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2D MODEL ON HEAT REGULATION IN HUMAN BODY WITH DERMAL TUMOR

KHALID NAZIR, MUKHTAR A. KHANDAY, BASHIR A. GANAI

Abstract. To predict the thermal fluctuations of a finite biological tissue in presence of tumor, an attempt has been made to formulate a 2D mathematical model based on the cross sectional temperature distribution in the tissues of the human limbs and variational finite element approach has been employed to establish the solution of the model. It is assumed that the dermal region of the human body is hosting tumor/cancerous cells. Thermal distribution at the tumor region with respect to different input parameters has been computed using MATLAB software. The physiological and bio-physical parameters like metabolic heat generation, blood mass flow rate and thermal conductivity are assumed to vary in the sub regions independently. The model describes the exchange of heat between the internal biological tissues and other surrounding media. Thermal fluctuations at the targeted regions were obtained with respect to various power densities of the heating sources. The results obtained may be helpful for various cases of practical interest especially in the treatment of cancerous tumors and in local hyperthermic therapies.

1. Introduction

Skin is an important element in the mechanism of temperature regulation, it acts as a barrier between internal system of the body and the surroundings by preventing excessive loss of water, organic and inorganic materials that are necessary for the maintenance of internal homoeostasis and normal functioning of the cells. In 1962, the structure and the functioning of the skin was first studied by Montagna [7]. The physiology of the skin also plays a vital role for various practical interests. Depending upon the physiological properties of the skin, epidermis and dermis are its main layers. However, below dermis subcutaneous tissues also plays an important role in thermoregulation.

Skin temperature also plays a vital role to keep the body temperature within certain stable range, even when the surrounding temperature is unstable. The hypothalamus regulates the body temperature by thermoregulatory mechanisms such as vasomotion, shivering and regulatory sweating. It receives inputs from the central and peripheral temperature receptors, situated at the core and the outer shell respectively. The control of body temperature depends upon the balance between the cooling and heating effects. The normal temperature of the body is

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conducted in blood stream to the skin, where it is released into the environment by means of conduction, convection and radiation and evaporation. Conduction takes place within the tissue governed by the Fourier law. Radiation takes place at the outer most surface governed by Stefan's law. Convection transfer by the blood circulation through the vessels is the main mode of transfer of heat from the core to the periphery and thermal conductivity solely depends upon rate of blood flow through the cardiovascular system. Since densities of blood vessels are very high, it is hard to compute the detailed temperature in even a small part of the body. Fortunately, the effects of blood vessels on body tissue temperature has been described collectivily by various researchers with some success.

In 1948, Pennes [10] devised a bio-heat equation, were he described the effect of blood perfusion and metabolic heat generation on heat transfer within the living tissue. The purpose of Pennes' study [10] was "to evaluate the applicability of heat flow theory to the forearm of the human body in basic terms of local rate of tissue heat production and volume flow of blood". In 1958, the paper by Pennes [10] was revisited by Wisler [16] because the experimental data seem to be at a variance with the theoretical results. In 1986, Knudsen and Overgaard [9] identified the new thermal model for human tissue. They have calculated temperature profiles within the human forearm which were in agreement with the results obtained by Pennes [10]. In 1987, Saxena and Bindra [11] used quadratic shape functions in variational finite element method to study heat distribution in cutaneous and subcutaneous tissues (SST). Recently Khanday et al. [6] has thoroughly discussed and established few models on bio-heat equation through eigenvalue expansion approach. They have discussed the effect of atmospheric temperature on heat regulation at the peripheral regions of the human tissue.

Hyperthermia is the procedure of excessive tissue temperature of the body organ or the whole body. The effectiveness of hyperthermia depends upon the value of the evaluated temperature and exposure time. In order to determine accurately the temperature field over the entire region, many numerical and experimental methods have been developed in order to solve the bio-heat equation. Gupta et al. [2] obtained approximate analytic solution of Pennes' bio-heat equation in thermal therapy. Ng et al. [8] did the parametric analysis of thermal profiles within heated human skin using boundary element method. Kengne et al. [3] gave analytical and numerical solution of temperature distribution with oscillatory surface and spatial heating. In the last few decades attempts were made further by Saxena and Pardasani [12],[13] to study the effect of dermal tumors on temperature distribution in skin and subcutaneous tissue with variable blood flow. They have discussed the radial heat flow in skin and underlying tissue layers of spherical regions of human or animal body. The model was further extended and studied by Khanday et al. [4]. They established a mathematical model for the treatment of cancerous tumors based on local hyperthermia with external heat source. Agrawal et al. [1] also worked on the thermal disturbances in dermal regions of human limbs involving metastasis of tumors.

The proposed work in this paper has many applications in various heat-transfer related problems. The behavior of tumor tissue temperature with respect to ambient conditions and in relation with external heat source has been studied. It is assumed that dermal regions host tumor cells in the human body. Variational finite element method has been employed for the numerical solution of Pennes'

bio-heat equation with appropriate boundary conditions to understand the thermal processes in the tumor tissues.

2. Mathematical formulation

The bio-heat equation developed by Pennes [10] is one of the earliest models for energy transport in biological tissues. If $T(x, y, t)$ denotes the temperature at time t and at a position (x, y), then the heat flow in tissues for two dimensional unsteady state is given below:

$$\rho c \frac{\partial T}{\partial t} = k\Big(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\Big) + \rho_b m_b c_b (T_a - T) + S + Q_{\text{ex}}\,. \tag{2.1}$$

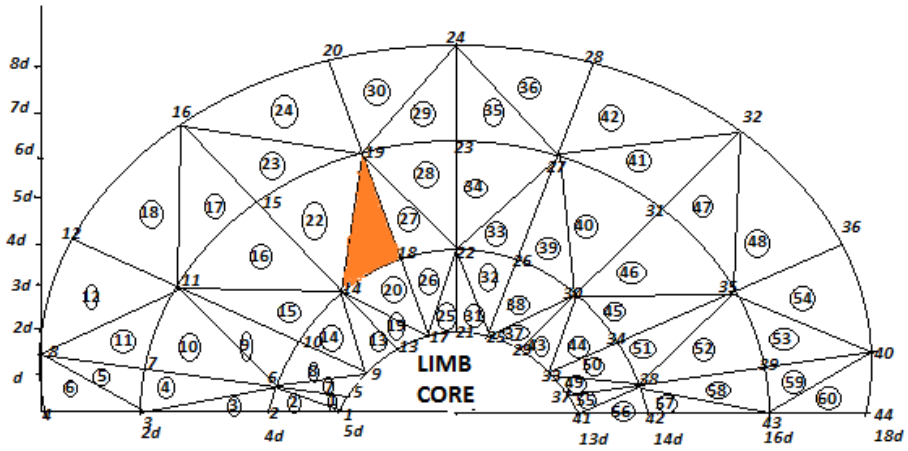

Figure 1. Triangular element arrangement for annular cross section of human limb.

The left-hand side represents the storage of heat through the tissue and the first terms on the R.H.S represents the conduction of heat through the 2D structure of tissue, the middle term represents the blood perfusion and the term S represents the heat generation due to metabolism. The last term denotes the external heat source $Q_{\text{ex}} = Q(x, y, t)$. The parameters ρ, c and k respectively denote density, specific heat and thermal conductivity of the tissue. m_b, c_b and T_a are the blood mass flow rate, specific heat of the blood and arterial blood temperature respectively. The outer surface of the tissue (skin) is exposed to the environment so there is continuous exchange of heat flux between the two media. Thus the boundary condition at the outer surface of the body is governed by Newton's cooling law given by

$$-k\frac{\partial T}{\partial n} = h(T - T_A) + LE\,, \tag{2.2}$$

where h is the heat transfer coefficient, T_A is atmospheric temperature and L, E are the latent heat and evaporation rates at the skin surface respectively.

Human beings maintain relatively constant temperature at the inner core of the limb, therefore the inner boundary conditions are given as

$$T_b = 37^{\circ}C, \quad \text{at } t \geq 0, \tag{2.3}$$

$$T(x,y) = T_b, \quad 5d \le x \le 13d, \quad 0 \le y \le 2d, \tag{2.4}$$

$$\frac{\partial T}{\partial y} = 0, \quad 0 \le x \le 5d, \quad 13d \le x \le 18d, \tag{2.5}$$

where the constant d denotes the length of the domain, and it can assume any value between $(0 \le d < 1)$ depending on the sample of the limb under study.

We assume that initially the temperature distribution is relatively equal to the body core temperature, i.e.,

$$T(x,0) = T_b\,. \tag{2.6}$$

3. Solution of the model

The domain of the underlying tissue is assumed to be annular in geometry. Keeping the complex structure of the dermal regions into account, the discretization of the domain has been approximated by the assembly of triangular elements of different sizes. Different types of parameters were considered for subregions such as epidermis, dermis and hypodermis. Also due to the complex structure of the domain, the exact solution of the formulated model is impossible to obtain. Therefore, the numerical solutions of boundary value problems are more effective and reasonable to get the solution of the formulated model. The variational finite element (VFEM) is one of the relevant numerical methods for handling irregular geometrical domains.

On comparing equation (2.1) with the Euler-Lagrange differential equation, the variational integral is given below

$$\begin{aligned} I = \frac{1}{2}\iint_A \Big[k\Big\{\Big(\frac{\partial T}{\partial x}\Big)^2 + \Big(\frac{\partial T}{\partial y}\Big)^2\Big\} + \rho_b c_b\Big(T - T_b\Big)^2 + \rho c \frac{\partial}{\partial t}T^2 \\ - 2SQ_{\rm ex}T\Big]\,dx\,dy + \int_\Omega \Big[h\Big(T - T_a\Big)^2 + 2LET\Big]d\Omega\,, \end{aligned} \tag{3.1}$$

where A and Ω is the cross sectional area and the boundary of the cylindrical limb respectively. The region under study has been divided into 60 triangular elements based on the anatomy of the dermal region. The element-wise variational integrals are given by

$$\begin{aligned} I_i = \frac{1}{2}\iint_A \Big[k_i\Big\{\Big(\frac{\partial T^i}{\partial x}\Big)^2 + \Big(\frac{\partial T^i}{\partial y}\Big)^2\Big\} + \rho_b c_b\Big(T^i - T_b\Big)^2 + \rho_i c_i \frac{\partial}{\partial t}\Big(T^i\Big)^2 \\ - 2S^i Q^i_{\rm ex}T^i\Big]\,dx\,dy + \int_\Omega \Big[h\Big(T^i - T_a\Big)^2 + 2LET^i\Big]d\Omega\,, \end{aligned} \tag{3.2}$$

where $i = 1, 2, 3, \ldots, 60$.

To solve the variational integral I_i, the following shape functions were used

$$T^i(x,y) = C^i_1 + C^i_2 x + C^i_3 y \tag{3.3}$$

In the matrix form, these equations can be written as

$$T^{(i)} = P^{(u)}C^{(i)}, \tag{3.4}$$

where $P^{(u)} = (1\ x\ y)$.

These values of C^i_1, C^i_2, C^i_3 can be obtained from the system of equations

$$\begin{pmatrix} T_i \\ T_j \\ T_k \end{pmatrix} = \begin{pmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{pmatrix} \begin{pmatrix} C^i_1 \\ C^i_2 \\ C^i_3 \end{pmatrix},$$

where T_i, T_j, T_k are the nodal temperatures of the i^{th} element.

Or,

$$U = P^{(i)}C^{(i)}, \tag{3.5}$$

where

$$U = \begin{pmatrix} T_i \\ T_j \\ T_k \end{pmatrix}, \quad P^{(i)} = \begin{pmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{pmatrix}, \quad C^{(i)} = \begin{pmatrix} C_1^i \\ C_2^i \\ C_3^i \end{pmatrix}.$$

On solving equation (3.5) for C^i, we get

$$C^{(i)} = (P^{(i)})^{-1}U\,. \tag{3.6}$$

Now from equation (3.4) we have

$$T^{(i)} = P^{(u)}(P^{(i)})^{-1}U\,. \tag{3.7}$$

Using this equation to evaluate I_i over each element and later assembling these integrals to obtain $I = \sum_1^{60} I_i$. I can be optimized with respect to the nodal values T_i to get the system of algebraic equations, written in matrix form as

$$XT + Y\dot{T} = Z, \tag{3.8}$$

where X and Y are the matrices of order (44×44), Z, T and $\dot{T}$ are the vectors of order (44×1) defined as

$$X = \begin{pmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,44} \\ a_{2,1} & a_{2,2} & \dots & a_{2,44} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ a_{44,1} & a_{44,2} & \dots & a_{44,44} \end{pmatrix}, \quad Y = \begin{pmatrix} b_{1,1} & b_{1,2} & \dots & b_{1,44} \\ b_{2,1} & b_{2,2} & \dots & b_{2,44} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ b_{44,1} & b_{44,2} & \dots & b_{44,44} \end{pmatrix},$$

$T = \begin{pmatrix} T_1 & T_2 & \dots & T_{44} \end{pmatrix}'$, $\dot{T} = \begin{pmatrix} \dot{T_1} & \dot{T_2} & \dots & \dot{T_{44}} \end{pmatrix}'$, $Z = \begin{pmatrix} Q_1 & Q_2 & \dots & Q_{44} \end{pmatrix}'$, where primes denotes the transpose of the vectors and dot denotes the derivative of temperature with respect to time.

Now to solve the system of ordinary equations, the Crank Nicholson implicit scheme has been employed to obtain

$$\Big(X + \frac{\Delta t}{2}Y\Big)T^{i+1} = \Big(X - \frac{\Delta t}{2}Y\Big)T^i + \Delta t Z\,. \tag{3.9}$$

On solving this equation, we obtain the values of the nodal temperatures for the different values of the ambient conditions $t_{\text{ex}} = 30s, 40s$ as given in Table 3.

4. Numerical results

The numerical and physiological values of the parameters used in the solution of the model are given in the Tables 1 and 2.

Table 1. Variants of Heating

Variant No.	Power density$[MW/m^2]$	Heating duration$[s]$
1	7.0	30, 40
2	3.5	30, 40
3	1.0	30, 40

TABLE 2. Numerical and Physiological values of Parameters

Parameters	Value	Units
Thermal conductivity of Epidermis(k_1)	0.030	$cal/cm - min^0C$
Specific heat of Epidermis(c_1)	0.83	cal/g
Density of Epidermis(ρ_1)	1.05	g/cm^3
Thermal conductivity of Dermis(k_2)	0.060	$cal/cm - min^0C$
Specific heat of Dermis(c_2)	0.95	cal/g
Density of Dermis(ρ_2)	1.09	g/cm^3
Thermal conductivity of SST(k_3)	0.090	$cal/cm - min^0C$
Specific heat of SST(c_3)	0.95	cal/g
Density of SST(ρ_3)	1.15	g/cm^3
Heat transfer coefficient(h)	0.009	$cal/cm^2 - min^0C$
Body core temperature($T_4 = T_b$)	37	^0C
Artrial temperature(T_a)	37	^0C
Density of blood(ρ_b)	1060	kg/m^3
Specific heat of blood(c_b)	3500	$Ws/kg/m^3$
Latent heat(L)	2.42	J/g
Rate of sweat evaporation(E)	0.001	$g/cm^3/min$

TABLE 3. Nodal temperatures of the tissue at different atmospheric temperatures when exposed to external heat for $t_{\text{ex}} = 30s$.

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8
$T_A = 25°$C	37.78	35.50	29.91	24.96	37.89	34.59	28.99	24.99
$T_A = 30°$C	37.80	35.62	29.93	29.91	37.97	34.96	33.98	29.97
	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	T_{15}	T_{16}
$T_A = 25°$C	36.99	34.41	28.98	25.01	37.00	44.01	25.00	37.00
$T_A = 30°$C	37.00	35.49	34.01	29.89	37.03	44.50	29.76	37.23
	T_{17}	T_{18}	T_{19}	T_{20}	T_{21}	T_{22}	T_{23}	T_{24}
$T_A = 25°$C	43.02	41.50	24.78	36.98	36.98	35.23	29.99	24.75
$T_A = 30°$C	44.01	43.50	29.99	37.01	37.01	35.60	33.98	29.98
	T_{25}	T_{26}	T_{27}	T_{28}	T_{29}	T_{30}	T_{31}	T_{32}
$T_A = 25°$C	36.92	35.00	29.01	24.88	37.01	35.32	28.34	24.67
$T_A = 30°$C	36.98	35.72	33.76	29.99	37.03	35.14	33.23	29.98
	T_{33}	T_{34}	T_{35}	T_{36}	T_{37}	T_{38}	T_{39}	T_{40}
$T_A = 25°$C	37.03	34.45	28.54	24.93	36.68	34.76	28.93	24.95
$T_A = 30°$C	37.15	35.07	32.92	29.97	36.78	35.01	32.73	29.95
	T_{41}	T_{42}	T_{43}	T_{44}				
$T_A = 25°$C	36.95	35.35	29.76	24.98				
$T_A = 30°$C	37.00	35.01	32.27	29.98				

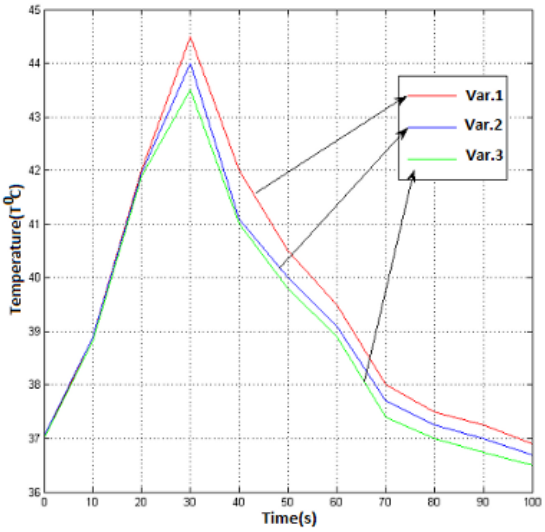

FIGURE 2. Thermal distribution of tumor tissue (21^{th} element) at $t_{\text{ex}} = 30s$.

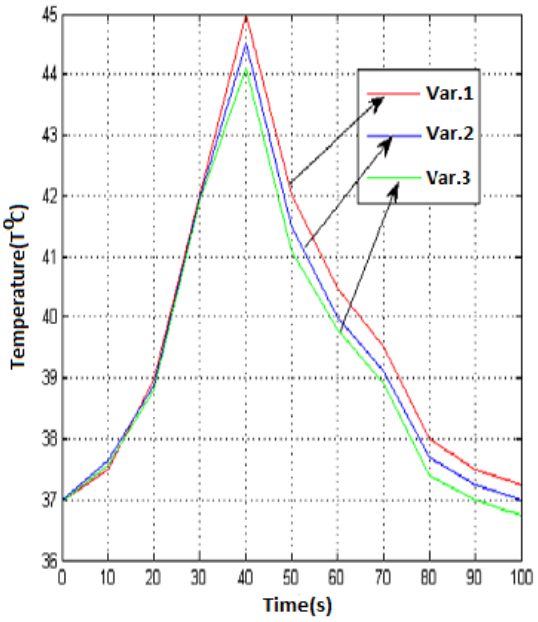

FIGURE 3. Thermal distribution of tumor tissue (21^{th} element) at $t_{\text{ex}} = 40s$.

5. Discussion and conclusion

The thermal behaviour of biological tissues hosting tumor cells at the dermal regions has been analysed using local hyperthermic therapy. Variational finite element method has been employed to solve the 2D mathematical model based on

Pennes' bio-heat equation. Laser therapy is one of the most commonly used therapy in which the local part of the underlying tissue is being heated using some spherical concave lenses and all the energy radiations can be focused on the target area of the tissue. In this connection the more effective heating source is the point heating source which is commonly written in terms of dirac delta function as

$$Q(x, y, t) = P(t)\delta(x - x_0, y - y_0)$$

where $P(t)$ is the point heating power for the treatment of local tumors, δ is the Dirac delta function and (x_0, y_0) is the position of the point heating source. This type of heating gains much more importance in the hyperthermic treatment. In this type of heating style only the selected tissue (tumor tissue) is subjected to the heating source, where as the temperature of the surrounding healthy tissue remains under the threshold temperature. In this study, we have taken different values of $P(t)$ (see Table-(1)). Also, it has been assumed that the heating region is the 21^{th} element of our discretized domain. The variation of temperature at different values of power density and for different exposing times on the tumour region is shown in Figures 2 and 3. It is clear from figures that during heating of the tissue all curves are almost similar up to the peak value, as soon the time crosses the exposing time the curves show a little difference and in case of long heating, the maximum tissue temperature also differs.

The main application of the study is that it gives some realistic values of temperature profiles as compared to the numerical results established by various researchers including Weinbaum et al. [14], Saxena and Pardasani [13]. The estimation of the temperature profiles may be helpful for various clinical purposes especially during the treatment of cancer through radiotherapy and other local hyperthermic approaches. The proposed model can contribute to medical application by understanding the macro-scale tissue response under the external heating. These results are also useful in the design and characterization of hyperthermic treatment. Moreover the model may be useful for optimization of a treatment by maximizing the therapeutic effect and minimizing side effect, proposing new treatment strategies and evaluation of their outcomes.

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Khalid Nazir
Department of Mathematics, University of Kashmir, Srinagar, Jammu and Kashmir, 190006, India
E-mail address: khldnzr99@gmail.com

Mukhtar A. Khanday (corrresponding author)
Department of Mathematics, University of Kashmir, Srinagar, Jammu and Kashmir, 190006, India
E-mail address: khanday@gmail.com

Bashir A. Ganai
CORD, University of Kashmir, Srinagar, Jammu and Kashmir, 190006, India
E-mail address: bbcganai@gmail.com