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STRUCTURED CONDITIONING OF MATRIX FUNCTIONS*

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Abstract. The existing theory of conditioning for matrix functions $f(X)\colon \mathbb{C}^{n\times n} \to \mathbb{C}^{n\times n}$ does not cater for structure in the matrix X. An extension of this theory is presented in which when X has structure, all perturbations of X are required to have the same structure. Two classes of structured matrices are considered, those comprising the Jordan algebra $\mathbb{J}$ and the Lie algebra $\mathbb{L}$ associated with a nondegenerate bilinear or sesquilinear form on $\mathbb{R}^n$ or $\mathbb{C}^n$. Examples of such classes are the symmetric, skew-symmetric, Hamiltonian and skew-Hamiltonian matrices. Structured condition numbers are defined for these two classes. Under certain conditions on the underlying scalar product, explicit representations are given for the structured condition numbers. Comparisons between the unstructured and structured condition numbers are then made. When the underlying scalar product is a sesquilinear form, it is shown that there is no difference between the values of the two condition numbers for (i) all functions of $X \in \mathbb{J}$, and (ii) odd and even functions of $X \in \mathbb{L}$. When the underlying scalar product is a bilinear form then equality is not guaranteed in all these cases. Where equality is not guaranteed, bounds are obtained for the ratio of the unstructured and structured condition numbers.

Key words. Matrix functions, Fréchet derivative, Condition numbers, Bilinear forms, Sesquilinear forms, Structured Matrices, Jordan algebra, Symmetric matrices, Lie algebra, Skew-symmetric matrices.

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