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Superconnections: an interpretation of the standard model *

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Dedicated to Eyvind H. Wichmann on his 70th birthday

Abstract

The mathematical framework of superbundles as pioneered by D. Quillen suggests that one considers the Higgs field as a natural constituent of a superconnection. I propose to take as superbundle the exterior algebra obtained from a Hermitian vector bundle of rank n, when $n = 2$ for the electroweak theory and when $n = 5$ for the full Standard Model. The present setup is similar to but avoids the use of non-commutative geometry.

1 Introduction

The key to our present-day understanding of the electroweak interactions is the spontaneous breakdown of local gauge symmetries. However, the mass generating mechanism requires the introduction of the so-called Higgs field. A long-standing problem is to give meaning to scalar fields as natural ingredients of a gauge theory. The subject has received special attention, since, up to now, the Higgs particle has not been observed in experiments. It would be impossible to provide a coherent account of all attempts to interpret the Higgs field within the context of supersymmetry or non-commutative geometry, nor shall I try to review the history of the Standard Model, or discuss its details. In the present approach, which I believe is new, I continue the work begun in [4, 5] and concentrate on one aspect only: the possible use of Quillen's concept of a superconnection [3, 5] in physics, since it became increasingly clear to me that Euclidean field theory is the study of G superbundles. The goals that motivate such a study are:

- To reduce the number of free parameters of the Standard Model
- To think of the Higgs field as some extension of the conventional gauge potential

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- To naturally explain the form of the Higgs potential
- To unite the gauge coupling and the Yukawa coupling to fermions in one Lagrangian, $\bar{\psi} i\mathcal{D}\psi$, where $\mathcal{D}$ is a generalized Dirac operator
- To predict the mass of the Higgs boson
- To predict the number of fermion generations and the structure of the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

Let us start with a few definitions. By a *superspace* we mean a $\mathbb{Z}_2$-graded vector space $V = V^+ \oplus V^-$. Elements of $V^\pm$ are said to be

– even/odd,
– right-handed/left-handed,
– positive/negative,
– matter-/antimatter-like, or
– bosonic/fermionic

depending on their use in physics. Examples of spaces with such a structure are abundant in the theory of elementary particles. In most instances, $\dim V^+ = \dim V^-$. For brevity we shall refer to the even(odd)ness indicated by the $\pm$ sign as the *parity* of elements in V. Notice also that the $\mathbb{Z}_2$-grading carries over to direct sums and tensor products of graded vector spaces in an obvious manner.

A *superalgebra* is a superspace whose product respects the grading, i.e. the even(odd)ness of its elements. Examples are:

– the exterior algebra of an ungraded vector space,
– the Clifford algebra of an ungraded vector space,
– the endomorphism algebra of a superspace.

Exterior algebras will be seen to play a particular role in what follows. We therefore remind the reader that the exterior algebra $\bigwedge E$ of a vector space E is $\mathbb{Z}$-graded by the degree p of the exterior power and $\mathbb{Z}_2$-graded by the parity $(-1)^p$. Notice that $\dim \bigwedge^+ E = \dim \bigwedge^- E = 2^{n-1}$ where $n = \dim E$.

Within a superalgebra A the *supercommutator* is defined as follows:

$$[a,b] = \begin{cases} ab + ba & \text{if } a, b \text{ are odd} \\ ab - ba & \text{otherwise} \end{cases} \qquad (a, b \in A).$$

Hence, the supercommutator of a pair of odd elements is in fact their anticommutator. From now on brackets $[\cdot,\cdot]$ will always denote the supercommutator, provided the parity of its arguments are unambiguously defined. By construction, any exterior algebra is supercommutative, i.e., all brackets vanish. One calls a *supertrace* any linear functional that vanishes on supercommutators. With exterior algebras any linear functional is a supertrace.

When it comes to studying differential operators on manifolds, the concept of derivations in a superalgebra will be essential. Such derivations may be even/odd depending on whether they preserve parity or not. Even derivations are defined as usual. By contrast, an odd *derivation* D of a superalgebra satisfies

$$D(ab) = \begin{cases} (Da)b + a(Db) & \text{if } a \text{ is even} \\ (Da)b - a(Db) & \text{if } a \text{ is odd.} \end{cases}$$

Inner derivations are given by supercommutators $D = [c, \cdot]$ where c is fixed. Moreover, the linear space of all derivations is a Lie superalgebra since any bracket $[D, D']$ is a derivation, too.

We shall frequently use tensor products. It is important to realize that tensor products of superalgebras are special. Generally speaking, if X and Y are $\mathbb{Z}_2$-graded algebras, the multiplication in $X \otimes Y$ is given by

$$(x \otimes y)(x' \otimes y') = \begin{cases} -xx' \otimes yy' & \text{if } x' \text{ and } y \text{ are odd} \\ xx' \otimes yy' & \text{otherwise.} \end{cases}$$

In physics, such tensor products are familiar constructions when dealing with Fock spaces of different fermions. For, if E and F are two vector spaces, there is a natural isomorphism

$$\bigwedge(E \oplus F) \cong \bigwedge E \otimes \bigwedge F \ .$$

2 Superconnections and the Higgs Field

Let M now be a (connected, oriented) differentiable manifold. It is helpful to think of M as a model of Euclidean spacetime. Later, we shall assume that its dimension is even. By a *superbundle* we mean a vector bundle on M whose fibers are superspaces. Examples are:

– the bundle $\bigwedge T^*M$ of exterior differentials,
– the Clifford bundle $C(M)$ of a Riemannian manifold,
– the endomorphism bundle of a superbundle.

Sections of a superbundle B obviously form a superspace $\Gamma(B)$.

The most common object for integration on manifolds is the exterior algebra of differential forms (a supercommutative algebra),

$$\Omega = \Gamma(\bigwedge T^*M).$$

Elements of Ω of degree p are said to be p-forms on M. They are even (odd) if p is even (odd). The even elements constitute a commutative subalgebra Ω^+ of Ω. There is a canonical odd derivation d on Ω, commonly known as the *exterior derivative*, mapping p-forms into $(p+1)$-forms such that $d^2 = 0$, which reduces to the ordinary derivative df on functions $f : M \to \mathbb{R}$.

In gauge theory one chooses a compact Lie group G, called the gauge group, and some principal G bundle P over M. A vector bundle, which is an associated G bundle, may be obtained from any representation ρ of the group G. The choice of ρ is dictated by the multiplet of particles (or fields) one wishes to describe.

Here we shall be interested in representations spaces (real or complex) carrying a $\mathbb{Z}_2$-grading respected by ρ. This in particular implies that ρ has subrepresentions $\rho^{\pm}$ of same dimension.

Let B some G superbundle obtained in the above manner. We will then consider the superspace of B-valued differential forms,

$$S(B) = \Gamma(\textstyle\bigwedge T^*M \otimes B),$$

and also the superalgebra of *local operators* on $S(B)$,

$$A(B) = \Gamma(\textstyle\bigwedge T^*M \otimes \operatorname{End} B).$$

As opposed to a differential operator, a local operator preserves fibers, that is to say, it commutes with the multiplication by functions $f \in C^\infty(M)$. Since the algebra Ω acts fiberwise on the vector space $S(B)$ in an obvious manner, there is a natural embedding $\Omega \to A(B)$. The following notion, due to D. Quillen, generalizes the concept of a covariant derivative. See also [1] for details.

Definition. A *superconnection* on B is a (first-order) differential operator $\mathbb{D}$ on $S(B)$ of odd type satisfying the Leibniz rule

$$[\mathbb{D}, \omega)] = d\omega, \qquad \omega \in \Omega \subset A(B).$$

A few observations are immediate.

1. If $\mathbb{D}$ and $\mathbb{D}'$ are two different superconnections, their difference supercommutes with ω and so is a local operator of odd type: superconnections form an affine space modeled on the vector space $A^-(B)$.

2. $\mathbb{D}^2 = \frac{1}{2}[\mathbb{D}, \mathbb{D}]$ is even. From the generalized Jacobi identity and the relation $[\mathbb{D}[\mathbb{D}, \omega] = d^2\omega = 0$ we see that $\mathbb{D}^2$ commutes with ω and hence is a local operator. We call $\mathbb{F} = \mathbb{D}^2 \in A^+(B)$ the *curvature* of the superbundle B.

3. Bianchi's identity $[\mathbb{D}, \mathbb{F}] = 0$ holds.

4. Any superconnection gives rise to an odd derivation of the superalgebra $A(B)$, again denoted $\mathbb{D}$, in a way consistent with the Leibniz rule: $\mathbb{D}a = [\mathbb{D}, a]$ $(a \in A(B))$. Thus, $\mathbb{D}\mathbb{F} = 0$ is another way to write Bianchi's identity.

It is not difficult to prove the following structure theorem. Any superconnection decomposes as $\mathbb{D} = D + L$ where D is a covariant derivative on B while $L \in A^-(B)$ (with no further restriction on L). Thus, D maps p-forms into $(p+1)$-forms and, in local coordinates,

$$D = dx^\mu \big(\partial_\mu + A_\mu(x)\big)$$

where $A_\mu(x)$ is the gauge potential, taking values in some representation of the Lie algebra of G, and

$$L = L(x) + \sum_{p\geq 2} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_p} L_{\mu_1\cdots\mu_p}(x) \tag{1}$$

with scalar field $L(x)$ (the $p = 0$ contribution) and tensor fields $L_{\mu_1\cdots\mu_p}(x)$ of degree $p \geq 2$. Fields in L are thought of as sections of the endomorphism bundle $\mathrm{End}^- B$ if p is even or $\mathrm{End}^+ B$ if p is odd. The idea of superconnections has thus provided new fields other than the gauge potential with a definite behavior under gauge transformations. We shall refer to the scalar field $L(x)$ as the *Higgs field* of the superconnection $I\!\!D$. At present, we need not introduce tensor fields of degree $p \geq 2$ in a superconnection if we merely wish to accommodate the particles of the Standard Model, and we will assume from now on that the series (1) truncates after the zeroth order term:

$$L = L(x) \in \Gamma(\mathrm{End}^- B).$$

With respect to the grading $B = B^+ \oplus B^-$, we may conveniently represent any superconnection as a matrix of operators:

$$I\!\!D = \begin{pmatrix} D^+ & i\Phi^* \\ i\Phi & D^- \end{pmatrix} \qquad L = \begin{pmatrix} 0 & i\Phi^* \\ i\Phi & 0 \end{pmatrix} .$$

Clearly, $D^\pm$ are covariant derivatives on $B^\pm$. We also assume that B is a Hermitian vector bundle and $I\!\!D$ is skew-selfadjoint in the sense that

$$(I\!\!Dv, w) + (v, I\!\!Dw) = d(v, w), \qquad v, w \in S(B)$$

where $(v, w) \in C^\infty(M)$ denotes the induced scalar product of sections. At each point $x \in M$, the field $\Phi^*(x)$ is the adjoint of the field $\Phi(x)$ and may be looked upon as an $n \times n$ matrix if the bundle B has rank $2n$ and some frame has been chosen. With no further restrictions on L, the Higgs field has n^2 independent components.

The curvature decomposes as

$$I\!\!F = D^2 + [D, L] + L^2$$

where the 2-form $F = D^2$ is referred to as the *field strength*, the 1-form $[D, L]$ is the covariant derivative of the Higgs field, and the 0-form L^2 determines the *Higgs potential*, once a scalar product $(I\!\!F, I\!\!F)$ has been defined (details in [4]).

3 Constructing the Standard Model

Assume now that P is a principal G bundle where the gauge group G is either the unitary group $U(n)$ or a subgroup thereof. Since G acts on P but also on $\mathbb{C}^n$ (equipped with the standard scalar product), we may construct the associated G bundle

$$V = P \times_G \mathbb{C}^n$$

having fibers isomorphic to $\mathbb{C}^n$. Though there is no natural graded structure on V, the exterior algebra $B = \bigwedge V$ is in fact a G superbundle of rank 2^n. The representation $\bigwedge$ of G acting on its fibers respects parity and has subrepresentations $\bigwedge^{\pm}$ of equal dimension. By construction, V is a Hermitian vector bundle and so is $\bigwedge V$. We will be mainly concerned with the following two cases:

$n = 2$	$G = U(2)$	electro-weak theory
$n = 5$	$G \subset U(5)$	Standard Model.

To introduce fermions into the theory we need a few more assumptions. Let M now be a Riemannian manifold of dimension $2m$ and $C(M)$ be its Clifford bundle (canonically associated with the cotangent bundle T^*M). Its construction formalizes Dirac's notion of an "algebra of γ matrices connected with space-time." Let $c : T^*M \to \operatorname{End} S$ be a spin^c-structure on M, i.e., S is a complex vector bundle of rank 2^m on M, called the *spinor bundle*, and the bundle map c satisfies $c(v)^2 + (v, v) = 0$ with respect to the scalar product $(\cdot, \cdot)$ on T^*M induced by the Riemannian structure. It may be shown that c extends to an algebraic isomorphism $C(M) \to \operatorname{End} S$ and thus gives S the structure of a Clifford module. The γ matrices are locally recovered by setting $\gamma^\mu = c(dx^\mu)$. Clifford modules formalize Dirac's concept of a "space on which the γ's act." The eigenvalues ± 1 of the chirality operator $\gamma_5 = i^m \gamma^1 \gamma^2 \cdots \gamma^{2m}$ give S the structure of a superbundle.

In order to incorporate gauge symmetries we consider the *twisted spinor bundle*,

$$E = \bigwedge V \otimes S.$$

Since both S and $\bigwedge V$ are superbundles, so is E. In particular,

$$E^+ = (\bigwedge{}^+ V \otimes S^+) \oplus (\bigwedge{}^- V \otimes S^-).$$

Dirac fields describing leptons and quarks are thought of as components of *one* master field $\psi \in \Gamma(E^+)$. The restriction to E^+ couples the helicity of S to the parity of the exterior algebra. Note that the master field ψ is capable of describing 2^n elementary fermion fields. Left- and right-handed fields count as different components. The fact that fermion fields are Grassmann variables in Euclidean field theory will not be discussed. Nevertheless, the reader should be aware that $\bar\psi(x)$ and $\psi(x)$ anticommute and, contrary to the situation in Minkowski field theory, are unrelated.

The fermionic part of the Lagrangian is taken to be $\bar\psi i \mathcal{D} \psi$ where $\mathcal{D}$ is a generalized Dirac operator. We shall not go into the details here except to say that $\mathcal{D}$ is constructed from the superconnection $\mathbb{D}$ in very much the same way as the conventional Dirac operator $\not{D}$ is constructed from the covariant derivative D. Formally, $\mathcal{D}$ is a (first-order) differential operator on $\Gamma(E)$ of odd type satisfying $[\mathcal{D}, f] = c(df)$ for all $f \in C^\infty(M)$. Being odd in particular means that a generalized Dirac operator cannot contain a "mass term." Leptons and quarks must acquire their masses by the Higgs mechanism. Our ansatz for the Lagrangian takes care of both the Yukawa and the gauge coupling of fermions.

Let us first turn to the $U(2)$ model describing weak isospin (quantum number I) along with hypercharge (quantum number Y). It goes without saying that $U(1)_Y$ is considered the center of the group $U(2)$. But, as a matter of convention, the generator of $U(1)_Y$ is taken here as the *negative* hypercharge. Irreducible representations are characterized by I and Y subject to the restriction $2I + Y$ =even. After symmetry breaking the residual gauge group will be

$$U(1)_Q = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix},\ 0 \le \alpha < 2\pi \right\}$$

giving rise to the notion of the electric charge Q as a conserved quantity. Likewise, the generator of $U(1)_Q$ in any representation is taken as $-Q$. By construction, the charge then satisfies the relation $Q = I_3 + \frac{1}{2}Y$ of Gell-Mann-Nishijima.

The master field ψ in the $U(2)$ model has four components,

$$\psi = (\nu_{eR}, e_R, \nu_{eL}, e_{eL}) \in \Gamma(E^+),$$

associated to three invariant subspaces of $\bigwedge\mathbb{C}^2$. As indicated, the components describe the electron (and the accompanying neutrino). There is another master field for the muon and one for the τ lepton. Left- and right-handed fields have different properties under gauge transformations:

$$\begin{array}{rllll} \nu_{eR} \to \bigwedge^0\mathbb{C}^2 & \text{singlet,} & Y=0 & I=0 & Q=0 \\ e_R \to \bigwedge^2\mathbb{C}^2 & \text{singlet,} & Y=-2 & I=0 & Q=-1 \\ \nu_{eL}, e_{eL} \to \bigwedge^1\mathbb{C}^2 & \text{doublet,} & Y=-1 & I=\frac{1}{2} & Q=0,-1. \end{array}$$

The appearance of a right-handed neutrino field, foreign to most weak interaction theories, signalizes that the neutrino is assumed to acquire a small mass after symmetry breaking.

The bosonic sector has a spin-one gauge field of four components, corresponding to the photon, the Z, and the $W^\pm$. In addition, there are two Higgs doublets of opposite hypercharge. If the Lagrangian is at most quadratic in the curvature $I\!\!F$ and gauge invariant, there are only very few free parameters left that enter the action functional.

We now turn to another Lie group G with Lie algebra

$$\mathrm{Lie}\, G \cong \mathbf{su(3)} \oplus \mathbf{su(2)} \oplus \mathbf{u(1)} \tag{2}$$

large enough to enable us to incorporate quark fields and strong interactions. In addition, we require that G be a subgroup of $SU(5)$, i.e., we define

$$G = \{(u,v) \in \mathrm{U}(3) \times \mathrm{U}(2) \mid \det u \cdot \det v = 1\} \tag{3}$$

and let the embedding $G \to SU(5)$ be given by

$$(u,v) \mapsto \begin{pmatrix} u & 0 \\ 0 & v \end{pmatrix}\ .$$

There are in fact three basic symmetry groups involved in our model. Note that they are related by the following exact sequence

$$1 \longrightarrow \mathrm{SU}(3) \xrightarrow{j} G \xrightarrow{s} \mathrm{U}(2) \longrightarrow 1 \tag{4}$$

where $j(u) = (u, 1)$ and $s(u, v) = v$. Though there is the isomorphism (2) between Lie algebras, the group G cannot be identified with the direct product $SU(3) \times U(2)$. It is correct to say that the color group $SU(3)$ of quantum chromodynamics is embedded in G as a subgroup. But the gauge group $U(2)$ of leptons is recovered only as the quotient $G/SU(3)$. This fact influences our idea of what the hypercharge Y should be. To see the point more clearly we consider the exact sequence

$$1 \longrightarrow \mathbb{Z}_3 \xrightarrow{j} \tilde{U}(1)_Y \xrightarrow{s} U(1)_Y \longrightarrow 1 \tag{5}$$

obtained from (4) by restricting to the centers. In this way we learn that the group $\tilde{U}(1)_Y$, a threefold cover of $U(1)_Y$, may also be looked upon as a one-dimensional closed subgroup of the two-torus:

$$\tilde{U}(1)_Y = \{(e^{i\beta}, e^{i\alpha}) \mid 3\beta + 2\alpha = 0 \bmod 2\pi\}.$$

As before, $U(1)_Y$ defines the hypercharge. So does $\tilde{U}(1)_Y$ by the local isomorphism s whose inverse is

$$s^{-1}(e^{i\alpha}) = (e^{-i2\alpha/3}, e^{i\alpha}). \tag{6}$$

Locally, the group $\tilde{U}(1)_Y$ is represented by a phase factor $e^{-i\alpha Y}$ in any unitary irreducible representation of G. Notice, however, that $Y \in \mathbb{Q}$ in general.

The vector bundle V is now modeled on the fiber space

$$\mathbb{C}^5 = \mathbb{C}^3 \oplus \mathbb{C}^2$$

with subspaces $\mathbb{C}^3$ and $\mathbb{C}^2$ carrying the *fundamental representations* of the color group SU(3) and the weak-isospin group $SU(2)$ respectively. As explained above, passage to the exterior algebra $\bigwedge \mathbb{C}^5$ is very essential, the fiber space of the superbundle $\bigwedge V$ carrying the reducible representation $\bigwedge$ of G. From the natural isomorphism $\bigwedge(\mathbb{C}^3 \oplus \mathbb{C}^2) \cong \bigwedge \mathbb{C}^3 \otimes \bigwedge \mathbb{C}^2$ we obtain $\bigwedge(u, v) = \bigwedge u \otimes \bigwedge v$ for $(u, v) \in G$ and hence

$$\textstyle\bigwedge^r(u, v) = \sum_{p+q=r} \bigwedge^p u \otimes \bigwedge^q v, \qquad r = 0, \ldots, 5.$$

Consequently, any fundamental fermion (quark or lepton) must belong to one of the irreducible representations of G,

$$\textstyle\bigwedge^{p,q} = \bigwedge^p \otimes \bigwedge^q \qquad p = 0, 1, 2, 3, \qquad q = 0, 1, 2$$

whose dimension is $\binom{p}{3}\binom{q}{2}$. To find its hypercharge we use Eq.(6):

$$e^{-i\alpha Y} = \textstyle\bigwedge^{p,q}\big(s^{-1}(e^{i\alpha})\big) = \exp(-i2p\alpha/3 + iq\alpha)$$

and thus obtain the fundamental relation

$$Y = \frac{2}{3}p - q\,. \tag{7}$$

Clearly, Y is integer valued if $p = 0 \bmod 3$ (for leptons) and fractional otherwise (for quarks). A similar statement holds for the electric charge Q.

The master field ψ has $2^5 = 32$ components. Dirac fields that enter ψ are characterized by three different "parities" owing to the $\mathbb{Z}_2$-gradings of $\bigwedge\mathbb{C}^5$, $\bigwedge\mathbb{C}^3$, and $\bigwedge\mathbb{C}^2$. Their interpretation is as follows:

$p+q =$even	: right-handed	$p+q =$odd	: left-handed
$p =$even	: matter	$p =$odd	: antimatter
$q =$even	: singlets	$q =$odd	: doublets.

Hence, there are left-handed and right-handed fields of equal number. Likewise, there are matter fields and antimatter fields of equal number. Each doublet is accompanied by two singlets. The following table shows the details.

	$q=0$	$q=2$	$q=1$	
$p=0$	ν_{eR}	e_R	ν_{eL}	e_L
$p=2$	u_{1R}	d_{1R}	u_{1L}	d_{1L}
	$-u_{2R}$	$-d_{2R}$	$-u_{2L}$	$-d_{2L}$
	u_{3R}	d_{3R}	u_{3L}	d_{3L}
$p=1$	d^c_{1L}	u^c_{1L}	d^c_{1R}	$-u^c_{1R}$
	d^c_{2L}	u^c_{2L}	d^c_{2R}	$-u^c_{2R}$
	d^c_{3L}	u^c_{3L}	d^c_{3R}	$-u^c_{3R}$
$p=3$	e^c_L	ν^c_{eL}	e^c_R	$-\nu^c_{eR}$

Quarks fields such as u(up) and d(down) come in three *colors*: $i = 1, 2, 3$. The upper index ^{c} is used to indicate antimatter. For instance, d^c is the charge conjugate field obtained from the Dirac field d. Charge conjugation passes from $\bigwedge^{p,q}$ to $\bigwedge^{3-p,2-q}$ and therefore reverses the electric charge, the hypercharge, and the helicity:

$$d^c_L := (d^c)_L = (d_R)^c, \qquad d^c_R := (d^c)_R = (d_L)^c.$$

Obviously, the operations C, P (charge conjugation and parity) are well defined on ψ, though they need not be symmetries. The introduction of fields together with their charge conjugates in one multiplet is welcome, because it eliminates $\bar\psi$ as an independent variable in the Lagrangian. Contrary to a wide-spread assumption, the fields for the antiquarks, in the present scheme, are assigned the defining representation **3** of $SU(3)$, while the fields for the quarks are assigned the complex conjugate representation $\bar{\mathbf{3}}$. Interchanging the role of the two fundamental representations, however, has no physical implication.

We emphasize once more that there is a natural place for the right-handed neutrino field, ν_{eR}, as well as for and its charge conjugate field, ν^c_{eL}. Both will be needed if the neutrino acquires a nonzero mass. The $SU(5)$ gauge model of Georgi and Glashow, however, discards this possibility leaving the representations $\bigwedge^{0,0}$ and $\bigwedge^{3,2}$ (trivial representations of $SU(3) \times SU(2)$) unoccupied. It seems that nature provides several generations of fundamental fermions. We offer no explanation for this fact, but mention that each generation has to be introduced by a separate master field.

We have presented a systematic and well-motivated analysis of some structural aspects of the Standard Model, leaving out all quantitative results: some of them have already been published [4]. Others are deferred to future investigations.

References

[1] N. Berline, E. Getzler, M. Vergne: Heat Kernels and Dirac Operators, Springer, Berlin 1996.

[2] V. Mathai, D. Quillen, Superconnections, Thom Classes, and Equivariant Differential Forms, Topology **25**, 85 (1986).

[3] D. Quillen, Superconnections and the Chern Character, Topology **24**, 89 (1985).

[4] G. Roepstorff, Superconnections and the Higgs Field, hep-th/9801040 published in J.Math.Phys. **40**, 2698 (1999).

[5] G. Roepstorff, Superconnection and Matter, hep-th/9801045.

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