Electron. Commun. Probab. **18** (2013), no. 96, 1–7.
DOI: 10.1214/ECP.v18-2851
ISSN: 1083-589X

ELECTRONIC COMMUNICATIONS in PROBABILITY

Poisson point processes: large deviation inequalities for the convex distance

Matthias Reitzner*

Abstract

An analogue of Talagrand's convex distance for binomial and Poisson point processes is defined. A corresponding large deviation inequality is proved.

Keywords: Keywords: Poisson point process; convex distance; large deviation inequality..
AMS MSC 2010: 60F10; 60E15; 60G55..
Submitted to ECP on June 4, 2013, final version accepted on December 24, 2013.
Supersedes arXiv:1306.0693.

1 Introduction and statement of results

Concentration and large deviations have been active topics for many years. Apart from theoretical interest much additional interest in these questions comes from applications in combinatorial optimization, stochastic geometry, and many others. For these problems a deviation inequality due to Talagrand [10] turned out to be extremely useful. It combines the notion of *convex distance* with an elegant proof of a corresponding dimension free deviation inequality.

Let $(E, \mathcal{B}(E), \mathbb{P})$ be a probability space. Choose n points $x_1, \dots, x_n \in E$, $x = (x_1, \dots, x_n) \in E^n$, and assume that $A \subset E^n$ is measurable. Talagrand defines his convex distance by

$$d_T(x, A) = \sup_{\|\alpha\|_2=1} \inf_{y\in A} \sum_{1\le i\le n} \alpha_i \mathbb{1}(x_i \neq y_i) \tag{1.1}$$

where $\alpha = (\alpha_1, \dots, \alpha_n)$ is a vector in S^{n-1}. For $A \subset E^n$ denote the s-blowup of A with respect to the convex distance by $A_s := \{x : d_T(x, A) \le s\}$. Talagrand proves that for all $n \in \mathbb{N}$

$$\mathbb{P}^{\otimes n}(X \in A)\mathbb{P}^{\otimes n}(X \notin A_s) \le e^{-\frac{s^2}{4}} \tag{1.2}$$

where $X = (X_1, \dots, X_n)$ is a random vector with iid random variables $X_1, \dots, X_n$.

To extend this to point processes denote by $\bar{N}(E)$ the set of all finite counting measures $\xi = \sum_1^k \delta_{x_i}$, $x_i \in E$, $k \in \mathbb{N}_0$, or equivalently finite point sets $\{x_1, x_2, \dots, x_k\}$ eventually with multiplicity.

For a function $\alpha : E \to \mathbb{R}$ we denote by $\|\alpha\|_{2,\xi}$ the 2-norm of α with respect to the measure ξ. For two counting measures ξ and ν the (set-)difference $\xi\backslash\nu$ is defined by

$$\xi\backslash\nu = \sum_{x:\,\xi(x)>0} (\xi(x) - \nu(x))_+ \, \delta_x. \tag{1.3}$$

*Universität Osnabrück, Germany. E-mail: matthias.reitzner@uni-osnabrueck.de

As will be shown in Section 4, the natural extension of d_T to counting measures $\eta \in \bar{N}(E)$ with $\eta(E) < \infty$ acts on $\bar{N}(E)$ and is defined by

$$d_T^{\pi}(\eta, A) = \sup_{\|\alpha\|_{2,\eta}^2 \le 1} \inf_{\nu \in A} \int \alpha \, d(\eta \backslash \nu) \tag{1.4}$$

for $A \subset \bar{N}(E)$. Here the supremum is taken over all nonnegative functions $\alpha : E \to \mathbb{R}$.

The main result of this paper is an extension of Talagrand's isoperimetric inequality to Poisson point processes on lcscH (locally compact second countable Hausdorff) spaces. If η is a Poisson point process then the random variable $\eta(A)$ is Poisson distributed for each set $A \subset E$ and the expectation $\mathbb{E}\eta(A)$ is the intensity measure of the point process. For $A \subset \bar{N}(E)$ we denote by $A_s^{\pi} := \{x : d_T^{\pi}(x, A) \le s\}$ the s-blowup of A with respect to the convex distance d_T^{π}.

Theorem 1.1. *Let E be a lcscH space and let η be a Poisson point process on E with finite intensity measure $\mathbb{E}\eta(E) < \infty$. Then for any measurable subset $A \subset \bar{N}(E)$ we have*

$$\mathbb{P}(\eta \in A)\mathbb{P}\left(\eta \notin A_s^{\pi}\right) \le e^{-\frac{s^2}{4}}.$$

It is the aim of this paper to stimulate further investigations on this topic. Of high interest would be an extension of this theorem to the case of point processes of possible infinite intensity measure. On the way to such a result one has to extend the notion of convex distance to locally finite counting measures with $\xi(E) = \infty$. It is unclear whether (1.4) is the correct way to define convex distance in general, see the short discussion in Section 4.

Our method of proof consists of an extension of Talagrand's large deviation inequality first to binomial processes and then to Poisson point processes. It would also be of interest to give a proof of our theorem using only methods from the theory of point processes. We have not been able to find such a direct proof for Theorem 1.1.

In general it seems that there are only few concentration inequalities for Poisson processes and Poisson measures. Apart from the pioneering work of Bobkov and Ledoux [2] concerning concentration for Poisson measures, we mention the concentration inequalities by Ané and Ledoux [1], Wu [11], Breton, Houdre and Privault [4] and the recent contribution by Eichelsbacher, Raic and Schreiber [5]. Concentration inequalities play an important role in application, for an example considering intensity estimation see Reynaud-Bouret [9]. As a general reference on recent developments we mention the book by Boucheron, Lugosi, and Massart [3].

Our investigations are motivated by a problem in stochastic geometry. In [8] Theorem 1.1 is used to prove a large deviation inequality for the length of the Gilbert graph.

2 Binomial point processes

Assume that μ is a probability measure on $(E, \mathcal{B}(E))$. The sets $\hat{A}, B$ considered in the following are measurable.

We call a point process ξ_n a *binomial* point process on E of intensity $t\mu$ with parameter n, $0 \le t \le 1$, $n \in \mathbb{N}_0$, if for any $B \subset E$ we have

$$\mathbb{P}(\xi_n(B) = k) = \binom{n}{k} (t\mu(B))^k (1 - t\mu(B))^{n-k}. \tag{2.1}$$

To link n iid points in E to a binomial point process ξ_n we consider the following natural construction. We choose n independent points in E according to the underlying probability measure μ and for each point we decide independently with probability t if it occurs in the process or not. To make this precise we add to E an artificial element $\triangle$ at infinity (containing of all points which have been deleted), define $\hat{E} = E \cup \{\triangle\}$ and extend μ to $\hat{E}$ by

$$\hat{\mu}(\hat{B}) = t\mu(\hat{B}\backslash\triangle) + (1-t)\ \delta_\triangle(\hat{B}) \ \text{ for } \hat{B} \subset \hat{E}.$$

Hence a random point $X_i \in \hat{E}$ chosen according to $\hat{\mu}$ is in E with probability t and equals $\triangle$ with probability $1-t$. Define the projection π of $x \in \hat{E}^n$ unto $\bar{N}(E)$ by 'deleting' all points $x_i = \triangle$, i.e.

$$\pi(x) = \pi((x_1,\dots,x_n)) = \sum_{i=1}^n \mathbb{1}(x_i \neq \triangle)\delta_{x_i} \in \bar{N}(E).$$

and define the point process $\xi_n \in \bar{N}(E)$ by

$$\xi_n = \pi(X) = \sum_{i=1}^n \mathbb{1}(X_i \neq \triangle)\delta_{X_i}. \tag{2.2}$$

If $B \subset E$, any set of n iid random points $X_1,\dots,X_n$ chosen according to $\hat{\mu}$ satisfies

$$\mathbb{P}(\xi_n(B) = k) = \mathbb{P}\Big(\pi(X_1,\dots,X_n)(B) = k\Big) = \binom{n}{k}(t\mu(B))^k(1-t\mu(B))^{n-k}$$

and thus by (2.1) the process ξ_n is a binomial point process.

Assume that $\hat{A} \subset \hat{E}^n$ is a symmetric set, i.e. if $y = (y_1,\dots y_n) \in \hat{A}$ then also $(y_{\sigma(1)},\dots,y_{\sigma(n)}) \in \hat{A}$ for all permutations $\sigma \in \mathcal{S}_n$. Here $\mathcal{S}_I$ is the group of permutations of a set $I \subset \mathbb{N}$, and we write $\mathcal{S}_n$ if $I = \{1,\dots,n\}$. It is immediate that a symmetric set is the preimage of a set $A \subset \bar{N}(E)$ under the projection π where $\pi(\hat{A}) = \bigcup_{y\in\hat{A}} \pi(y) \subset \bar{N}(E)$. As shown above for a random vector $X = (X_1,\dots,X_n)$ with iid coordinates we have

$$\mathbb{P}(X \in \hat{A}) = \mathbb{P}(\pi(X) \in \pi(\hat{A})) = \mathbb{P}(\xi_n \in A). \tag{2.3}$$

The essential observation is that the convex distance $d_T(x,\hat{A})$ defined in (1.1) for $x \in \hat{E}^n$ is compatible with the projection π and yields the convex distance

$$d_T^n(\xi_n, A) = \sup_{\|\alpha\|^2_{2,\xi_n}\le 1} \inf_{\nu\in A} \Big[\int \alpha d(\xi_n\backslash\nu) + \frac{(\nu(E)-\xi_n(E))_+}{(n-\xi_n(E))^{\frac12}}(1-\|\alpha\|^2_{2,\xi_n})^{\frac12}\Big] \tag{2.4}$$

on the space $\bar{N}(E)$.

Lemma 2.1. *Assume $x \in \hat{E}^n$ and that $\hat{A} \subset \hat{E}^n$ is a symmetric set. Then for $\xi_n = \pi(x)$ and $A = \pi(\hat{A})$ we have*

$$d_T(x,\hat{A}) = d_T^n(\xi_n, A).$$

Proof. Since $\hat{A}$ is a symmetric set for any function f

$$\inf_{y\in\hat{A}} f(y_1,\dots,y_n) = \inf_{y\in\hat{A},\sigma\in\mathcal{S}_n} f(y_{\sigma(1)},\dots,y_{\sigma(n)}),$$

and we can rewrite the convex distance on $\hat{E}^n$ given by (1.1) as

$$d_T(x,\hat{A}) = \sup_{\|\alpha\|_2\le 1} \inf_{y\in\hat{A}} \inf_{\sigma\in\mathcal{S}_n} \sum_{1\le i\le n} \alpha_i \mathbb{1}(x_i \neq y_{\sigma(i)}).$$

We write $\xi_n = \pi(x), \nu = \pi(y)$. It is immediate by the symmetry of $\hat{A}$ that $d_T(x, \hat{A})$ is invariant under any permutation of $x_1, \dots x_n$. Hence we assume w.l.o.g. that x_i are sorted in such a way that $x_i \neq \triangle$ for $i = 1, \dots, \xi_n(E)$ and $x_i = \triangle$ for $i \geq \xi_n(E) + 1$.

$$d_T(x, \hat{A}) = \sup_{\|\alpha\|_2=1} \inf_{y \in \hat{A}} \inf_{\sigma \in S_n} \Big[\sum_{i=1}^{\xi_n(E)} \alpha_i \mathbb{1}(x_i \neq y_{\sigma(i)}) + \sum_{i=\xi_n(E)+1}^{n} \alpha_i \mathbb{1}(y_{\sigma(i)} \neq \triangle) \Big]$$

Here the second summand is zero if $\nu(E) \leq \xi(E)$. For fixed x and y we decrease the summands if we assume that the permutation acts in such a way that the maximal number of $\triangle$'s in x and y coincide. If $\nu(E) \leq \xi_n(E)$ this means that the minimum over $\mathcal{S}_n$ is attained if $y_{\sigma(i)} = \triangle$ for all $\sigma(i) \geq \xi_n(E)$ which coincides with the fact that the second summand in this case vanishes. If $\nu(E) > \xi_n(E)$ then $y_{\sigma(i)} = \triangle$ implies $\sigma(i) \geq \xi_n(E)$. To make things more visible we take in this case the infimum over additional permutations $\tau \in \mathcal{S}_{[\xi_n(E)+1,n]}$ of the second summand.

$$\begin{aligned} d_T(x, \hat{A}) \quad = \quad & \sup_{\|\alpha\|_2=1} \inf_{y \in \hat{A}} \inf_{\sigma \in S_n} \Big[\sum_{i=1}^{\xi_n(E)} \alpha_i \mathbb{1}(x_i \neq y_{\sigma(i)}) \\ & + \inf_{\tau \in S_{[\xi_n(E)+1,n]}} \sum_{i=\xi_n(E)+1}^{n} \alpha_i \mathbb{1}(y_{\tau(\sigma(i))} \neq \triangle) \Big] \end{aligned} \tag{2.5}$$

The second summand equals the sum of the $(\nu(E) - \xi_n(E))_+$ smallest α_i's in $\{\alpha_{\xi_n(E)+1}, \dots, \alpha_n\}$. We set $\alpha_{\xi_n(E)+i} = \beta_i$ for $i = 1, \dots, n - \xi_n(E)$ and denote by $\beta_{(1)} \leq \dots \leq \beta_{(n-\xi_n(E))}$ the order statistic of the β_i. We obtain

$$d_T(\xi, A) \quad = \quad \sup_{\|\alpha\|_2^2 + \|\beta\|_2^2 = 1} \inf_{y \in \hat{A}} \inf_{\sigma \in S_n} \Big[\sum_{i=1}^{\xi_n(E)} \alpha_i \mathbb{1}(x_i \neq y_{\sigma(i)}) + \sum_{j=1}^{(\nu(E)-\xi_n(E))_+} \beta_{(j)} \Big]$$

where from now on $\|\alpha\|_2^2 = \sum_1^{\xi_n(E)} \alpha_i^2$. The β_j^2 sum up to $1 - \|\alpha\|_2^2$ so that the sum of the k-th smallest is at most $(1 - \|\alpha\|_2^2)k/(n - \xi_n(E))$. Hölder's inequality yields

$$\begin{aligned} d_T(x, \hat{A}) \quad \leq \quad & \sup_{\|\alpha\|_2^2 + \|\beta\|_2^2 = 1} \inf_{y \in \hat{A}} \inf_{\sigma \in S_n} \Big[\sum_{i=1}^{\xi_n(E)} \alpha_i \mathbb{1}(x_i \neq y_{\sigma(i)}) \\ & + \Big((\nu(E) - \xi_n(E))_+ \sum_{j=1}^{(\nu(E)-\xi_n(E))_+} \beta_{(j)}^2 \Big)^{\frac{1}{2}} \Big] \\ \leq \quad & \sup_{\|\alpha\|_2^2 \leq 1} \inf_{y \in \hat{A}} \inf_{\sigma \in S_n} \Big[\sum_{i=1}^{\xi_n(E)} \alpha_i \mathbb{1}(x_i \neq y_{\sigma(i)}) \\ & + \frac{(\nu(E) - \xi_n(E))_+}{(n - \xi_n(E))^{\frac{1}{2}}} (1 - \|\alpha\|_2^2)^{\frac{1}{2}} \Big]. \end{aligned}$$

On the other hand if we take $\beta_j^2 = (1 - \|\alpha\|_2^2)/(n - \xi_n(E))$ we have $\|\beta\|_2^2 = 1 - \|\alpha\|_2^2$ and the supremum is bounded from below by setting β_j equal to these values.

$$\begin{aligned} d_T(x, \hat{A}) \quad \geq \quad & \sup_{\|\alpha\|_2^2 \leq 1} \inf_{y \in \hat{A}} \inf_{\sigma \in S_n} \Big[\sum_{i=1}^{\xi_n(E)} \alpha_i \mathbb{1}(x_i \neq y_{\sigma(i)}) \\ & + \frac{(\nu(E) - \xi_n(E))_+}{(n - \xi_n(E))^{\frac{1}{2}}} (1 - \|\alpha\|_2^2)^{\frac{1}{2}} \Big]. \end{aligned}$$

Both bounds coincide so that d_T equals the right hand side. Define the function $\alpha : E \to \mathbb{R}$ by

$$\alpha(x) = \begin{cases} \alpha_i & \text{if } x = x_i \\ 0 & \text{otherwise} \end{cases}$$

so that $\|\alpha\|^2_{2,\xi} = \int \alpha^2 d\xi = \sum_{i=1}^{\xi_n(E)} \alpha_i^2 = \|\alpha\|_2^2$ and by the definition (1.3) of $\xi\backslash\nu$

$$\inf_{\sigma \in \mathcal{S}_{\xi_n(E)}} \sum_{i=1}^{\xi_n(E)} \alpha_i \mathbb{1}(x_i \neq y_{\sigma(i)}) = \int \alpha d(\xi\backslash\nu).$$

This proves

$$d_T(x, \hat{A}) = \sup_{\|\alpha\|^2_{2,\xi_n} \le 1} \inf_{\nu \in A} \Big[\int \alpha d(\xi\backslash\nu) + \frac{(\nu(E) - \xi_n(E))_+}{(n - \xi_n(E))^{\frac{1}{2}}} (1 - \|\alpha\|^2_{2,\xi_n})^{\frac{1}{2}} \Big].$$

□

By (2.3) we have $\mathbb{P}(X \in \hat{A}) = \mathbb{P}(\xi_n \in A)$ for any measurable symmetric subset $\hat{A}$ of E^n. Recall that $\xi_n = \pi(X)$ and $A = \pi(\hat{A})$. Lemma (2.1) shows that

$$d_T(X, \hat{A}) \ge s \ \text{ iff } \ d_T^n(\xi_n, A) \ge s,$$

so that $X \notin \hat{A}_s$ iff $\xi_n \notin A_s^n$. Here we denote by A_s^n the blowup with respect to the distance d_T^n. Again by (2.3) this yields $\mathbb{P}(X \notin \hat{A}_s) = \mathbb{P}(\xi_n \notin A_s^n)$. Combining this with Talagrand's large deviation inequality (1.2),

$$\mathbb{P}(X \in \hat{A})\mathbb{P}(X \notin \hat{A}_s) \le e^{-\frac{s^2}{4}}$$

we obtain a large deviation inequality for the binomial process.

Theorem 2.2. *Assume ξ_n is a binomial point process with parameter n on E. Then we have*

$$\mathbb{P}(\xi_n \in A)\mathbb{P}\left(\xi_n \notin A_s^n\right) \le e^{-\frac{s^2}{4}}$$

for any $A \subset \bar{N}(E)$.

3 Poisson point processes

We extend Theorem 2.2 to Poisson point processes using the usual approximation of a Poisson point process by Binomial point processes. Assume that the state space E is a lcscH space (locally compact second countable Hausdorff space) and that μ is a probability measure on $(E, \mathcal{B}(E))$. As usual, the space $\bar{N}(E)$ is endowed with the σ-field $\mathcal{N}(E)$ generated by the evaluation mappings $\eta \mapsto \eta(B)$ with $\eta \in \bar{N}(E)$ and $B \in \mathcal{B}(E)$, see [7, Chapter 12]).

Fix some $t > 0$ and recall that μ is a probability measure on E. Set $t_n = t/n$ for $n \in \mathbb{N}$, $t \ge 0$ and assume that n is sufficiently large such that $t/n \le 1$. The following Lemma is known in great generality (see Jagers [6], or Theorem 16.18 in Kallenberg [7]), we include its proof for the sake of completeness.

Lemma 3.1. *The sequence of binomial point processes ξ_n defined in (2.2) with intensity measure $t_n\mu$ and parameter n converges in distribution to a Poisson point process η with intensity measure $t\mu$ as $n \to \infty$. I.e.,*

$$\mathbb{P}(\xi_n \in A) \to \mathbb{P}(\eta \in A). \tag{3.1}$$

for any $A \in \mathcal{N}(E)$.

Proof of Lemma 3.1. Put $\xi_{ni} = \mathbb{1}(X_i \in E)\delta_{X_i}$ such that $\xi_n = \sum_{i=1}^n \xi_{ni}$. Here X_i is in E with probability $t_n = t/n$. Since ξ_{ni}, $i \in \{1, \dots n\}$ are independent for given n and $\sup_j \mathbb{E}(\min\{\xi_{nj}(B), 1\}) \leq t_n \to 0$ for all measurable $B \subset E$, the random measures ξ_{ni} form a null array. By Theorem 16.18 in Kallenberg [7] on an lcscH space E the point process $\sum \xi_{ni}$ converges to a Poisson point process η with intensity measure $t\mu$ if $\sum_i \mathbb{P}(\xi_{ni}(B) > 0) \to t\mu(B)$, and $\sum_i \mathbb{P}(\xi_{ni}(B) > 1) \to 0$. Both is immediate for all $B \in \mathcal{B}(E)$ from the definition of ξ_{ni}. □

The distance d_T^n depends on n and has to be extended from binomial to Poisson point processes as $n \to \infty$. As stated in the introduction we use as a suitable definition

$$d_T^\pi(\xi, A) = \sup_{\|\alpha\|_{2,\xi_n}^2 \leq 1} \inf_{\nu \in A} \int \alpha d(\xi \backslash \nu)$$

This is motivated by the more detailed investigations in Section 4. For $A \subset \bar{N}(E)$ let A_s^π be the blowup of A with respect to the distance d_T^π. It is immediate that

$$d_T^\pi(\xi, A) \leq d_T^n(\xi, A). \tag{3.2}$$

This implies $A_s^\pi \supset A_s^n$ and thus for a binomial point process ξ_n

$$\mathbb{P}(\xi_n \notin A_s^\pi) \leq \mathbb{P}(\xi_n \notin A_s^n). \tag{3.3}$$

The following theorem is an immediate consequence of Theorem 2.2 and formulae (3.3) and (3.1).

Theorem 3.2. *Assume η is a Poisson point process on some lcscH space E with $\mathbb{E}\eta(E) < \infty$. Then we have*

$$\mathbb{P}(\eta \in A)\mathbb{P}\left(\eta \notin A_s^\pi\right) \leq e^{-\frac{s^2}{4}}$$

for any $A \subset \bar{N}(E)$.

4 The convex distance

In this section we collect some facts about the convex distances d_T^n and d_T^π on $\bar{N}(E)$. To start with we show that d_T^n not only gives the lower bound (3.2) for the distance d_T^π defined in (1.4). We also prove that $d_T^n \to d_T^\pi$ as $n \to \infty$ which shows that there is essentially no other natural choice for d_T^π.

We start with the representation (2.4) of the convex distance for binomial point processes. Assume $\xi \in \bar{N}(E)$ satisfies $\xi(E) < \infty$. Set

$$D_\alpha = \inf_{\nu \in A} \left[\int \alpha d(\xi \backslash \nu) + \frac{(\nu(E) - \xi(E))_+}{(n - \xi(E))^{\frac{1}{2}}} (1 - \|\alpha\|_{2,\xi}^2)^{\frac{1}{2}} \right]$$

so that $d_T^n(\xi, A) = \sup\{D_\alpha : \ \|\alpha\|_{2,\xi}^2 \leq 1\}$.

Since ξ is finite, the map $\nu \to \xi \backslash \nu$ can take only finitely many 'values' $\mu_1, \dots, \mu_m \in \bar{N}(E)$. Write $A_1, \dots, A_m \subset A$ for the preimage of the measures μ_i under this map. Denote for $i = 1, \dots, m$ by ν_i one of the counting measures in A_i with minimal $\nu(E)$. Note that these minimizers are independent of α. Assume that $\nu_i(E) \leq \dots \leq \nu_m(E)$. We compute the infimum over $\nu \in A$ by taking the infimum over $\nu \in A_i$ and then the minimum over $i = 1, \dots, m$.

$$\begin{aligned} D_\alpha &= \min_{i=1,\dots,m} \left[\int \alpha d\mu_i + \frac{\inf_{\nu \in A_i} (\nu(E) - \xi(E))_+}{(n - \xi(E))^{\frac{1}{2}}} (1 - \|\alpha\|_{2,\xi}^2)^{\frac{1}{2}} \right] \\ &= \min_{i=1,\dots,m} \left[\int \alpha d\mu_i + \frac{(\nu_i(E) - \xi(E))_+}{(n - \xi(E))^{\frac{1}{2}}} (1 - \|\alpha\|_{2,\xi}^2)^{\frac{1}{2}} \right]. \end{aligned}$$

Since $\nu_i(E)$ is bounded by $\nu_m(E)$ we have

$$\min_{i=1,\dots,m}\int \alpha d\mu_i \le D_\alpha \le \min_{i=1,\dots,m}\int \alpha d\mu_i + \frac{(\nu_m(E)-\xi(E))_+}{(n-\xi(E))^{\frac{1}{2}}}$$

which shows that for $n\to\infty$ the distance d_T^n converges to

$$d_T^\pi(\xi,A) = \sup_{\|\alpha\|_{2,\xi}^2\le 1}\inf_{\nu\in A}\int \alpha d(\xi\backslash\nu).$$

Note that this is only a pseudo-distance since $d_T^\pi(\xi,A)=0$ does not imply that $\xi\in A$. For $d_T^\pi(\xi,A)=0$ it suffices that A containes some counting measure of the form $\xi+\nu$ with $\nu\in\bar{N}(E)$ because then $\xi\backslash\nu=0$.

It would be nice to have a definition of d_T^π which is a distance for counting measures and which indicates extensions to point processes with possibly unbounded $\mathbb{E}\xi(E)$. One could also use the distance d_T^π given in (1.4) as a definition but we could not relate it to the distance d_T for binomial processes. In applications it would be of high importance to have such a representation and a large deviation inequality at least for Poisson point processes on $\mathbb{R}^d$. To the best of our knowledge even recent results like the one by Wu [11] or Eichelsbacher, Raic and Schreiber [5] cannot be easily extended to our setting.

References

[1] Ané C. and Ledoux M.: On logarithmic Sobolev inequalities for continuous time random walks on graphs. *Probab. Theory Related Fields* **116**, (2000), 573–602. MR-1757600

[2] Bobkov S. G. and Ledoux M.: On modified logarithmic Sobolev inequalities for Bernoulli and Poisson measures. *J. Funct. Anal.* **156**, (1998), 347–365. MR-1636948

[3] Boucheron S. and Lugosi G. and Massart P.: Concentration Inequalities: A Nonasymptotic Theory of Inependence. *Oxford University Press*, New York, 2013.

[4] Breton J. and Houdré C. and Privault N.: Dimension free and infinite variance tail estimates on Poisson space. *Acta Appl. Math.* **95**, (2007), 151–203. MR-2317609

[5] Eichelsbacher P. and Raic M. and Schreiber T.: Moderate deviations for stabilizing functionals in geometric probablity. arXiv:1010.1665

[6] Jagers P.: On the weak convergence of superpositions of point processes. *Z. Wahrsch. Verw. Geb.* **22**, (1972), 1–7. MR-0309191

[7] Kallenberg O.: Foundations of Modern Probability, Second Edition. Probability and its Applications, *Springer*, New York, 2002. MR-1876169

[8] Reitzner M. and Schulte M. and Thäle C.: Limit theory for the Gilbert graph. arXiv:1312.4861

[9] Reynaud-Bouret P.: Adaptive estimation of the intensity of inhomogeneous Poisson processes via concentration inequalities. *Probab. Theory Related Fields* **126**, (2003), 103–153. MR-1981635

[10] Talagrand M.: Concentration of measure and isoperimetric inequalities in product spaces. *Inst. Hautes Études Sci. Publ. Math.* **81**, (1995), 73–205. MR-1361756

[11] Wu L.: A new modified logarithmic Sobolev inequality for poisson point processes and several applications. *Probab. Theory Relat. Fields* **118**, (2000), 427–438. MR-1800540

Acknowledgments. This work was partially supported by ERC Advanced Research Grant no 267165 (DISCONV). The author thanks Imre Barany and the Renyi Institute of Mathematics, Hungarian Academy of Sciences, for the kind invitation and a fruitful meeting in Budapest, where this work got started.

Electronic Journal of Probability
Electronic Communications in Probability

Advantages of publishing in EJP-ECP

- Very high standards
- Free for authors, free for readers
- Quick publication (no backlog)

Economical model of EJP-ECP

- Low cost, based on free software (OJS[1])
- Non profit, sponsored by IMS[2], BS[3], PKP[4]
- Purely electronic and secure (LOCKSS[5])

Help keep the journal free and vigorous

- Donate to the IMS open access fund[6] (click here to donate!)
- Submit your best articles to EJP-ECP
- Choose EJP-ECP over for-profit journals

[1] OJS: Open Journal Systems http://pkp.sfu.ca/ojs/
[2] IMS: Institute of Mathematical Statistics http://www.imstat.org/
[3] BS: Bernoulli Society http://www.bernoulli-society.org/
[4] PK: Public Knowledge Project http://pkp.sfu.ca/
[5] LOCKSS: Lots of Copies Keep Stuff Safe http://www.lockss.org/
[6] IMS Open Access Fund: http://www.imstat.org/publications/open.htm