Electronic Journal of Linear Algebra ISSN 1081-3810
A publication of the International Linear Algebra Society
Volume 21, pp. 98-109, October 2010

ELA
http://math.technion.ac.il/iic/ela

CLASSES OF NON-HERMITIAN OPERATORS WITH REAL EIGENVALUES*

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Abstract. Classes of non-Hermitian operators that have only real eigenvalues are presented. Such operators appear in quantum mechanics and are expressed in terms of the generators of the Weyl-Heisenberg algebra. For each non-Hermitian operator A, a Hermitian involutive operator $\hat{J}$ such that A is $\hat{J}$-Hermitian, that is, $\hat{J}A = A^*\hat{J}$, is found. Moreover, we construct a positive definite Hermitian Q such that A is Q-Hermitian, allowing for the standard probabilistic interpretation of quantum mechanics. Finally, it is shown that the considered matrices are similar to Hermitian matrices.

Key words. Infinite matrices, pseudo-Hermitian matrices, creation and annihilation operators, Krein spaces.

AMS subject classifications. 47B50, 47A63.

*Received by the editors on June 10, 2009. Accepted for publication on July 31, 2010. Handling Editors: Roger A. Horn and Fuzhen Zhang.

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