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EXISTENCE OF POSITIVE GLOBAL RADIAL SOLUTIONS TO NONLINEAR ELLIPTIC SYSTEMS

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Dedicated to the memory of John W. Nueberger

Abstract. In this article we obtain global positive and radially symmetric solutions to the system of nonlinear elliptic equations

$$\operatorname{div}\big(\phi_j(|\nabla u|)\nabla u\big) + a_j(x)\phi_j(|\nabla u|)|\nabla u| = p_j(x)f_j(u_1(x),\dots,u_k(x))\,,$$

and in particular to Laplace's equation

$$\Delta u_j(x) = p_j(x)f_j(u_1(x),\dots,u_k(x))\,,$$

where $j=1,\dots,k$, $x\in\mathbb{R}^N$, $N\geq 3$, Δ is the Laplacian operator, and ∇ is the gradient. Also we state conditions for solutions to be bounded, and to be unbounded. With an example we illustrate our results.

1. Introduction

In this article we study the existence and asymptotic behavior of positive radial solutions to the system

$$\operatorname{div}\big(\phi_j(|\nabla u|)\nabla u\big) + a_j(x)\phi_j(|\nabla u|)|\nabla u| = p_j(x)f_j(u_1(x),\dots,u_k(x)), \tag{1.1}$$

and in particular to the system

$$\Delta u_j(x) = p_j(x)f_j(u_1(x),\dots,u_k(x)) \tag{1.2}$$

for $x\in\mathbb{R}^N$, $N\geq 3$, and $j=1,\dots,k$. Here Δ is the Laplacian operator, ∇ is the gradient, a_j,p_j are radially symmetric functions, and ϕ_j is a continuously differentiable function. Let $r=|x|$ be the Euclidean norm of x in $\mathbb{R}^N$. We use the same symbol to indicate a radial function in terms of $x\in\mathbb{R}^N$ and in terms of r.

We prove that (1.1) has positive solutions which are global and radially symmetric. See Theorem 2.4 below. By doing this, we extend the existing results from 2 equations to k equations. The main difficulty is finding the proper function h that allows us bounding the iterated solutions, see inequality (2.5) and hypothesis (A5). Also we provide a small improvement in the solution estimates which allows us stating conditions in a much simpler way than in the references; see Remarks 2.3 and 2.7 below.

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The motivation for this article comes the following references: Lair [8, 7] considered the system

$$\Delta u = p_1(|x|)v^{\alpha}$$
$$\Delta v = p_2(|x|)u^{\beta}\,,$$

for $x \in \mathbb{R}^N$. Li, Zhang, and Zhang [9], and Zhang [14] studied the system

$$\Delta u = p_1(|x|)f(v)$$
$$\Delta v = p_2(|x|)g(u)\,,$$

for $x \in \mathbb{R}^N$. Covei [2] considered the system

$$\begin{gathered} \Delta u = p_1(|x|)f(u,v) \\ \Delta v = p_2(|x|)g(u)\,, \end{gathered} \tag{1.3}$$

for $x \in \mathbb{R}^N$. Othman and Chemman [10] used a fixed point approach by assuming the existence of subsolution and supersolution to study the existence of a large solution to the system $\Delta_p u = f_1(x,u,v)$, $\Delta_q v = f_2(x,u,v)$. This is done in a smooth bounded domain Ω of $\mathbb{R}^N$, $N \geq 2$, $u, v > 0$ in Ω, $u|_{\partial\Omega} = +\infty$ and $v|_{\partial\Omega} = +\infty$, where $1 < p, q < \infty$ and $\Delta_t w = \operatorname{div}(|\nabla w|^{t-2}\nabla w)$ for any $1 < t < \infty$. In an another work, Alves and de Holanda [1] studied the system $\Delta u = F_u(x,u,v)$, $\Delta v = F_v(x,u,v)$ from a variational point of view. However this approach does not apply to (1.2). Zhou [15] considered the system

$$\operatorname{div}\big(\phi_1(|\nabla u|)|\nabla u|\big) + a_1(|x|)\phi_1(|\nabla u|)|\nabla u| = p_1(|x|)f_1(u,v)$$
$$\operatorname{div}\big(\phi_2(|\nabla v|)|\nabla v|\big) + a_2(|x|)\phi_2(|\nabla v|)|\nabla v| = p_2(|x|)f_2(u,v)\,,$$

for $x \in \mathbb{R}^N$. By using a monotone iterative technique and the Arzela-Ascoli theorem, Yang et al. [13], studied the positive entire bounded radial solutions of the Schrödinger elliptic system

$$\operatorname{div}(\mathcal{G}(|\nabla y|^{p-2})\nabla y) = b_1(|x|)\psi(y) + h_1(|x|)\varphi(z), \quad x \in \mathbb{R}^n \ (n \geq 3),$$
$$\operatorname{div}(\mathcal{G}(|\nabla z|^{p-2})\nabla z) = b_2(|x|)\psi(z) + h_2(|x|)\varphi(y), \quad x \in \mathbb{R}^n,$$

where $\mathcal{G}$ is a nonlinear operator. García-Melián [4] studied the system

$$\Delta u = p_1(|x|)u^{\alpha}v^{\beta}$$
$$\Delta v = p_2(|x|)u^{\gamma}v^{\eta},$$

for $x \in \Omega \subset \mathbb{R}^N$. For more results on elliptic boundary value problems see [3, 5, 6, 9, 12] and the references therein.

2. Results

To obtain a solution to (1.1), we build a sequence of functions, and then show that the sequence converges to a solution. First using the integrating factor $r^{N-1}\mu_j(r)$, we obtain a radial version of (1.1),

$$\begin{gathered} \Big(r^{N-1}\mu_j(r)\phi_j\big(u_j'(r)\big)u_j'(r)\Big)' = r^{N-1}\mu_j(r)p_j(r)f_j(u_1(r),\dots,u_k(r)), \\ u_j(0) = \alpha_j \geq 0, \quad u_j'(0) = 0, \quad j = 1,\dots,k, \end{gathered} \tag{2.1}$$

where

$$\mu_j(r) = \exp\Big(\int_0^r a_j(s)\,ds\Big).$$

To study this problem we use the following assumptions:

(A1) $p_j : [0,\infty) \to [0,\infty)$ are continuous and radially symmetric functions. The initial values satisfy $\alpha_j \geq 0$ with $\alpha_j > 0$ for at least one index j.

(A2) $f_j : [0,\infty)^k \to [0,\infty)$ are continuous and non-decreasing with respect to each one of their arguments.

(A3) $\phi_j \in C^1([0,\infty),[0,\infty))$ for each j, and
$$\psi_j(r) := r\phi_j(r) \quad \text{satisfies} \quad \psi_j'(r) > 0 \quad \text{for } r > 0\,.$$

(A4) There exist positive constants $\tilde{B}_1, \tilde{B}_2$ and $\beta_2 \geq \beta_1 \geq 1$ such that $\tilde{B}_1 t^{\beta_1} \leq \psi(t) \leq \tilde{B}_2 t^{\beta_2}$ for $t > 0$. By a contradiction argument we can show that that there are positive constants B_1 and B_2 such that
$$B_2 y^{1/\beta_2} \leq \psi_j^{-1}(y) \leq B_1 y^{1/\beta_1} \quad \text{for } y > 0,\ j = 1,\dots,k\,.$$

(A5) There exists a continuous function $h : [0,\infty) \to [0,\infty)$ such that
$$f_j(u_1,\dots,u_k) \leq h(u_1 + \dots + u_k), \quad \text{for } 1 \leq j \leq k\,;$$
for the β_1 in (A4),
$$\int_1^\infty \frac{1}{h^{1/\beta_1}(t)}\,dt = \infty\,;$$
and for each positive t_0, there is positive constant h_0, such that $h_0 \leq h(t)$ for all $t \geq t_0$.

Examples of functions satisfying (A3) include $\phi_j(t) = 1$ which yields the Laplacian operator, and $\phi_j(t) = t^{p-2}$ which yields the p-Laplacian operator. See more examples in [15]. We do not need to define ϕ_j for negative values because we show later that $u_{j,n}' \geq 0$. Assumption (A4) was stated as $\beta_1 \leq \frac{t\psi'(t)}{\psi(t)} \leq \beta_2$ in [15]. As an example of a functions satisfying (A2) and (A5) we have $f_j(u_1,\dots,u_k) = u_1^{\delta_1}\cdots u_k^{\delta_k}$ with $\max\{\delta_i : 1 \leq i \leq k\} \leq \beta_1$, and $h(t) = \max\big\{1, \big(\sum_i^k u_i\big)^{\max\{\delta_i : 1 \leq i \leq k\}}\big\}$.

We define a sequence of functions converging to a solution of (2.1) as follows. First we integrate (2.1) to obtain
$$\begin{gathered}
u_j'(r) = \psi_j^{-1}\Big(\frac{r^{1-N}}{\mu_j(r)}\int_0^r s^{N-1}\mu_j(s)p_j(s)f_j\big(u_1(s),\dots,u_k(s)\big)\,ds\Big),\\
u_j(r) = \alpha_j + \int_0^r \psi_j^{-1}\Big(\frac{t^{1-N}}{\mu_j(t)}\int_0^t s^{N-1}\mu_j(s)p_j(s)f_j\big(u_1(s),\dots,u_k(s)\big)\,ds\Big)dt\,.
\end{gathered} \tag{2.2}$$
For $n = 0$, we define $u_{1,0} = \alpha_1$, $u_{2,0} = \alpha_2$, ..., $u_{k,0} = \alpha_k$. And for $n \geq 1$, we define
$$u_{j,n}(r) = \alpha_j + \int_0^r \psi_j^{-1}\Big(\frac{t^{1-N}}{\mu_j(t)}\int_0^t s^{N-1}\mu_j(s)p_j(s)f_j\big(u_{1,n-1}(s),\dots,u_{k,n-1}(s)\big)\,ds\Big)dt\,. \tag{2.3}$$
We use a simultaneous iteration. To compute $u_{j,n}$ we use $u_{j,n-1}$ for $j = 1\dots,k$ (all $u_{j,n-1}$ are replaced by $u_{j,n}$ at the same time). However it is possible to use successive iterations: the $u_{j,n}$ are used as they become available. Compute $u_{j,n}$ in ascending order of j, and for computing $u_{j+1,n}$ use $u_{j,n}$, instead of $u_{j,n-1}$. This technique called the Gauss-Seidel method when doing numerical approximations.

Lemma 2.1. *Under assumptions* (A1)–(A3)*, the sequence of functions* $\{u_{j,n}\}$ *is non-decreasing with respect to* n*, and each function is non-decreasing.*

Proof. To show that the sequence is non-decreasing we use induction on n. Since f_j, p_j, μ_j, and ψ_j are non-negative, we have

$$u_{j,0}(r) \le \alpha_j + \int_0^r \psi_j^{-1}\Big(\frac{t^{1-N}}{\mu_j(t)}\int_0^t s^{N-1}\mu_j(s)p_j(s)f_j(\alpha_1,\dots,\alpha_k)\,ds\Big)\,dt = u_{j,1}(r)$$

for all $r \ge 0$, which is the base step for induction. Now we assume that $u_{j,n-1}(s) \le u_{j,n}(s)$ for $s \ge 0$. As f_j is non-decreasing in each argument, ψ_j is increasing, and $\mu_j > 0$, we have

$$\begin{aligned} u_{j,n}(r) &= \alpha_j + \int_0^r \psi_j^{-1}\Big(\frac{t^{1-N}}{\mu_j(t)}\int_0^t s^{N-1}\mu_j(s)p_j(s)f_j\big(u_{1,n-1}(s),\dots,u_{k,n-1}(s)\big)\,ds\Big)dt \\ &\le \alpha_j + \int_0^r \psi_j^{-1}\Big(\frac{t^{1-N}}{\mu_j(t)}\int_0^t s^{N-1}\mu_j(s)p_j(s)f_j\big(u_{1,n}(s),\dots,u_{k,n}(s)\big)\,ds\Big)dt\,, \end{aligned}$$

for all $r \ge 0$, which completes the induction step.

To show that these functions are non-decreasing, we use

$$u'_{j,n}(r) = \psi_j^{-1}\Big(\frac{r^{1-N}}{\mu_j(r)}\int_0^r s^{N-1}\mu_j(s)p_j(s)f_j\big(u_{1,n}(s),\dots,u_{k,n}(s)\big)\,ds\Big) \ge 0\,,$$

which indicates that $u_{j,n}(r)$ is non-decreasing with respect to r. □

Lemma 2.2. *Under assumptions* (A1)–(A5)*, the sequence* $\{u_{j,n}(r)\}$ *is uniformly bounded on each interval* $0 \le r \le r_1$*.*

Proof. Using (2.2), (A4), and that $h(\sum\alpha_i) \ge f_j(\alpha_1,\dots,\alpha_k)$, we have

$$\begin{aligned} u'_j(r) &\le \psi_j^{-1}\Big(h\big(u_{1,n}(r)+\dots+u_{k,n}(r)\big)\frac{r^{1-N}}{\mu_j(r)}\int_0^r s^{N-1}\mu_j(s)p_j(s)\,ds\Big) \\ &\le B_1\Big(h\big(u_{1,n}(r)+\dots+u_{k,n}(r)\big)\frac{r^{1-N}}{\mu_j(r)}\int_0^r s^{N-1}\mu_j(s)p_j(s)\,ds\Big)^{1/\beta_1}. \end{aligned} \tag{2.4}$$

Dividing by B_1h^{1/β_1}, and summing over j, we have

$$\begin{aligned} &\frac{1}{B_1h^{1/\beta_1}\big(u_{1,n}(r)+\dots+u_{k,n}(r)\big)}\sum_{j=1}^k u'_{j,n}(r) \\ &\le \sum_{j=1}^k\Big(\frac{r^{1-N}}{\mu_j(r)}\int_0^r s^{N-1}\mu_j(s)p_j(s)\,ds\Big)^{1/\beta_1}. \end{aligned} \tag{2.5}$$

Integrating from $r=0$ to $r=r_1$ yields

$$\int_{\sum\alpha_i}^{\sum u_{i,n}(r_1)}\frac{1}{B_1h^{1/\beta_1}(t)}\,dt \le \sum_{j=1}^k\int_0^r\Big(\frac{t^{1-N}}{\mu_j(t)}\int_0^t s^{N-1}\mu_j(s)p_j(s)\,ds\Big)^{1/\beta_1}dt\,. \tag{2.6}$$

Note that the right-hand side is independent of n, and finite as long as $r_1 < \infty$. Recall that by (A5), $h(\sum\alpha_i)$ is bounded below by a positive constant because $\alpha_i > 0$ at least one index i. Then we can find a constant $M(r_1)$ such that the upper limit of integration satisfies

$$\sum_{j=1}^k u_{j,n}(r_1) \le M(r_1), \quad \forall n \ge 1\,. \tag{2.7}$$

Since $0 \le \alpha_j = u_{j,0}(r) \le u_{j,n}(r) \le u_{j,n}(r_1)$ for all $r \in [0, r_1]$, the sequence $\{u_{j,n}(\cdot)\}$ is uniformly bounded for $1 \le j \le k$ and $1 \le n$ on $[0, r_1]$. □

Remark 2.3. In [2] and [11], the quantity $u_{i,n}(s)$ in the integrand in (2.2) is substituted using the inequality

$$\begin{aligned} u_{i,n}(r) &\le \alpha_i + \int_0^r s^{1-N} \int_0^t s^{N-1} p_i(s) f_i\big(u_{1,n}(s), \dots, u_{k,n}(s)\big)\, ds\, dt \\ &\le M_i f_i(u_{1,n}(r), \dots, u_{k,n}(r)) \Big(1 + \int_0^r s^{1-N} \int_0^t s^{N-1} p_i(s)\Big), \end{aligned}$$

where $M_i = \max\{1, \alpha_i / f_i(\alpha_1, \dots, \alpha_k)\}$. This inequality comes from Lemma 2.1 with $\psi_j(r) = r$, and $\mu_j = 1$. This substitution does not improve estimate (2.2); on the contrary it makes the estimate less accurate. By avoiding this substitution, our estimates yield a small improvement and make our estimates simpler than theirs. Also we use the upper bound h in (A5), while [2] and [11] used the estimate

$$f_j\big(u_{1,n}, \dots, u_{k,n}\big) \le f_j\big(u_{1,n} + \dots + u_{k,n},\ \dots,\ u_{1,n} + \dots + u_{k,n}\big),$$

which can be larger than the $h(u_{1,n} + \dots + u_{k,n})$ used in (A5). Therefore, our estimates provide another small improvement.

Theorem 2.4. *Under assumptions* (A1)–(A5) *there is a positive solution to* (2.1) *for all* $r \ge 0$*, and hence a global radially symmetric solution to* (1.1).

Proof. First we show that $\{u_{j,n}\}$ is equi-continuous, by finding a uniform bound for $\{u'_{j,n}(\cdot)\}$ on an interval $[0, r_1]$. From the continuity of f_j, p_j, and ψ_j, we have that the bound for $\{u_{j,n}\}$ in Lemma 2.2 provides a bound for the right-hand side of (2.4). Therefore, $\{u'_{j,n}(\cdot)\}$ is uniformly bounded on $[0, r_1]$. By (1.1) and the continuity of h, from (2.5), there exists a constant $\tilde{M}(r_1)$ such that $0 \le u'_{j,n}(r) \le \tilde{M}(r_1)$ for all r in $[0, r_1]$; i.e., $u'_{j,n}$ is uniformly bounded on $[0, r_1]$. For $x, y \in [0, r_1]$, by the mean value theorem,

$$|u_{j,n}(y) - u_{j,n}(x)| \le \tilde{M}(r_1)|y - x| \quad \text{for } 1 \le j \le k,\ 1 \le n\,.$$

Given $\epsilon > 0$, we select $\delta \le \epsilon / \tilde{M}(r_1)$. If $|x - y| < \delta$, then $|u_{j,n}(y) - u_{j,n}(x)| < \epsilon$ which shows the equi-continuity of $u_{j,n}$ for $1 \le j \le k$ and $1 \le n$.

Then by the Arzela-Ascoli theorem, there exists a subsequence of $\{u_{j,n}\}$ that converges uniformly to a function u_j. Since $\{u_{j,n}\}$ is non-decreasing in n, the whole sequence converges uniformly to u_j. Therefore,

$$u_j(r) = \alpha_j + \int_0^r \psi_j^{-1}\Big(\frac{t^{1-N}}{\mu_j(t)} \int_0^t s^{N-1} \mu_j(s) p_j(s) f_j\big(u_1(s), \dots, u_k(s)\big)\, ds\Big) dt\,.$$

which provides a solution to (2.2) on $[0, r_1]$. Noting that r_1 can be arbitrarily large, we complete the proof. □

Regarding the asymptotic behavior of solutions we have the following result.

Theorem 2.5. *The solution obtained in Theorem 2.4 satisfies the following: (1) if*

$$\sum_{j=1}^k \int_a^\infty \Big(\frac{t^{1-N}}{\mu_j(t)} \int_a^t s^{N-1} \mu_j(s) p_j(s)\, ds\Big)^{1/\beta_1} dt < \infty \quad \textit{for some } a > 0\,, \tag{2.8}$$

then $\lim_{r\to\infty} u_j(r) < \infty$ *for each* $j \in \{1, \dots, k\}$.

(2) If for an index j, $\int_0^\infty \Big(\frac{t^{1-N}}{\mu_j(t)}\int_0^t s^{N-1}\mu_j(s)p_j(s)\,ds\Big)^{1/\beta_1}\,dt<\infty$, and $f_j(\dots)$ is bounded, then $\lim_{r\to\infty}u_j(r)<\infty$.

(3) If for an index j,
$$\int_a^\infty \Big(\frac{t^{1-N}}{\mu_j(t)}\int_a^t s^{N-1}\mu_j(s)p_j(s)\,ds\Big)^{1/\beta_2}\,dt=\infty\,, \tag{2.9}$$
then $\lim_{t\to\infty}u_j(t)=\infty$.

Proof. (1) Under assumption (2.8), the right-hand side of (2.6) remains bounded when $r\to\infty$, so the bound in (2.7) can be made independent of n and r_1. That is there exists a constant M such that
$$\sum_{j=1}^k u_j(r)\le M,\quad \forall r\ge 0\,.$$
Using that $u_j(\cdot)$ is non-decreasing, we have the result in part (1).

(2) From the assumptions, we define two bounds: $f_j(\dots)\le D_j$ and
$$\int_0^r \Big(\frac{t^{1-N}}{\mu_j(t)}\int_0^t s^{N-1}\mu_j(s)p_j(s)\,ds\Big)^{1/\beta_1}\,dr\le \tilde D_j,\ r\ge 0\,.$$
Then from (2.2), (A2), and (A4), we have
$$u_j(r)\le \alpha_j+B_1\big(D_j\big)^{1/\beta_1}\tilde D_j\quad\text{for } r\ge 0\,.$$
Then (2) follows from u_j begin non-decreasing.

(3) From (2.2), (A2), and (A4), we have
$$u_j(r)\ge \alpha_j+B_2\big(f_j(\alpha_1,\dots,\alpha_k)\big)^{1/\beta_2}\int_0^r\Big(\frac{t^{1-N}}{\mu_j(t)}\int_0^t s^{N-1}\mu_j(s)p_j(s)\,ds\Big)^{1/\beta_2}dt\,.$$
Because at least one α_i is positive, $f_j(\alpha_1,\dots,\alpha_k)>0$. Then the right-hand side increases to infinity as $r\to\infty$, and conclusion (3) follows. □

Remark 2.6. The results from Theorems 2.4 and 2.5 apply to (1.2), by setting $\phi_j(r)=1$, $a_j=0$, and $B_1=B_2=\beta_1=\beta_2=1$.

Remark 2.7. Conditions $\bar P(\infty)<\infty$ and $\bar Q(\infty)<\infty$ on [2, page 88] are equivalent to (2.8) with $j=1,2$, $\psi(r)=r$, $\mu_j=1$ and $\beta_1=1$. Also condition [11, ineq. (22)] is equivalent to (2.8). However our assumption (2.8) is much easier to verify than theirs.

Example 2.8. Consider the system
$$\begin{gathered}\Big(r^{N-1}r^{0.1}\big(u_1'(r)\big)^3\Big)'=r^{N-1}r^{0.1}\frac{1}{1+r^{2.1}}u_1(r)u_2(r),\quad r\ge 0,\\ \Big(r^{N-1}r^{0.2}\big(u_2'(r)\big)^3\Big)'=r^{N-1}r^{0.2}\frac{1}{1+r^{2.2}}\big(u_1(r)u_2(r)\big)^{1/2},\quad r\ge 0,\\ u_1(0)=1,\quad u_2(0)=0,\quad u_1'(0)=u_2'(0)=0\,.\end{gathered}\tag{2.10}$$
First we check that the assumptions in Theorem 2.5 are satisfied. Certainly functions $p_1(r)=\frac{1}{1+r^{2.1}}$ and $p_2(r)=\frac{1}{1+r^{2.2}}$ satisfy (A1). Also $f_1(u_1,u_2)=u_1u_2$ and $f_1(u_1,u_2)=(u_1u_2)^{1/2}$ satisfy (A2).

To check (A3), we use $\phi_1(r)=\phi_2(r)=r^2$ so $\psi_1(r)=\psi_2(r)=r^3$, and $\psi_1'(r)=3r^2>0$ for $r>0$.

To check (A4), we have $\psi_1^{-1}(y) = y^{1/3}$, thus $y^{1/3} \le \psi_1^{-1}(y) \le y^{1/3}$ and $B_1 = B_2 = 1$, $\beta_1 = \beta_2 = 3$.

Now we check (A5). Let us set $h(t) = \max\{1, t^2\}$. Then

$$f_1(u_1, u_2) = u_1 u_2 \le (u_1 + u_2)^2 \le \max\{1, (u_1 + u_2)^2\} = h(u_1 + u_2)$$

and

$$\begin{aligned} f_2(u_1, u_2) = (u_1 u_2)^{1/2} &\le \big((u_1 + u_2)^2\big)^{1/2} = u_1 + u_2 \\ &\le \max\{1, (u_1 + u_2)^2\} = h(u_1 + u_2)\,. \end{aligned}$$

Moreover,

$$\int_1^\infty \frac{1}{h^{1/\beta_1}(t)}\, dt = \int_1^\infty \frac{1}{t^{2/3}}\, dt = \infty$$

and (A5) is satisfied.

Now we check (2.8). For $j = 1$, the inner integral is

$$\int_a^r s^{N-1} s^{0.1} \frac{1}{1 + s^{2.1}}\, ds \le \int_0^r s^{N-1} s^{0.1} \frac{1}{s^{2.1}}\, ds = \frac{1}{N-2} r^{N-2}\,.$$

While the outer integral in (2.8) is

$$\int_a^\infty r^{1-N} \frac{1}{r^{0.1}} \frac{1}{N-2} r^{N-2}\, dr = \frac{1}{N-2} \int_a^\infty \frac{1}{r^{1.1}}\, dr < \infty.$$

For $j = 2$, the inner integral in (2.8) is

$$\int_a^r s^{N-1} s^{0.2} \frac{1}{1 + s^{2.2}}\, ds \le \int_0^r s^{N-1} s^{0.2} \frac{1}{s^{2.2}}\, ds = \frac{1}{N-2} r^{N-2}\,.$$

While the outer integral in (2.8) is

$$\int_a^\infty r^{1-N} \frac{1}{r^{0.2}} \frac{1}{N-2} r^{N-2}\, dr = \frac{1}{N-2} \int_a^\infty \frac{1}{r^{1.2}}\, dr < \infty.$$

So (2.8) is satisfied. Therefore, by Theorems 2.4 and 2.5, there is a global solution for which both u_1 and u_2 remain bounded as $r \to \infty$.

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