

## POSITIVE DEFINITE SOLUTION OF THE MATRIX EQUATION

$$X = Q + A^H(I \otimes X - C)^{-\delta} A^*$$

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**Abstract.** We consider the nonlinear matrix equation  $X = Q + A^H(I \otimes X - C)^{-\delta} A$  ( $0 < \delta \leq 1$ ), where  $Q$  is an  $n \times n$  positive definite matrix,  $C$  is an  $mn \times mn$  positive semidefinite matrix,  $I$  is the  $m \times m$  identity matrix, and  $A$  is an arbitrary  $mn \times n$  matrix. We prove the existence and uniqueness of the solution which is contained in some subset of the positive definite matrices under the condition that  $I \otimes Q > C$ . Two bounds for the solution of the equation are derived. This equation is related to an interpolation problem when  $\delta = 1$ . Some known results in interpolation theory are improved and extended.

**Key words.** Nonlinear matrix equation, Positive definite solution, Interpolation theory.

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