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POSITIVE DEFINITE SOLUTION OF THE MATRIX EQUATION
$$X = Q + A^H(I \otimes X - C)^{-\delta}A^*$$

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Abstract. We consider the nonlinear matrix equation $X = Q + A^H(I \otimes X - C)^{-\delta}A$ $(0 < \delta \leq 1)$, where Q is an $n \times n$ positive definite matrix, C is an $mn \times mn$ positive semidefinite matrix, I is the $m \times m$ identity matrix, and A is an arbitrary $mn \times n$ matrix. We prove the existence and uniqueness of the solution which is contained in some subset of the positive definite matrices under the condition that $I \otimes Q > C$. Two bounds for the solution of the equation are derived. This equation is related to an interpolation problem when $\delta = 1$. Some known results in interpolation theory are improved and extended.

Key words. Nonlinear matrix equation, Positive definite solution, Interpolation theory.

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