Electronic Journal of Linear Algebra ISSN 1081-3810
A publication of the International Linear Algebra Society
Volume 14, pp. 32-42, February 2005

ELA
www.math.technion.ac.il/iic/ela

GRAPHS WHOSE MINIMAL RANK IS TWO: THE FINITE FIELDS CASE*

WAYNE BARRETT†, HEIN VAN DER HOLST‡, AND RAPHAEL LOEWY§

Abstract. Let F be a finite field, $G = (V, E)$ be an undirected graph on n vertices, and let $S(F, G)$ be the set of all symmetric $n \times n$ matrices over F whose nonzero off-diagonal entries occur in exactly the positions corresponding to the edges of G. Let $\mathrm{mr}(F, G)$ be the minimum rank of all matrices in $S(F, G)$. If F is a finite field with p^t elements, $p \neq 2$, it is shown that $\mathrm{mr}(F, G) \leq 2$ if and only if the complement of G is the join of a complete graph with either the union of at most $(p^t+1)/2$ nonempty complete bipartite graphs or the union of at most two nonempty complete graphs and of at most $(p^t-1)/2$ nonempty complete bipartite graphs. These graphs are also characterized as those for which 9 specific graphs do not occur as induced subgraphs. If F is a finite field with 2^t elements, then $\mathrm{mr}(F, G) \leq 2$ if and only if the complement of G is the join of a complete graph with either the union of at most $2^t + 1$ nonempty complete graphs or the union of at most one nonempty complete graph and of at most 2^{t-1} nonempty complete bipartite graphs. A list of subgraphs that do not occur as induced subgraphs is provided for this case as well.

Key words. Rank 2, Minimum rank, Symmetric matrix, Forbidden subgraph, Bilinear symmetric form, Finite field.

AMS subject classifications. 05C50, 05C75, 15A03, 15A57.

*Received by the editors 22 November 2004. Accepted for publication 13 January 2005. Handling Editor: Richard A. Brualdi.

†Department of Mathematics, Brigham Young University, Provo, UT 84602, USA (wayne@math.byu.edu).

‡ Faculty of Mathematics and Computer Science, Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands (Hein.van.der.Holst@cwi.nl).

§ Department of Mathematics, Technion-Israel Institute of Technology, Haifa 32000, Israel (loewy@techunix.technion.ac.il). The research of this author was supported by the M. and M. Bank Mathematics Research Fund.