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Errata to

Fluctuations of a Surface Submitted to a Random Average Process

There follow corrections to Section 3 of Ferrari and Fontes (1998). We thank Beat Niederhauser for pointing out to us a mistake in (3.14) of that publication.

The first correction concerns an assertion in the first full paragraph of page 13. The one but last sentence starting "The second moment condition..." should end "... is then seen to be recurrent *for* $d = 1$ *and* 2*, and transient for* $d \geq 3$".

The remaining corrections are for arguments in the proof of Lemma 3.2. One (in (3.11)) is just an inversion. The others originate in a misquote to Spitzer (1964). They all concentrate in the paragraphs of equations (3.11)-(3.18). The correct paragraphs should read as follows.

"Thus,

$$\frac{f(s)}{g(s)} = \left[\frac{1+(1-\gamma)s\phi_T(s)}{1+(1-\gamma')s\phi_{\tilde{T}}(s)}\right]^{-1} = \left[\frac{[1/\phi_{\tilde{T}}(s)]+(1-\gamma)s\,[\phi_T(s)/\phi_{\tilde{T}}(s)]}{[1/\phi_{\tilde{T}}(s)]+(1-\gamma')s}\right]^{-1}. \tag{3.11}$$

Now,

$$\phi_T(s) = \sum_{x\neq 0}\phi_{T_x}(s)p_x, \quad \phi_{\tilde{T}}(s) = \sum_{x\neq 0}\phi_{T'_x}(s)p'_x \stackrel{*}{=} \sum_{x\neq 0}\phi_{T_x}(s)p'_x$$

where T_x is the hitting time of the origin of the process D_n starting at x, and p_x, $x \neq 0$, is the distribution of the jump from the origin of the same process. T'_x and p'_x are the analogues of T_x and p_x for the process H_n. Also, $\phi_{T_x} = \sum_{n\geq 0}\mathbf{P}(T_x > n)s^n$, and similarly for $\phi_{T'_x}$. Since T_x and T'_x have the same distribution for all $x \neq 0$, we have that $\phi_{T_x} = \phi_{T'_x}$ for all $x \neq 0$, and the identity $*$ above is justified.

Now, we have, on the one hand, that

$$\lim_{s\to 1}\phi_{\tilde{T}}(s) = \mathbf{E}(\tilde{T}) = \infty \tag{3.12}$$

and, on the other hand, by **P32.1** on p.378 and **P32.2** on p.379 of Spitzer (1964), that

$$\lim_{s\to 1}\phi_{T_x}(s)/\phi_{\tilde{T}_0}(s)=a(x), \tag{3.13}$$

for all x, where $\tilde{T}_0=\inf\{n\geq 1: H_n=0\}$, $\phi_{\tilde{T}_0}(s)=\sum_{n\geq 0}\mathbf{P}(\tilde{T}_0>n|H_0=0)s^n$ and a is defined in Chapter VII of Spitzer (1964).

We have that a is integrable with respect to (p_x) and (p'_x) in all dimensions. For $d\geq 3$, this follows from the boundedness of a (which we let the reader check). In $d=1$ and 2, it follows from the bounds on $a(x)$, in terms of $|x|$, in **P12.3** and **P28.4** of Spitzer (1964), pp.124 and 345, respectively, and the fact that (p_x) and (p'_x) both have absolute first moments (this follows from assumption (2.1)).

To be able to apply the dominated convergence theorem to conclude that

$$\lim_{s\to 1}\frac{f(s)}{g(s)}=\frac{(1-\gamma')\sum_x a(x)p'_x}{(1-\gamma)\sum_x a(x)p_x}, \tag{3.14}$$

we need to find $b(\cdot)$ integrable with respect to (p_x) and (p'_x) such that

$$\phi_{T_x}(s)/\phi_{\tilde{T}_0}(s)\leq b(x) \tag{3.15}$$

for all $x\neq 0$ and $s<1$. For that, let $\mathcal{N}$ denote the set of nearest neighboring sites to the origin. Let

$$\wp := \min_{e\in\mathcal{N}}\gamma_H(0,e), \tag{3.16}$$

$$\tau := \max_{e\in\mathcal{N}}T_e. \tag{3.17}$$

By (2.2), $\wp>0$. Notice first that

$$\phi_{T_x}(s)/\phi_{\tilde{T}_0}(s)\leq(\phi_{T_x}(s)/\wp)\sum_{e\in\mathcal{N}}\phi_{T_e}(s). \tag{3.18}$$

"

To conclude, we remark that, since $p_x=\gamma(0,x)/(1-\gamma)$ and $p'_x=\gamma_H(0,x)/(1-\gamma')$, $x\neq 0$, the right hand side of (3.14) becomes $\sum_x\gamma_H(0,x)a(x)/\sum_x\gamma(0,x)a(x)$. By the definition of a and the Markov property of H_n, it can be shown that $\sum_x\gamma_H(0,x)a(x)=1$. Thus, finally,

$$\lim_{s\to 1}\frac{f(s)}{g(s)}=\left[\sum_x\gamma(0,x)a(x)\right]^{-1}.$$

References

- P.A. Ferrari, L.R.G. Fontes (1998), Fluctuations of a surface submitted to a random average process, *Electronic Journal of Probability* **3**, paper #6.

- F. Spitzer (1964) *Principles of Random Walk*, Academic Press.

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