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NODAL DOMAIN THEOREMS AND BIPARTITE SUBGRAPHS[*]

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Abstract. The Discrete Nodal Domain Theorem states that an eigenfunction of the k-th largest eigenvalue of a generalized graph Laplacian has at most k (weak) nodal domains. The number of strong nodal domains is shown not to exceed the size of a maximal induced bipartite subgraph and that this bound is sharp for generalized graph Laplacians. Similarly, the number of weak nodal domains is bounded by the size of a maximal bipartite minor.

Key words. Graph Laplacian, Nodal Domain Theorem, Eigenvectors, Bipartite Graphs.

AMS subject classifications. 05C50, 05C22, 05C83.

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