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SCHUR COMPLEMENTS OF MATRICES WITH ACYCLIC BIPARTITE GRAPHS*

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Abstract. Bipartite graphs are used to describe the generalized Schur complements of real matrices having no square submatrix with two or more nonzero diagonals. For any matrix A with this property, including any nearly reducible matrix, the sign pattern of each generalized Schur complement is shown to be determined uniquely by the sign pattern of A. Moreover, if A has a normalized LU factorization $A = LU$, then the sign pattern of A is shown to determine uniquely the sign patterns of L and U, and (with the standard LU factorization) of L^{-1} and, if A is nonsingular, of U^{-1}. However, if A is singular, then the sign pattern of the Moore-Penrose inverse $U^{\dagger}$ may not be uniquely determined by the sign pattern of A. Analogous results are shown to hold for zero patterns.

Key words. Schur complement, LU factorization, Bipartite graph, Sign pattern, Zero pattern, Nearly reducible matrix, Minimally strongly connected digraph.

AMS subject classifications. 05C50, 15A09, 15A23.

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