

A DIAZ–METCALF TYPE INEQUALITY FOR POSITIVE LINEAR MAPS AND ITS APPLICATIONS*

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Abstract. We present a Diaz–Metcalf type operator inequality as a reverse Cauchy–Schwarz inequality and then apply it to get some operator versions of Pólya–Szegő’s, Greub–Rheinboldt’s, Kantorovich’s, Shisha–Mond’s, Schweitzer’s, Cassels’ and Klamkin–McLenaghan’s inequalities via a unified approach. We also give some operator Grüss type inequalities and an operator Ozeki–Izumino–Mori–Seo type inequality. Several applications are included as well.

Key words. Diaz–Metcalf type inequality, Reverse Cauchy–Schwarz inequality, Positive map, Ozeki–Izumino–Mori–Seo inequality, Operator inequality.

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