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MATRIX INVERSION AND DIGRAPHS: THE ONE FACTOR CASE*

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Abstract. The novel concept of a cyclic sequence of a digraph that has precisely one factor is defined, and is used to characterize the entries of the inverse of a matrix with such a digraph. This leads to a characterization of a strongly sign-nonsingular matrix in terms of cyclic sequences. Nonsingular nearly reducible matrices are a well-known class of matrices having precisely one nonzero diagonal, and a simple expression for the entries of the inverse of such a matrix in terms of cyclic sequences is derived. A consequence is that a nonsingular nearly reducible matrix is strongly sign-nonsingular. Several conditions that are equivalent to the inverse of a nonsingular nearly reducible matrix being nearly reducible are obtained.

Key words. Inverse matrix, Digraph, Sign pattern, Strongly sign-nonsingular matrix, Nearly reducible matrix, Minimally strongly connected digraph.

AMS subject classifications. 05C50, 15A09, 05C20.

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