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GRAPHS WHOSE MINIMAL RANK IS TWO*

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Abstract. Let F be a field, $G = (V, E)$ be an undirected graph on n vertices, and let $S(F, G)$ be the set of all symmetric $n \times n$ matrices whose nonzero off-diagonal entries occur in exactly the positions corresponding to the edges of G. For example, if G is a path, $S(F, G)$ consists of the symmetric irreducible tridiagonal matrices. Let $\mathrm{mr}(F, G)$ be the minimum rank over all matrices in $S(F, G)$. Then $\mathrm{mr}(F, G) = 1$ if and only if G is the union of a clique with at least 2 vertices and an independent set. If F is an infinite field such that $\operatorname{char} F \neq 2$, then $\mathrm{mr}(F, G) \leq 2$ if and only if the complement of G is the join of a clique and a graph that is the union of at most two cliques and any number of complete bipartite graphs. A similar result is obtained in the case that F is an infinite field with $\operatorname{char} F = 2$. Furthermore, in each case, such graphs are characterized as those for which 6 specific graphs do not occur as induced subgraphs. The number of forbidden subgraphs is reduced to 4 if the graph is connected. Finally, similar criteria is obtained for the minimum rank of a Hermitian matrix to be less than or equal to two. The complement is the join of a clique and a graph that is the union of any number of cliques and any number of complete bipartite graphs. The number of forbidden subgraphs is now 5, or in the connected case, 3.

Key words. Rank 2, Minimum rank, Symmetric matrix, Forbidden subgraph, Bilinear symmetric form.

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