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HYPONORMAL MATRICES AND SEMIDEFINITE INVARIANT SUBSPACES IN INDEFINITE INNER PRODUCTS*

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Abstract. It is shown that, for any given polynomially normal matrix with respect to an indefinite inner product, a nonnegative (with respect to the indefinite inner product) invariant subspace always admits an extension to an invariant maximal nonnegative subspace. Such an extension property is known to hold true for general normal matrices if the nonnegative invariant subspace is actually neutral. An example is constructed showing that the extension property does not generally hold true for normal matrices, even when the nonnegative invariant subspace is assumed to be positive. On the other hand, it is proved that the extension property holds true for hyponormal (with respect to the indefinite inner product) matrices under certain additional hypotheses.

Key words. Indefinite inner products, Semidefinite invariant subspaces, Hyponormal matrices, Normal matrices.

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