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SPECTRAL GRAPH THEORY AND THE INVERSE EIGENVALUE PROBLEM OF A GRAPH*

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Abstract. Spectral Graph Theory is the study of the spectra of certain matrices defined from a given graph, including the adjacency matrix, the Laplacian matrix and other related matrices. Graph spectra have been studied extensively for more than fifty years. In the last fifteen years, interest has developed in the study of generalized Laplacian matrices of a graph, that is, real symmetric matrices with negative off-diagonal entries in the positions described by the edges of the graph (and zero in every other off-diagonal position).

The set of all real symmetric matrices having nonzero off-diagonal entries exactly where the graph G has edges is denoted by $\mathcal{S}(G)$. Given a graph G, the problem of characterizing the possible spectra of B, such that $B \in \mathcal{S}(G)$, has been referred to as the *Inverse Eigenvalue Problem of a Graph*. In the last fifteen years a number of papers on this problem have appeared, primarily concerning trees.

The adjacency matrix and Laplacian matrix of G and their normalized forms are all in $\mathcal{S}(G)$. Recent work on generalized Laplacians and Colin de Verdière matrices is bringing the two areas closer together. This paper surveys results in Spectral Graph Theory and the Inverse Eigenvalue Problem of a Graph, examines the connections between these problems, and presents some new results on construction of a matrix of minimum rank for a given graph having a special form such as a 0,1-matrix or a generalized Laplacian.

Key words. Spectral Graph Theory, Minimum Rank, Generalized Laplacian, Inverse Eigenvalue Problem, Colin de Verdière matrix.

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