 that

$$\overline{x_1(t)}^* \le \frac{r_1-\beta_1}{a_{11}} \quad \text{a.s.}$$

Combining inequality (3.4) and Lemma 2.6, from (3.1) we deduce that

$$a_{11}\overline{x_1(t)}_* \ge \liminf_{t\to\infty}\big\{r_1-\beta_1-\frac{\ln(x_1(t)/x_1(0))}{t}-a_{12}\overline{x_2(t)}+\frac{\sigma_1B_1(t)}{t}+\frac{Q_1(t)}{t}\big\}$$

$$
\begin{aligned}
&\ge r_1-\beta_1-\limsup_{t\to\infty}\frac{\ln x_1(t)}{t}-a_{12}\overline{x_2(t)}^*\\
&\ge r_1-\beta_1-a_{12}\frac{-r_2-\beta_2+\frac{a_{21}}{b_1}}{a_{22}}.
\end{aligned}
$$

So,

$$
\frac{r_1-\beta_1-a_{12}\frac{-r_2-\beta_2+\frac{a_{21}}{b_1}}{a_{22}}}{a_{11}}\le\overline{x_1(t)}_*\le\overline{x_1(t)}^*\le\frac{r_1-\beta_1}{a_{11}}\quad\text{a.s.}
$$

Finally, we shall prove case (iv). From (3.3),

$$
\frac{\ln(x_3(t)/x_3(0))}{t}\le -r_3-\beta_3+\frac{a_{32}}{b_2}-a_{33}\overline{x_3(t)}+\frac{\sigma_3B_3(t)}{t}+\frac{Q_3(t)}{t}.
$$

By case (1) in Lemma 2.7, we deduce that

$$
\overline{x_3(t)}^*\le\frac{-r_3-\beta_3+\frac{a_{32}}{b_2}}{a_{33}}\quad\text{a.s.}
$$

Again from (3.3),

$$
\frac{\ln(x_3(t)/x_3(0))}{t}\ge -r_3-\beta_3-a_{33}\overline{x_3(t)}+\frac{\sigma_3B_3(t)}{t}+\frac{Q_3(t)}{t}.
$$

Using case (2) in Lemma 2.7, we have

$$
\overline{x_3(t)}_*\ge\frac{-r_3-\beta_3}{a_{33}}\quad\text{a.s.}
$$

Therefore,

$$
\frac{-r_3-\beta_3}{a_{33}}\le\overline{x_3(t)}_*\le\overline{x_3(t)}^*\le\frac{-r_3-\beta_3+\frac{a_{32}}{b_2}}{a_{33}}\quad\text{a.s.}\tag{3.5}
$$

Through (3.2),

$$
\frac{\ln(x_2(t)/x_2(0))}{t}\le -r_2-\beta_2+\frac{a_{21}}{b_1}-a_{22}\overline{x_2(t)}+\frac{\sigma_2B_2(t)}{t}+\frac{Q_2(t)}{t}.
$$

According to case (1) in Lemma 2.7, we have

$$
\overline{x_2(t)}^*\le\frac{-r_2-\beta_2+\frac{a_{21}}{b_1}}{a_{22}}\quad\text{a.s.}
$$

Combining inequality (3.5), (3.2) and Lemma 2.6, we can deduce that

$$
\begin{aligned}
a_{22}\overline{x_2(t)}_*&\ge\liminf_{t\to\infty}\Big\{-r_2-\beta_2-\frac{\ln(x_2(t)/x_2(0))}{t}-a_{23}\overline{x_3(t)}+\frac{\sigma_2B_2(t)}{t}+\frac{Q_2(t)}{t}\Big\}\\
&\ge -r_2-\beta_2-\limsup_{t\to\infty}\frac{\ln x_2(t)}{t}-a_{23}\overline{x_3(t)}^*\\
&\ge -r_2-\beta_2-a_{23}\frac{-r_3-\beta_3+\frac{a_{32}}{b_2}}{a_{33}}.
\end{aligned}
$$

Therefore,

$$
\frac{-r_2-\beta_2-a_{23}\frac{-r_3-\beta_3+\frac{a_{32}}{b_2}}{a_{33}}}{a_{22}}\le\overline{x_2(t)}_*\le\overline{x_2(t)}^*\le\frac{-r_2-\beta_2+\frac{a_{21}}{b_1}}{a_{22}}\quad\text{a.s.}
$$

The estimation for the ultimate infimum and ultimate supremum of $\overline{x_1(t)}$ is similar with one in case (iii), hence it is omitted. □

Remark 3.2. When $a_{22} = a_{33} = 0$, we easily check the conclusion in the case (i) and case (ii) of Theorem 3.1 still holds. This means that the conclusion can be applied to the model (1.1) with stochastic effects and see the case from Example 4.1 later.

Furthermore, we will give a condition weaker than the one given in the above case (iv) to discuss the stochastic permanence in the sense of probability for the stochastic system (1.3).

Theorem 3.3. *If* (A3) *holds, then system* (1.3) *is stochastically permanent.*

Proof. We define a Lyapunov function $V(x(t)) := \frac{1}{x_1(t)+x_2(t)+x_3(t)}$, where $x(t) = (x_1(t), x_2(t), x_3(t))^T$ is any positive solution of (1.3). By generalized Itô's formula, we obtain

$$
\begin{aligned}
dV =& \Big\{ - V^2(x)[x_1(r_1 - a_{11}x_1 - \frac{a_{12}}{1+b_1x_1}x_2) \\
&+ x_2(-r_2 + \frac{a_{21}}{1+b_1x_1}x_1 - a_{22}x_2 - \frac{a_{23}}{1+b_2x_2}x_3) \\
&+ x_3(-r_3 + \frac{a_{32}}{1+b_2x_2}x_2 - a_{33}x_3)] + V^3(x)(\sum_{i=1}^3 \sigma_i x_i)^2 \\
&+ \int_{\mathbb{Y}} (\frac{1}{\sum_{i=1}^3 x_i(1+\gamma_i)} - V(x))\lambda(du)\Big\} dt - V^2(x)\sum_{i=1}^3 \sigma_i x_i dB_i(t) \\
&+ \int_{\mathbb{Y}} (\frac{1}{\sum_{i=1}^3 x_i(1+\gamma_i)} - V)\widetilde{N}(dt, du).
\end{aligned}
$$

Note that

$$
\begin{aligned}
&\lim_{\theta\to 0^+} \Big\{ \frac{\max_{i=1,2,3}\sigma_i^2}{2}\theta + \int_{\mathbb{Y}} [\frac{1}{\theta \min_{i=1,2,3}(1+\gamma_i(u))^\theta} - \frac{1}{\theta}]\lambda(du)\Big\} \\
&= \int_{\mathbb{Y}} \ln \frac{1}{\min_{i=1,2,3}(1+\gamma_i(u))}\lambda(du) \\
&= -\int_{\mathbb{Y}} \ln[\min_{i=1,2,3}(1+\gamma_i(u))]\lambda(du).
\end{aligned}
$$

By (A3), we can find a sufficiently small $\theta > 0$ such that

$$
\begin{aligned}
\min r_1 - \frac{\max \sigma_1^2}{2}(1+\theta) - \int_{\mathbb{Y}} [\frac{1}{\theta \min(1+\gamma_1(u))^\theta} - \frac{1}{\theta}]\lambda(du) > 0, \\
-\max_{i=2,3} r_i - \frac{\max_{i=2,3}\sigma_i^2}{2}(1+\theta) - \int_{\mathbb{Y}} [\frac{1}{\theta \min_{i=2,3}(1+\gamma_i(u))^\theta} - \frac{1}{\theta}]\lambda(du) > 0.
\end{aligned}
$$

Then there is a small positive η such that

$$
\begin{aligned}
\min r_1 - \frac{\max \sigma_1^2}{2}(1+\theta) - \int_{\mathbb{Y}} [\frac{1}{\theta \min(1+\gamma_1(u))^\theta} - \frac{1}{\theta}]\lambda(du) > \frac{\eta}{\theta}, \\
-\max_{i=2,3} r_i - \frac{\max_{i=2,3}\sigma_i^2}{2}(1+\theta) - \int_{\mathbb{Y}} [\frac{1}{\theta \min_{i=2,3}(1+\gamma_i(u))^\theta} - \frac{1}{\theta}]\lambda(du) > \frac{\eta}{\theta}.
\end{aligned} \tag{3.6}
$$

We define another Lyapunov function, $U(x) = e^{\eta t} V^{\theta}(x)$. Then

$$
\begin{aligned}
dU(x) = e^{\eta t}\Big\{ & F(V(x))dt - \theta V^{\theta-1}(x)V^2(x)\sum_{i=1}^{3}\sigma_i x_i dB_i(t) \\
& + \int_{\mathbb{Y}} [(\frac{1}{\sum_{i=1}^{3} x_i(1+\gamma_i)})^{\theta} - V^{\theta}(x)]\widetilde{N}(dt,du)\Big\},
\end{aligned}
\tag{3.7}
$$

where

$$
\begin{aligned}
& F(V(x)) \\
=& \eta V^{\theta}(x) - \theta V^{\theta-1}(x)V^2(x)[x_1(r_1 - a_{11}x_1 - \frac{a_{12}}{1+b_1x_1}x_2) \\
& + x_2(-r_2 + \frac{a_{21}}{1+b_1x_1}x_1 - a_{22}x_2 - \frac{a_{23}}{1+b_2x_2}x_3) \\
& + x_3(-r_3 + \frac{a_{32}}{1+b_2x_2}x_2 - a_{33}x_3)] + \theta V^{\theta-1}(x)V^3(x)\Big(\sum_{i=1}^{3}\sigma_i x_i\Big)^2 \\
& + \frac{\theta(\theta-1)}{2}V^{\theta-2}(x)V^4(x)(\sum_{i=1}^{3}\sigma_i x_i)^2 + \int_{\mathbb{Y}}[(\frac{1}{\sum_{i=1}^{3} x_i(1+\gamma_i)})^{\theta} - V^{\theta}(x)]\lambda(du).
\end{aligned}
$$

By (A3), we see that $-r_i \geq B + \frac{\sigma_i^2}{2} - \int_{\mathbb{Y}} \ln(1+\gamma_i(u))\lambda(du)$ $(i = 2,3)$ and $r_1 \geq B + \frac{\sigma_1^2}{2} - \int_{\mathbb{Y}} ln(1+\gamma_1(u))\lambda(du)$. Thus, we can find constants θ and η satisfying (3.6) such that

$$
\begin{aligned}
& B\theta - \frac{\theta^2}{2}\max_{i=1,2,3}\sigma_i^2 - \int_{\mathbb{Y}}\Big[\frac{1}{\min_{i=1,2,3}(1+\gamma_i(u))^{\theta}} - 1 \\
& + \theta\ln\min_{i=1,2,3}(1+\gamma_i(u))\Big]\lambda(du) > \eta > 0.
\end{aligned}
\tag{3.8}
$$

Accordingly,

$$
\begin{aligned}
& F(V(x)) \\
& \leq \eta V^{\theta}(x) - \theta V^{\theta-1}(x)V(x)\sum_{i=1}^{3}x_i(B - \int_{\mathbb{Y}} ln(1+\gamma_i(u))\lambda(du))V(x) \\
& \quad - \theta V^{\theta-1}(x)V(x)\sum_{i=1}^{3}x_i\frac{\sigma_i^2}{2}V(x) + \theta V^{\theta-1}(x)V^2(x)\sum_{i=1}^{3}a_{ii}x_i^2 \\
& \quad + \theta V^{\theta-1}(x)V^2(x)(|a_{12}-a_{21}|x_1x_2 + |a_{23}-a_{32}|x_2x_3) \\
& \quad + \theta V^{\theta-1}(x)V(x)\Big(\sum_{i=1}^{3}\sigma_i x_i\Big)^2 V^2(x) \\
& \quad + \frac{\theta(\theta-1)}{2}V^{\theta-2}(x)V^2(x)\Big(\sum_{i=1}^{3}\sigma_i x_i\Big)^2 V^2(x) \\
& \quad + \int_{\mathbb{Y}}[(\frac{1}{\sum_{i=1}^{3}x_i(1+\gamma_i)})^{\theta} - V^{\theta}]\lambda(du) \\
& := \mathcal{O}(V^{\theta}(x))V^{\theta}(x) + G(V(x)),
\end{aligned}
\tag{3.9}
$$

where $\lim_{V\to+\infty}\frac{G(V(x))}{V^\theta(x)}=0$. Since $0\le V^2(x)\sum_{i=1}^3 a_{ii}x_i^2\le \max_{i=1,2,3}a_{ii}$, $0\le V^2(x)(x_1x_2+x_2x_3)\le\frac12$, we obtain

$$\begin{aligned}\mathcal{O}(V^\theta(x)) =&\eta-\theta\sum_{i=1}^3 x_i(B-\int_{\mathbb{Y}} ln(1+\gamma_i(u))\lambda(du))V(x)-\theta\sum_{i=1}^3 x_i\frac{\sigma_i^2}{2}V(x)\\ &+\frac{\theta(\theta+1)}{2}(\sum_{i=1}^3\sigma_ix_i)^2V^2(x)+\int_{\mathbb{Y}}[(\frac{x_1+x_2+x_3}{\sum_{i=1}^3 x_i(1+\gamma_i)})^\theta-1]\lambda(du).\end{aligned}$$

In view of Jensen's inequality and (3.8), we deduce that

$$\begin{aligned}\mathcal{O}(V^\theta)&\le\eta-B\theta+\theta\int_{\mathbb{Y}}\sum_{i=1}^3 x_iln(1+\gamma_i(u))V\lambda(du)+\frac{\theta^2}{2}\Big(\sum_{i=1}^3\sigma_ix_i\Big)^2V^2(x)\\ &\quad+\int_{\mathbb{Y}}\big[(\frac{x_1+x_2+x_3}{\sum_{i=1}^3 x_i(1+\gamma_i)})^\theta-1\big]\lambda(du)\\ &\le\eta-B\theta+\frac{\theta^2}{2}\max_{i=1,2,3}\sigma_i^2+\int_{\mathbb{Y}}\sum_{n=2}^\infty\frac{\theta^n}{n!}(\ln\frac{x_1+x_2+x_3}{\sum_{i=1}^3 x_i(1+\gamma_i)})^n\lambda(du)\\ &\le\eta-B\theta+\frac{\theta^2}{2}\max_{i=1,2,3}\sigma_i^2+\int_{\mathbb{Y}}\big[\frac{1}{\min_{i=1,2,3}(1+\gamma_i(u))^\theta}-1\\ &\quad+\theta\ln\min_{i=1,2,3}(1+\gamma_i(u))\big]\lambda(du)<0.\end{aligned}\tag{3.10}$$

From (3.7), (3.9) and (3.10), there exists $H(\theta)>0$ such that

$$E[e^{\eta t}V^\theta(x(t))]-V^\theta x((0))\le E\int_0^t e^{\eta s}H(\theta)ds=\frac{H(\theta)}{\eta}(e^{\eta t}-1).$$

So we have

$$\limsup_{t\to\infty}E(V^\theta(x(t)))\le\frac{H(\theta)}{\eta}.$$

In light of $\frac{1}{|x(t)|^\theta}\le 2^{\frac{\theta}{2}}V^\theta(t)$, we obtain

$$\limsup_{t\to\infty}E(\frac{1}{|x(t)|^\theta})\le 2^{\theta/2}\frac{H(\theta)}{\eta}.$$

Based on Chebyshev's inequality, for any $\epsilon>0$, there exists $H=\frac{\sqrt2}{2}(\frac{\eta\epsilon}{H(\theta)})^{1/\theta}>0$ such that

$$\limsup_{t\to\infty}P\{|x(t)|<H\}=\limsup_{t\to\infty}P\big\{\frac{1}{|x(t)|}>\frac1H\big\}\le H^\theta\limsup_{t\to\infty}E\big(\frac{1}{|x(t)|^\theta}\big)\le\epsilon.$$

Therefore,

$$\liminf_{t\to\infty}P\{|x(t)|\ge H\}\ge1-\epsilon.$$

Combining this and Lemma 2.5, it follows that (1.3) is stochastically permanent. □

Remark 3.4. According to Theorems 3.1 and 3.3, the dynamical behavior of system (1.3) may be changed by stochastic perturbations. In fact, when the deterministic system (1.2) is persistent, the species in the stochastic system (1.3) always trend to extinction if we take large enough white noise parameters σ_i^2 or large enough Lévy noise parameters $\gamma_i(\cdot)$ such that $r_1-\beta_1,-r_i-\beta_i<0, i=2,3$. Whereas, when the species in the deterministic system (1.2) becomes extinct, the stochastic system

(1.3) will become persistent by handling Lévy noise satisfying $B > 0$. However, our results may be unable to handle white noises to change the extinction for the deterministic system (1.2) into the persistence for the stochastic system (1.3) because $\sigma_i \geq 0$. That is, there should be different effects on dynamics of (1.3) between white noises and Lévy noise.

4. Examples and conclusions

In this section, we shall give some numerical examples to illustrate our theoretical results, and show the effects of white noise and Lévy noise to dynamical properties of the system.

Example 4.1. Consider the following stochastic system based on the Hastings and Powell's model (1.1)

$$\begin{aligned} dx_1(t) &= x_1(t)[(1 - x_1(t) - \frac{a_1 x_2(t)}{1 + b_1 x_1(t)})dt] \\ &\quad + \sigma_1 x_1 dB_1(t) + x_1(t^-)\int_{\mathbb{Y}} \gamma_1(u)\tilde{N}(dt, du), \\ dx_2(t) &= x_2(t)[(-r_2 + \frac{a_1 x_1(t)}{1 + b_1 x_1(t)} - \frac{a_2 x_3(t)}{1 + b_2 x_2(t)})dt] \\ &\quad + \sigma_2 x_2 dB_2(t) + x_2(t^-)\int_{\mathbb{Y}} \gamma_2(u)\tilde{N}(dt, du), \\ dx_3(t) &= x_3(t)[(-r_3 + \frac{a_2 x_2(t)}{1 + b_2 x_2(t)})dt] \\ &\quad + \sigma_3 x_3 dB_3(t) + x_3(t^-)\int_{\mathbb{Y}} \gamma_3(u)\tilde{N}(dt, du), \end{aligned}$$

which is a special example of system (1.3) with $r_1 = a_{11} = 1$, $a_{22} = a_{33} = 0$, $a_{12} = a_{21} = a_1$, $a_{23} = a_{32} = a_2$.

Take $a_1 = 5$, $a_2 = 0.1$, $b_1 = 3$, $b_2 = 2$, $r_2 = 0.4$, $r_3 = 0.01$. According to [7], the deterministic Hastings and Powell's model exhibits chaotic dynamics in long-term behavior (i.e., $\sigma_i = \gamma_i(\cdot) = 0, i = 1, 2, 3$). According to Theorem 3.1 and Remark 3.2, we shall show that the chaotic behaviors can be eliminated under certain stochastic perturbations by choosing different values of σ_i, γ_i and $\lambda(\mathbb{Y}) = 1$.

Case I. Let $\gamma_i(u) = 0$, $i = 1, 2, 3$, $\sigma_1 = 2$, $\sigma_2 = 1$, $\sigma_3 = 0.5$, then $r_1 - \beta_1 = -1$, $-r_2 - \beta_2 = -0.9$, $-r_3 - \beta_3 = -0.135$. From the case (i) in Theorem 3.1, we have all populations become extinct.

Case II. Let $\gamma_i(u) = 0, i = 1, 2, 3$, $\sigma_1 = 1$, $\sigma_2 = 2$, $\sigma_3 = 0.5$, then $r_1 - \beta_1 = 0.5$, $-r_2 - \beta_2 + \frac{a_{21}}{b_1} = -0.7333$, $-r_3 - \beta_3 = -0.135$. By the case (ii) in Theorem 3.1, the populations $x_2(t)$, $x_3(t)$ become extinct, $x_1(t)$ is stable in the mean and $\lim_{t\to\infty} \overline{x_1(t)} = 0.5$ a.s.

Case III. Let $\sigma_i = 0, i = 1, 2, 3$, $\gamma_1(u) = -0.8$, $\gamma_2(u) = -0.4$, $\gamma_3(u) = -0.3$, then $r_1 - \beta_1 = -0.6094$, $-r_2 - \beta_2 = -0.9108$, $-r_3 - \beta_3 = -0.3667$. It follows from the case (i) in Theorem 3.1 that all populations go to extinction.

Case IV. Let $\sigma_i = 0, i = 1, 2, 3$, $\gamma_1(u) = 0.2$, $\gamma_2(u) = -0.8$, $\gamma_3(u) = -0.6$, then $r_1 - \beta_1 = 1.1823$, $-r_2 - \beta_2 + \frac{a_{21}}{b_1} = -0.3428$, $-r_3 - \beta_3 = -0.9263$. From case (ii) in Theorem 3.1, we see that the populations $x_2(t)$, $x_3(t)$ become extinct, $x_1(t)$ is stable in the mean and $\lim_{t\to\infty} \overline{x_1(t)} = 1.1823$ a.s.

The above cases illustrate also that the chaotic dynamics can be suppressed by either white noises or Lévy noises.

Example 4.2. Consider the following stochastic system with white noises or Lévy noises

$$\begin{aligned} dx_1(t) &= x_1(t)[0.8 - 0.4x_1(t) - \frac{0.3x_2(t)}{1+3x_1(t)}]dt \\ &\quad + \sigma_1 x_1(t)dB_1(t) + x_1(t^-)\int_{\mathbb{Y}} \gamma_1(u)\tilde{N}(dt,du), \\ dx_2(t) &= x_2(t)[-0.5 + \frac{0.2x_1(t)}{1+3x_1(t)} - 0.4x_2(t) - \frac{0.2x_3(t)}{1+2x_2(t)}]dt \\ &\quad + \sigma_2 x_2(t)dB_2(t) + x_2(t^-)\int_{\mathbb{Y}} \gamma_2(u)\tilde{N}(dt,du), \\ dx_3(t) &= x_3(t)[-0.3 + \frac{0.1x_2(t)}{1+2x_2(t)} - 0.4x_3(t)]dt \\ &\quad + \sigma_3 x_3(t)dB_3(t) + x_3(t^-)\int_{\mathbb{Y}} \gamma_3(u)\tilde{N}(dt,du). \end{aligned} \tag{4.1}$$

In the following, we take different values of white noise and Lévy noise to show that the system (4.1) has different dynamical behaviors.

Case I. Let $\sigma_i = 0.2, \gamma_i(u) = 0, i = 1,2,3$, then $r_1 - \beta_1 = 0.78$, $-r_2 - \beta_2 + \frac{a_{21}}{b_1} = -0.4533$, $-r_3 - \beta_3 = -0.32$. From the second statement of Theorem 3.1, it follows that the meso-predator and super-predator become extinct while the prey is stable in the mean, and $\lim_{t\to\infty} \overline{x_1(t)} = 1.95$ a.s.

Case II. Let $\sigma_i = 0, i = 1,2,3, \gamma_1(u) = -0.6$, $\gamma_2(u) = -0.4$, $\gamma_3(u) = -0.2$, then $r_1 - \beta_1 = -0.1163$, $-r_2 - \beta_2 = -1.0108$, $-r_3 - \beta_3 = -0.5231$. The first statement of Theorem 3.1 exhibit that all populations go to extinction.

Case III. Let $\gamma_1(u) = 0.3$, $\gamma_2(u) = 0.8$, $\gamma_3(u) = 0.4$, then $r_1 - \beta_1 = 1.0624$, $-r_2 - \beta_2 = 0.0878$, $-r_3 - \beta_3 = 0.0365$, $r_1 - \beta_1 - a_{12}\frac{-r_2-\beta_2+\frac{a_{21}}{b_1}}{a_{22}} = 0.9466$, $-r_2 - \beta_2 - a_{23}\frac{-r_3-\beta_3+\frac{a_{32}}{b_2}}{a_{33}} = 0.0446$. The fourth statement of Theorem 3.1 show that all populations are strongly persistent in the mean.

Case IV. Let $\sigma_i = 0.2, i = 1,2,3$, $\gamma_1(u) = -0.6$, $\gamma_2(u) = -0.4$, $\gamma_3(u) = -0.2$, then $r_1 - \beta_1 = -0.1363$, $-r_2 - \beta_2 = -1.0308$, $-r_3 - \beta_3 = -0.5431$. From the first statement of Theorem 3.1, we can see that all populations go to extinction.

Case V. Let $\sigma_i = 0.2, i = 1,2,3$, $\gamma_1(u) = 0.6$, $\gamma_2(u) = -0.4$, $\gamma_3(u) = -0.2$, then $r_1 - \beta_1 = 1.25$, $-r_2 - \beta_2 + \frac{a_{21}}{b_1} = -0.9642$, $-r_3 - \beta_3 = -0.5431$. The second statement of Theorem 3.1 tells us that the populations $x_2(t)$ and $x_3(t)$ become extinct and the population $x_1(t)$ is stable in the mean, and $\lim_{t\to\infty} \overline{x_1(t)} = 3.125$ a.s.

Case VI. Let $\sigma_i = 0.2, i = 1,2,3$, $\gamma_1(u) = 0.3$, $\gamma_2(u) = 0.8$, $\gamma_3(u) = -0.2$, then $r_1 - \beta_1 = 1.0424$, $-r_2 - \beta_2 = 0.0678$, $-r_3 - \beta_3 + \frac{a_{32}}{b_2} = -0.4931$, $r_1 - \beta_1 - a_{12}\frac{-r_2-\beta_2+\frac{a_{21}}{b_1}}{a_{22}} = 0.9416$. From the third statement of Theorem 3.1, it follows that the population $x_3(t)$ becomes extinct and the populations $x_1(t)$ and $x_2(t)$ are strongly persistent in the mean.

Case VII. Let $\sigma_i = 0.2, i = 1,2,3$, $\gamma_1(u) = 0.3$, $\gamma_2(u) = 0.8$, $\gamma_3(u) = 0.4$, then $r_1 - \beta_1 = 1.0424$, $-r_2 - \beta_2 = 0.0678$, $-r_3 - \beta_3 = 0.0165$, $r_1 - \beta_1 - a_{12}\frac{-r_2-\beta_2+\frac{a_{21}}{b_1}}{a_{22}} = 0.9416$, $-r_2 - \beta_2 - a_{23}\frac{-r_3-\beta_3+\frac{a_{32}}{b_2}}{a_{33}} = 0.0346$. The fourth statement of Theorem 3.1 exhibit that all populations are strongly persistent in the mean.

Case VIII. Let $\sigma_i = 0.2, i = 1,2,3$, $\gamma_1(u) = -0.5$, $\gamma_2(u) = 0.7$, $\gamma_3(u) = 0.4$, then $r_1 - \beta_1 = 0.0869$, $-r_2 - \beta_2 = 0.0106$, $-r_3 - \beta_3 = 0.0165$. From Theorem 3.3 we can see that the system (4.1) is stochastically permanent.

From the above cases, we can switch dynamical behaviors between the extinction and the permanence by handling the parameters of Lévy noises for the stochastic system. We can also handle the parameters of white noises to change permanence into extinction, but our results are invalid to switch the dynamical behaviors from extinction to permanence by utilizing white noises. This may be because there are have different impacts on dynamical properties of the system between white noises and Lévy noises.

Conclusion. This paper formulated a Holling-II type three-species prey-predator system with white noise and Lévy noise. First of all, we showed that the system admits a unique global positive solution, and discuss stochastic ultimate boundedness of the solution. Next we obtained sufficient conditions for extinction, strongly persistence in the mean and stability in the mean of the population and stochastic permanence of the system. Finally, our theoretical analysis reveals that dynamical behaviors of the system are closely related to stochastic noises. That is, under stochastic perturbations the extinct species can become persistent and the persistent species can go to extinction, and there are different effects on dynamical properties between white noises and Lévy noises for the stochastic system. In addition, we found an interesting result that the chaotic dynamics can be supressed by stochastic noises for the Hastings and Powell's model. However, we didn't further investigate how to generate chaos by white noises and Lévy noises for the stochastic system. This leaves some interesting works to develop this direction in future.

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